One of the important uses of algebra is the solving of equations and inequalities. In this chapter we look at techniques for solving linear and nonlinear equations and inequalities. In addition, we consider a number of applications that can be solved using these techniques. Additional techniques for solving polynomial equations will be discussed in Chapter 3.

SECTION 1-1 Linear Equations and Applications

- Equations
- Solving Linear Equations
- A Strategy for Solving Word Problems
- Number and Geometric Problems
- Rate–Time Problems
- Mixture Problems
- Some Final Observations on Linear Equations

• Equations

An **algebraic equation** is a mathematical statement that relates two algebraic expressions involving at least one variable. Some examples of equations with x as the variable are

$$3x - 2 = 7 \qquad \frac{1}{1+x} = \frac{x}{x-2}$$

$$2x^2 - 3x + 5 = 0 \qquad \sqrt{x+4} = x - 1$$

The **replacement set,** or **domain,** for a variable is defined to be the set of numbers that are permitted to replace the variable.

Assumption

On Domains of Variables

Unless stated to the contrary, we assume that the domain for a variable is the set of those real numbers for which the algebraic expressions involving the variable are real numbers.

For example, the domain for the variable x in the expression

$$2x - 4$$

is R, the set of all real numbers, since $2x - 4$ represents a real number for all replacements of x by real numbers. The domain of x in the equation

$$\frac{1}{x} = \frac{2}{x-3}$$

is the set of all real numbers except 0 and 3. These values are excluded because the left member is not defined for $x = 0$ and the right member is not defined for $x = 3$.

The left and right members represent real numbers for all other replacements of x by real numbers.

The **solution set** for an equation is defined to be the set of elements in the domain of the variable that makes the equation true. Each element of the solution set is called a **solution,** or **root,** of the equation. To **solve an equation** is to find the solution set for the equation.

An equation is called an **identity** if the equation is true for all elements from the domain of the variable. An equation is called a **conditional equation** if it is true for certain domain values and false for others. For example,

$$2x - 4 = 2(x - 2) \qquad \text{and} \qquad \frac{5}{x^2 - 3x} = \frac{5}{x(x - 3)}$$

are identities, since both equations are true for all elements from the respective domains of their variables. On the other hand, the equations

$$3x - 2 = 5 \qquad \text{and} \qquad \frac{2}{x - 1} = \frac{1}{x}$$

are conditional equations, since, for example, neither equation is true for the domain value 2.

Knowing what we mean by the solution set of an equation is one thing; finding it is another. To this end we introduce the idea of equivalent equations. Two equations are said to be **equivalent** if they both have the same solution set for a given replacement set. A basic technique for solving equations is to perform operations on equations that produce simpler equivalent equations, and to continue the process until an equation is reached whose solution is obvious.

Application of any of the properties of equality given in Theorem 1 will produce equivalent equations.

Theorem 1

Properties of Equality

For a, b, and c any real numbers:

1. If $a = b$, then $a + c = b + c$. — **Addition Property**
2. If $a = b$, then $a - c = b - c$. — **Subtraction Property**
3. If $a = b$, then $ca = cb$, $c \neq 0$. — **Multiplication Property**
4. If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$. — **Division Property**
5. If $a = b$, then either may replace the other in any statement without changing the truth or falsity of the statement. — **Substitution Property**

• Solving Linear Equations

We now turn our attention to methods of solving *first-degree,* or *linear, equations* in one variable.

DEFINITION 1

Linear Equation in One Variable

Any equation that can be written in the form

$$ax + b = 0 \qquad a \neq 0 \qquad \textbf{Standard Form}$$

where a and b are real constants and x is a variable, is called a **linear,** or **first-degree, equation** in one variable.

$5x - 1 = 2(x + 3)$ is a linear equation, since it can be written in the standard form $3x - 7 = 0$.

EXAMPLE 1 Solving a Linear Equation

Solve $5x - 9 = 3x + 7$ and check.

Solution We use the properties of equality to transform the given equation into an equivalent equation whose solution is obvious.

$5x - 9 = 3x + 7$	Original equation
$5x - 9 + 9 = 3x + 7 + 9$	Add 9 to both sides.
$5x = 3x + 16$	Combine like terms.
$5x - 3x = 3x + 16 - 3x$	Subtract $3x$ from both sides.
$2x = 16$	Combine like terms.
$\frac{2x}{2} = \frac{16}{2}$	Divide both sides by 2.
$x = 8$	Simplify.

The solution set for this last equation is obvious:

Solution set: $\{8\}$

And since the equation $x = 8$ is equivalent to all the preceding equations in our solution, $\{8\}$ is also the solution set for all these equations, including the original equation. [*Note:* If an equation has only one element in its solution set, we generally use the last equation (in this case, $x = 8$) rather than set notation to represent the solution.]

Check

$5x - 9 = 3x + 7$	Original equation
$5(8) - 9 \stackrel{?}{=} 3(8) + 7$	Substitute $x = 8$.
$40 - 9 \stackrel{?}{=} 24 + 7$	Simplify each side.
$31 \stackrel{\checkmark}{=} 31$	A true statement

Matched Problem 1 Solve and check: $7x - 10 = 4x + 5$

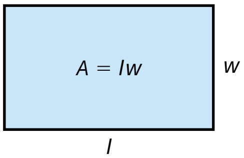


FIGURE 1 Area of a rectangle.

We frequently encounter equations involving more than one variable. For example, if l and w are the length and width of a rectangle, respectively, the area of the rectangle is given by (see Fig. 1).

$$A = lw$$

Depending on the situation, we may want to solve this equation for l or w. To solve for w, we simply consider A and l to be constants and w to be a variable. Then the equation $A = lw$ becomes a linear equation in w which can be solved easily by dividing both sides by l:

$$w = \frac{A}{l} \qquad l \neq 0$$

EXAMPLE 2 **Solving an Equation with More Than One Variable**

Solve for P in terms of the other variables: $A = P + Prt$

Solution

$$A = P + Prt \qquad \text{Think of } A, r, \text{ and } t \text{ as constants.}$$

$$A = P(1 + rt) \qquad \text{Factor to isolate } P.$$

$$\frac{A}{1 + rt} = P \qquad \text{Divide both sides by } 1 + rt.$$

$$P = \frac{A}{1 + rt} \qquad \text{Restriction: } 1 + rt \neq 0$$

Matched Problem 2 Solve for F in terms of C: $C = \frac{5}{9}(F - 32)$

• A Strategy for Solving Word Problems

A great many practical problems can be solved using algebraic techniques—so many, in fact, that there is no one method of attack that will work for all. However, we can formulate a strategy that will help you organize your approach.

Strategy for Solving Word Problems

1. Read the problem carefully—several times if necessary—that is, until you understand the problem, know what is to be found, and know what is given.

2. Let one of the unknown quantities be represented by a variable, say x, and try to represent all other unknown quantities in terms of x. This is an important step and must be done carefully.
3. If appropriate, draw figures or diagrams and label known and unknown parts.
4. Look for formulas connecting the known quantities to the unknown quantities.
5. Form an equation relating the unknown quantities to the known quantities.
6. Solve the equation and write answers to *all* questions asked in the problem.
7. Check and interpret all solutions in terms of the original problem—not just the equation found in step 5—since a mistake may have been made in setting up the equation in step 5.

The remaining examples in this section contain solutions to a variety of word problems illustrating both the process of setting up word problems and the techniques used to solve the resulting equations. It is suggested that you cover up a solution, try solving the problem yourself, and uncover just enough of a solution to get you going again in case you get stuck. After successfully completing an example, try the matched problem. After completing the section in this way, you will be ready to attempt a fairly large variety of applications.

• Number and Geometric Problems

The first examples introduce the process of setting up and solving word problems in a simple mathematical context. Following these, the examples are of a more substantive nature.

EXAMPLE 3 Setting Up and Solving a Word Problem

Find four consecutive even integers such that the sum of the first three exceeds the fourth by 8.

Solution Let x = the first even integer, then

$$x \qquad x + 2 \qquad x + 4 \qquad \text{and} \qquad x + 6$$

represent four consecutive even integers starting with the even integer x. (Remember, even integers increase by 2.) The phrase "the sum of the first three exceeds the fourth by 8" translates into an equation:

$$\begin{aligned} \text{Sum of the first three} &= \text{Fourth} + \text{Excess} \\ x + (x + 2) + (x + 4) &= (x + 6) + 8 \\ 3x + 6 &= x + 14 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

The four consecutive integers are 4, 6, 8, and 10.

Check

$$\begin{array}{r} 4 + 6 + 8 = 18 \\ -8 \\ \hline 10 \end{array} \quad \begin{array}{l} \text{Sum of first three} \\ \text{Excess} \\ \text{Fourth} \end{array}$$

Matched Problem 3 Find three consecutive odd integers such that 3 times their sum is 5 more than 8 times the middle one.

EXPLORE-DISCUSS 1

According to property 1 of Theorem 1, multiplying both sides of an equation by a nonzero number always produces an equivalent equation. By what number would you choose to multiply both sides of the following equation to eliminate all the fractions?

$$\frac{x+1}{3} - \frac{x}{4} = \frac{1}{2}$$

If you did not choose 12, the LCD of all the fractions in this equation, you could still solve the resulting equation, but with more effort. (For a discussion of LCDs and how to find them, see Section A-4.)

EXAMPLE 4 Using a Diagram in the Solution of a Word Problem

If one side of a triangle is one-third the perimeter, the second side is one-fifth the perimeter, and the third side is 7 meters, what is the perimeter of the triangle?

Solution Let p = the perimeter. Draw a triangle and label the sides, as shown in Figure 2. Then

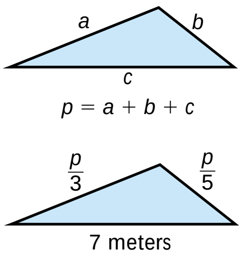


FIGURE 2

$$p = a + b + c$$

$$p = \frac{p}{3} + \frac{p}{5} + 7$$

$$15 \cdot p = 15\left(\frac{p}{3} + \frac{p}{5} + 7\right)$$

Multiply both sides by 15, the LCD. This and the next step usually can be done mentally.

$$15p = 15 \cdot \frac{p}{3} + 15 \cdot \frac{p}{5} + 15 \cdot 7$$

$$15p = 5p + 3p + 105$$

$$7p = 105$$

$$p = 15$$

The perimeter is 15 meters.

Check

$$\begin{array}{rll} \frac{p}{3} = \frac{15}{3} = & 5 & \text{Side 1} \\ \frac{p}{5} = \frac{15}{5} = & 3 & \text{Side 2} \\ & 7 & \text{Side 3} \\ \hline & 15 \text{ meters} & \text{Perimeter} \end{array}$$

Matched Problem 4 If one side of a triangle is one-fourth the perimeter, the second side is 7 centimeters, and the third side is two-fifths the perimeter, what is the perimeter?

CAUTION A very common error occurs about now—students tend to confuse *algebraic expressions* involving fractions with *algebraic equations* involving fractions.

Consider these two problems:

(A) Solve: $\frac{x}{2} + \frac{x}{3} = 10$ (B) Add: $\frac{x}{2} + \frac{x}{3} + 10$

The problems look very much alike but are actually very different. To solve the equation in (A) we multiply both sides by 6 (the LCD) to clear the fractions. This works so well for equations that students want to do the same thing for problems like (B). The only catch is that (B) is not an equation, and the multiplication property of equality does not apply. If we multiply (B) by 6, we simply obtain an expression 6 times as large as the original! Compare the following:

(A)

$$\frac{x}{2} + \frac{x}{3} = 10$$

$$\boxed{6 \cdot \frac{x}{2} + 6 \cdot \frac{x}{3} = 6 \cdot 10}$$

$$3x + 2x = 60$$

$$5x = 60$$

$$x = 12$$

(B)

$$\frac{x}{2} + \frac{x}{3} + 10$$

$$\boxed{= \frac{3 \cdot x}{3 \cdot 2} + \frac{2 \cdot x}{2 \cdot 3} + \frac{6 \cdot 10}{6 \cdot 1}}$$

$$= \frac{3x}{6} + \frac{2x}{6} + \frac{60}{6}$$

$$= \frac{5x + 60}{6}$$

• Rate–Time Problems

There are many types of quantity–rate–time problems and distance–rate–time problems. In general, if Q is the quantity of something produced (kilometers, words, parts, and so on) in T units of time (hours, years, minutes, seconds, and so on), then the formulas given in the box are relevant.

Quantity–Rate–Time Formulas

$$R = \frac{Q}{T} \qquad \text{Rate} = \frac{\text{Quantity}}{\text{Time}}$$

$$Q = RT \qquad \text{Quantity} = (\text{Rate})(\text{Time})$$

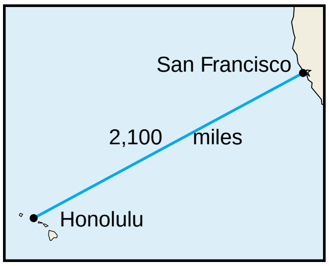
$$T = \frac{Q}{R} \qquad \text{Time} = \frac{\text{Quantity}}{\text{Rate}}$$

If Q is distance D, then

$$R = \frac{D}{T} \qquad D = RT \qquad T = \frac{D}{R}$$

[*Note:* R is an average or uniform rate.]

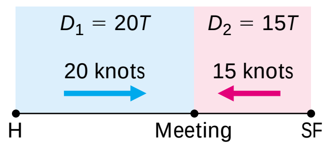
EXAMPLE 5 A Distance–Rate–Time Problem



The distance along a shipping route between San Francisco and Honolulu is 2,100 nautical miles. If one ship leaves San Francisco at the same time another leaves Honolulu, and if the former travels at 15 knots* and the latter at 20 knots, how long will it take the two ships to rendezvous? How far will they be from Honolulu and San Francisco at that time?

Solution

Let T = number of hours until both ships meet. Draw a diagram and label known and unknown parts. Both ships will have traveled the same amount of time when they meet.



$$\begin{pmatrix}\text{Distance ship 1}\\ \text{from Honolulu}\\ \text{travels to}\\ \text{meeting point}\end{pmatrix} + \begin{pmatrix}\text{Distance ship 2}\\ \text{from San Francisco}\\ \text{travels to}\\ \text{meeting point}\end{pmatrix} = \begin{pmatrix}\text{Total distance}\\ \text{from Honolulu}\\ \text{to San Francisco}\end{pmatrix}$$

$$\begin{aligned} D_1 + D_2 &= 2{,}100 \\ 20T + 15T &= 2{,}100 \\ 35T &= 2{,}100 \\ T &= 60 \end{aligned}$$

Therefore, it takes 60 hours, or 2.5 days, for the ships to meet.

*15 knots means 15 nautical miles per hour. There are 6,076.1 feet in 1 nautical mile.

$$\text{Distance from Honolulu} = 20 \cdot 60 = 1{,}200 \text{ nautical miles}$$

$$\text{Distance from San Francisco} = 15 \cdot 60 = 900 \text{ nautical miles}$$

Check $$1{,}200 + 900 = 2{,}100 \text{ nautical miles}$$

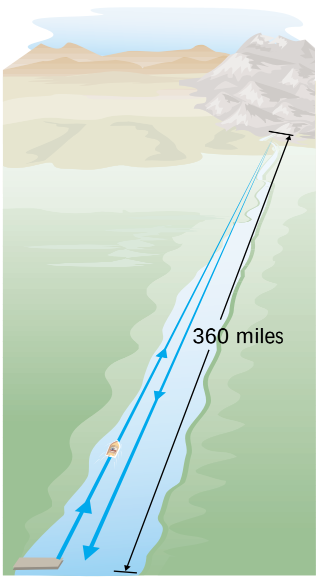
Matched Problem 5 An old piece of equipment can print, stuff, and label 38 mailing pieces per minute. A newer model can handle 82 per minute. How long will it take for both pieces of equipment to prepare a mailing of 6,000 pieces? [*Note:* The mathematical form is the same as in Example 5.]

Some equations involving variables in a denominator can be transformed into linear equations. We may proceed in essentially the same way as in the preceding example; however, we must exclude any value of the variable that will make a denominator 0. With these values excluded, we may multiply through by the LCD even though it contains a variable, and, according to Theorem 1, the new equation will be equivalent to the old.

EXAMPLE 6 A Distance–Rate–Time Problem

An excursion boat takes 1.5 times as long to go 360 miles up a river than to return. If the boat cruises at 15 miles per hour in still water, what is the rate of the current?

Solution Let



$$x = \text{Rate of current (in miles per hour)}$$

$$15 - x = \text{Rate of boat upstream}$$

$$15 + x = \text{Rate of boat downstream}$$

$$\text{Time upstream} = (1.5)(\text{Time downstream})$$

$$\frac{\text{Distance upstream}}{\text{Rate upstream}} = (1.5)\frac{\text{Distance downstream}}{\text{Rate downstream}} \qquad \text{Recall: } T = \frac{D}{R}$$

$$\frac{360}{15 - x} = (1.5)\frac{360}{15 + x} \qquad x \neq 15,\ x \neq -15$$

$$\frac{360}{15 - x} = \frac{540}{15 + x}$$

$$360(15 + x) = 540(15 - x) \qquad \text{Multiply both sides by } (15 - x)(15 + x).$$

$$5{,}400 + 360x = 8{,}100 - 540x$$

$$900x = 2{,}700$$

$$x = 3$$

Therefore, the rate of the current is 3 miles per hour. The check is left to the reader.

Matched Problem 6 A jetliner takes 1.2 times as long to fly from Paris to New York (3,600 miles) as to return. If the jet cruises at 550 miles per hour in still air, what is the average rate of the wind blowing in the direction of Paris from New York?

EXPLORE-DISCUSS 2 Consider the following solution:

$$\frac{x}{x-2} + 2 = \frac{2x-2}{x-2}$$
$$x + 2x - 4 = 2x - 2$$
$$x = 2$$

Is $x = 2$ a root of the original equation? If not, why? Discuss the importance of excluding values that make a denominator 0 when solving equations.

EXAMPLE 7 A Quantity–Rate–Time Problem

An advertising firm has an old computer that can prepare a whole mailing in 6 hours. With the help of a newer model the job is complete in 2 hours. How long would it take the newer model to do the job alone?

Solution Let x = time (in hours) for the newer model to do the whole job alone.

$$\begin{pmatrix}\text{Part of job completed}\\ \text{in a given length of time}\end{pmatrix} = (\text{Rate})(\text{Time})$$

$$\text{Rate of old model} = \frac{1}{6}\text{ job per hour}$$

$$\text{Rate of new model} = \frac{1}{x}\text{ job per hour}$$

$$\begin{pmatrix}\text{Part of job completed}\\ \text{by old model}\\ \text{in 2 hours}\end{pmatrix} + \begin{pmatrix}\text{Part of job completed}\\ \text{by new model}\\ \text{in 2 hours}\end{pmatrix} = 1 \text{ whole job}$$

$$\begin{pmatrix}\text{Rate of}\\ \text{old model}\end{pmatrix}\begin{pmatrix}\text{Time of}\\ \text{old model}\end{pmatrix} + \begin{pmatrix}\text{Rate of}\\ \text{new model}\end{pmatrix}\begin{pmatrix}\text{Time of}\\ \text{new model}\end{pmatrix} = 1 \qquad \text{Recall: } Q = RT$$

$$\frac{1}{6}(2) + \frac{1}{x}(2) = 1 \qquad x \neq 0$$

$$
\begin{aligned}
\frac{1}{3} + \frac{2}{x} &= 1 \\
x + 6 &= 3x \\
-2x &= -6 \\
x &= 3
\end{aligned}
$$

Therefore, the new computer could do the job alone in 3 hours.

Check

$$
\begin{aligned}
\text{Part of job completed by old model in 2 hours} = 2(\tfrac{1}{6}) &= \tfrac{1}{3} \\
\text{Part of job completed by new model in 2 hours} = 2(\tfrac{1}{3}) &= \underline{\tfrac{2}{3}} \\
\text{Part of job completed by both models in 2 hours} &= 1
\end{aligned}
$$

Matched Problem 7 Two pumps are used to fill a water storage tank at a resort. One pump can fill the tank by itself in 9 hours, and the other can fill it in 6 hours. How long will it take both pumps operating together to fill the tank?

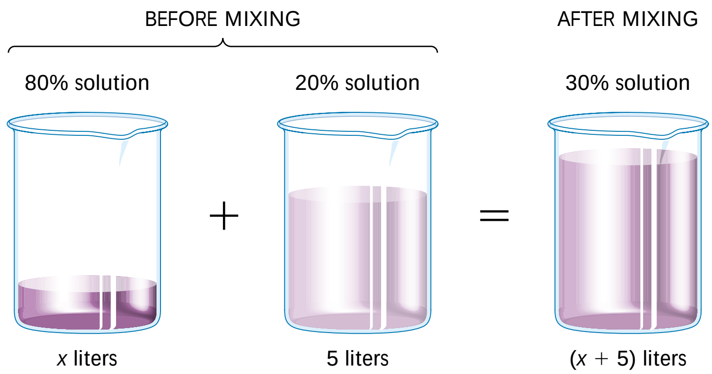
• Mixture Problems

A variety of applications can be classified as mixture problems. Even though the problems come from different areas, their mathematical treatment is essentially the same.

EXAMPLE 8 A Mixture Problem

How many liters of a mixture containing 80% alcohol should be added to 5 liters of a 20% solution to yield a 30% solution?

Solution Let x = amount of 80% solution used.



$$
\begin{pmatrix}\text{Amount of}\\ \text{alcohol in}\\ \text{first solution}\end{pmatrix} + \begin{pmatrix}\text{Amount of}\\ \text{alcohol in}\\ \text{second solution}\end{pmatrix} = \begin{pmatrix}\text{Amount of}\\ \text{alcohol in}\\ \text{mixture}\end{pmatrix}
$$

$$
0.8x + 0.2(5) = 0.3(x + 5)
$$

$$0.8x + 1 = 0.3x + 1.5$$
$$0.5x = 0.5$$
$$x = 1$$

Add 1 liter of the 80% solution.

Check

	Liters of solution	Liters of alcohol	Percent alcohol
First solution	1	$0.8(1) = 0.8$	80
Second solution	5	$0.2(5) = 1$	20
Mixture	6	1.8	$1.8/6 = 0.3$, or 30%

Matched Problem 8 A chemical storeroom has a 90% acid solution and a 40% acid solution. How many centiliters of the 90% solution should be added to 50 centiliters of the 40% solution to yield a 50% solution?

• Some Final Observations on Linear Equations

It can be shown that any equation that can be written in the form

$$ax + b = 0 \qquad a \neq 0 \tag{1}$$

with no restrictions on x, has exactly one solution, and the solution can be found as follows:

$$ax + b = 0$$
$$ax = -b \quad \text{Subtraction property of equality}$$
$$x = \frac{-b}{a} \quad \text{Division property of equality}$$

Requiring $a \neq 0$ in equation (1) is an important restriction, because without it we are able to write equations with first-degree members that have no solutions or infinitely many solutions. For example,

$$2x - 3 = 2x + 5$$

has no solution, and

$$3x - 4 = 5 + 3(x - 3)$$

has infinitely many solutions. Try to solve each equation to see what happens.

Answers to Matched Problems

1. $x = 5$ **2.** $F = \frac{9}{5}C + 32$ **3.** 3, 5, 7 **4.** 20 centimeters
5. 50 minutes **6.** 50 miles per hour **7.** 3.6 hours **8.** 12.5 centiliters

EXERCISE 1-1

A

In Problems 1–16, solve each equation.

1. $4(x+5)=6(x-2)$

2. $3(y-4)+2y=18$

3. $5+4(w-1)=2w+2(w+4)$

4. $4-3(t+2)+t=5(t-1)-7t$

5. $5-\dfrac{3a-4}{5}=\dfrac{7-2a}{2}$

6. $\dfrac{3b}{7}+\dfrac{2b-5}{2}=-4$

7. $\dfrac{x}{2}+\dfrac{2x-1}{3}=\dfrac{3x+4}{4}$

8. $\dfrac{x}{5}+\dfrac{3x-1}{2}=\dfrac{6x+5}{4}$

9. $0.1(t+0.5)+0.2t=0.3(t-0.4)$

10. $0.1(w+0.5)+0.2w=0.2(w-0.4)$

11. $0.35(s+0.34)+0.15s=0.2s-1.66$

12. $0.35(u+0.34)-0.15u=0.2u-1.66$

13. $\dfrac{2}{y}+\dfrac{5}{2}=4-\dfrac{2}{3y}$

14. $\dfrac{3+w}{6w}=\dfrac{1}{2w}+\dfrac{4}{3}$

15. $\dfrac{z}{z-1}=\dfrac{1}{z-1}+2$

16. $\dfrac{t}{t-1}=\dfrac{2}{t-1}+2$

B

In Problems 17–24, solve each equation.

17. $\dfrac{2m}{5}+\dfrac{m-4}{6}=\dfrac{4m+1}{4}-2$

18. $\dfrac{3(n-2)}{5}+\dfrac{2n+3}{6}=\dfrac{4n+1}{9}+2$

19. $1-\dfrac{x-3}{x-2}=\dfrac{2x-3}{x-2}$

20. $\dfrac{2x-3}{x+1}=2-\dfrac{3x-1}{x+1}$

21. $\dfrac{6}{y+4}+1=\dfrac{5}{2y+8}$

22. $\dfrac{4y}{y-3}+5=\dfrac{12}{y-3}$

23. $\dfrac{3a-1}{a^2+4a+4}-\dfrac{3}{a^2+2a}=\dfrac{3}{a}$

24. $\dfrac{1}{b-5}-\dfrac{10}{b^2-5b+25}=\dfrac{1}{b+5}$

In Problems 25–28, use a calculator to solve each equation to 3 significant digits.

25. $3.142x-0.4835(x-4)=6.795$

26. $0.0512x+0.125(x-2)=0.725x$

27. $\dfrac{2.32x}{x-2}-\dfrac{3.76}{x}=2.32$

28. $\dfrac{6.08}{x}+4.49=\dfrac{4.49x}{x+3}$

In Problems 29–36, solve for the indicated variable in terms of the other variables.

29. $a_n=a_1+(n-1)d$ for d (arithmetic progressions)

30. $F=\frac{9}{5}C+32$ for C (temperature scale)

31. $\dfrac{1}{f}=\dfrac{1}{d_1}+\dfrac{1}{d_2}$ for f (simple lens formula)

32. $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}$ for R_1 (electric circuit)

33. $A=2ab+2ac+2bc$ for a (surface area of a rectangular solid)

34. $A=2ab+2ac+2bc$ for c

35. $y=\dfrac{2x-3}{3x+5}$ for x

36. $x=\dfrac{3y+2}{y-3}$ for y

In Problems 37 and 38, imagine that the indicated "solutions" were given to you by a student whom you were tutoring in this class. Is the solution right or wrong? If the solution is wrong, explain what is wrong and show a correct solution.

37.
$$\frac{x}{x-3}+4=\frac{2x-3}{x-3}$$
$$x+4x-12=2x-3$$
$$x=3$$

38.
$$\frac{x^2+1}{x-1}=\frac{x^2+4x-3}{x-1}$$
$$x^2+1=x^2+4x-3$$
$$x=1$$

C

In Problems 39–41, solve for x.

39. $\dfrac{x-\dfrac{1}{x}}{1+\dfrac{1}{x}}=3$

40. $\dfrac{x-\dfrac{1}{x}}{x+1-\dfrac{2}{x}}=1$

41. $\dfrac{x+1-\dfrac{2}{x}}{1-\dfrac{1}{x}}=x+2$

42. Solve for y in terms of x: $\dfrac{y}{1-y} = \left(\dfrac{x}{1-x}\right)^3$

43. Solve for x in terms of y: $y = \dfrac{a}{1 + \dfrac{b}{x+c}}$

44. Let m and n be real numbers with m larger than n. Then there exists a positive real number p such that $m = n + p$. Find the fallacy in the following argument:

$$\begin{aligned} m &= n + p \\ (m-n)m &= (m-n)(n+p) \\ m^2 - mn &= mn + mp - n^2 - np \\ m^2 - mn - mp &= mn - n^2 - np \\ m(m-n-p) &= n(m-n-p) \\ m &= n \end{aligned}$$

APPLICATIONS

These problems are grouped according to subject area. As before, the most difficult problems are marked with two stars (★★), the moderately difficult problems are marked with one star (★), and the easier problems are not marked.

Numbers

45. Find a number such that 10 less than two-thirds the number is one-fourth the number.

46. Find a number such that 6 more than one-half the number is two-thirds the number.

47. Find four consecutive even integers so that the sum of the first three is 2 more than twice the fourth.

48. Find three consecutive even integers so that the first plus twice the second is twice the third.

Geometry

49. Find the dimensions of a rectangle with a perimeter of 54 meters, if its length is 3 meters less than twice its width.

50. A rectangle 24 meters long has the same area as a square that is 12 meters on a side. What are the dimensions of the rectangle?

51. Find the perimeter of a triangle if one side is 16 feet, another side is two-sevenths the perimeter, and the third side is one-third the perimeter.

52. Find the perimeter of a triangle if one side is 11 centimeters, another side is one-fifth the perimeter, and the third side is one-fourth the perimeter.

Business and Economics

53. The sale price on a camera after a 20% discount is \$72. What was the price before the discount?

54. A stereo store marks up each item it sells 60% above wholesale price. What is the wholesale price on a cassette player that retails at \$144?

55. One employee of a computer store is paid a base salary of \$2,150 a month plus an 8% commission on all sales over \$7,000 during the month. How much must the employee sell in 1 month to earn a total of \$3,170 for the month?

56. A second employee of the computer store in Problem 55 is paid a base salary of \$1,175 a month plus a 5% commission on all sales during the month.

(A) How much must this employee sell in 1 month to earn a total of \$3,170 for the month?

(B) Determine the sales level where both employees receive the same monthly income. If employees can select either of these payment methods, how would you advise an employee to make this selection?

Earth Science

★ **57.** In 1984, the Soviets led the world in drilling the deepest hole in the Earth's crust—more than 12 kilometers deep. They found that below 3 kilometers the temperature T increased 2.5°C for each additional 100 meters of depth.

(A) If the temperature at 3 kilometers is 30°C and x is the depth of the hole in kilometers, write an equation using x that will give the temperature T in the hole at any depth beyond 3 kilometers.

(B) What would the temperature be at 15 kilometers? (The temperature limit for their drilling equipment was about 300°C.)

(C) At what depth (in kilometers) would they reach a temperature of 280°C?

★ **58.** Because air is not as dense at high altitudes, planes require a higher ground speed to become airborne. A rule of thumb is 3% more ground speed per 1,000 feet of elevation, assuming no wind and no change in air temperature. (Compute numerical answers to 3 significant digits.)

(A) Let

V_S = Takeoff ground speed at sea level for a particular plane (in miles per hour)

A = Altitude above sea level (in thousands of feet)

V = Takeoff ground speed at altitude A for the same plane (in miles per hour)

Write a formula relating these three quantities.

(B) What takeoff ground speed would be required at Lake Tahoe airport (6,400 feet), if takeoff ground speed at San Francisco airport (sea level) is 120 miles per hour?

(C) If a landing strip at a Colorado Rockies hunting lodge (8,500 feet) requires a takeoff ground speed of 125 miles per hour, what would be the takeoff ground speed in Los Angeles (sea level)?
(D) If the takeoff ground speed at sea level is 135 miles per hour and the takeoff ground speed at a mountain resort is 155 miles per hour, what is the altitude of the mountain resort in thousands of feet?

★★ **59.** An earthquake emits a primary wave and a secondary wave. Near the surface of the Earth the primary wave travels at about 5 miles per second, and the secondary wave travels at about 3 miles per second. From the time lag between the two waves arriving at a given seismic station, it is possible to estimate the distance to the quake. Suppose a station measures a time difference of 12 seconds between the arrival of the two waves. How far is the earthquake from the station? (The *epicenter* can be located by obtaining distance bearings at three or more stations.)

★★ **60.** A ship using sound-sensing devices above and below water recorded a surface explosion 39 seconds sooner on its underwater device than on its above-water device. If sound travels in air at about 1,100 feet per second and in water at about 5,000 feet per second, how far away was the explosion?

Life Science

61. A naturalist for a fish and game department estimated the total number of trout in a certain lake using the popular capture–mark–recapture technique. She netted, marked, and released 200 trout. A week later, allowing for thorough mixing, she again netted 200 trout and found 8 marked ones among them. Assuming that the ratio of marked trout to the total number in the second sample is the same as the ratio of all marked fish in the first sample to the total trout population in the lake, estimate the total number of fish in the lake.

62. Repeat Problem 61 with a first (marked) sample of 300 and a second sample of 180 with only 6 marked trout.

Chemistry

★ **63.** How many gallons of distilled water must be mixed with 50 gallons of 30% alcohol solution to obtain a 25% solution?

★ **64.** How many gallons of hydrochloric acid must be added to 12 gallons of a 30% solution to obtain a 40% solution?

★ **65.** A chemist mixes distilled water with a 90% solution of sulfuric acid to produce a 50% solution. If 5 liters of distilled water is used, how much 50% solution is produced?

★ **66.** A fuel oil distributor has 120,000 gallons of fuel with 0.9% sulfur content, which exceeds pollution control standards of 0.8% sulfur content. How many gallons of fuel oil with a 0.3% sulfur content must be added to the 120,000 gallons to obtain fuel oil that will comply with the pollution control standards?

Rate–Time

★ **67.** An old computer can do the weekly payroll in 5 hours. A newer computer can do the same payroll in 3 hours. The old computer starts on the payroll, and after 1 hour the newer computer is brought on-line to work with the older computer until the job is finished. How long will it take both computers working together to finish the job? (Assume the computers operate independently.)

★ **68.** One pump can fill a gasoline storage tank in 8 hours. With a second pump working simultaneously, the tank can be filled in 3 hours. How long would it take the second pump to fill the tank operating alone?

★★ **69.** The cruising speed of an airplane is 150 miles per hour (relative to the ground). You wish to hire the plane for a 3-hour sightseeing trip. You instruct the pilot to fly north as far as he can and still return to the airport at the end of the allotted time.
(A) How far north should the pilot fly if the wind is blowing from the north at 30 miles per hour?
(B) How far north should the pilot fly if there is no wind?

★ **70.** Suppose you are at a river resort and rent a motor boat for 5 hours starting at 7 A.M. You are told that the boat will travel at 8 miles per hour upstream and 12 miles per hour returning. You decide that you would like to go as far up the river as you can and still be back at noon. At what time should you turn back, and how far from the resort will you be at that time?

Music

★ **71.** A major chord in music is composed of notes whose frequencies are in the ratio 4:5:6. If the first note of a chord has a frequency of 264 hertz (middle C on the piano), find the frequencies of the other two notes. [*Hint:* Set up two proportions using 4:5 and 4:6.]

★ **72.** A minor chord is composed of notes whose frequencies are in the ratio 10:12:15. If the first note of a minor chord is A, with a frequency of 220 hertz, what are the frequencies of the other two notes?

Psychology

73. In an experiment on motivation, Professor Brown trained a group of rats to run down a narrow passage in a cage to receive food in a goal box. He then put a harness on each rat and connected it to an overhead wire attached to a scale. In this way he could place the rat different distances from the food and measure the pull (in grams) of the rat toward the food. He found that the relationship between motivation (pull) and position was given approximately by the equation

$$p = -\tfrac{1}{5}d + 70 \qquad 30 \le d \le 170$$

where pull p is measured in grams and distance d in centimeters. When the pull registered was 40 grams, how far was the rat from the goal box?

74. Professor Brown performed the same kind of experiment as described in Problem 73, except that he replaced the food in the goal box with a mild electric shock. With the same kind of apparatus, he was able to measure the avoidance strength relative to the distance from the object to be avoided. He found that the avoidance strength a (measured in grams) was related to the distance d that the rat was from the shock (measured in centimeters) approximately by the equation

$$a = -\tfrac{4}{3}d + 230 \qquad 30 \le d \le 170$$

If the same rat were trained as described in this problem and in Problem 73, at what distance (to one decimal place) from the goal box would the approach and avoidance strengths be the same? (What do you think the rat would do at this point?)

Puzzle

75. An oil-drilling rig in the Gulf of Mexico stands so that one-fifth of it is in sand, 20 feet of it is in water, and two-thirds of it is in the air. What is the total height of the rig?

76. During a camping trip in the North Woods in Canada, a couple went one-third of the way by boat, 10 miles by foot, and one-sixth of the way by horse. How long was the trip?

★★ **77.** After exactly 12 o'clock noon, what time will the hands of a clock be together again?

SECTION 1-2 Systems of Linear Equations and Applications

- Systems of Equations
- Substitution
- Applications

• Systems of Equations

In the preceding section, we solved word problems by introducing a single variable representing one of the unknown quantities and then tried to represent all other unknown quantities in terms of this variable. In certain word problems, it is more convenient to introduce several variables, find equations relating these variables, and then solve the resulting system of equations. For example, if a 12-foot board is cut into two pieces so that one piece is 2 feet longer than the other, then letting

x = Length of the longer piece

y = Length of the shorter piece

we see that x and y must satisfy both the following equations:

$$x + y = 12$$
$$x - y = 2$$

We now have a system of two linear equations in two variables. Thus, we can solve this problem by finding all pairs of numbers x and y that satisfy both equations.

In general, we are interested in solving linear systems of the type:

$$\begin{aligned} ax + by &= h \\ cx + dy &= k \end{aligned}$$ System of two linear equations in two variables