Mathematics

Singapore-Cambridge General Certificate of Education Advanced Level Higher 2

(Syllabus 9758)

(Updated for examination from 2021)

CONTENTS

The Common Last Topics highlighted in yellow will not be examined in 2021 A-Level national examination.

Significant changes to the syllabus are indicated by black vertical lines either side of the text.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

PREAMBLE

Mathematics is a basic and important discipline that contributes to the developments and understandings of sciences and other disciplines. It is used by scientists, engineers, business analysts and psychologists, etc. to model, understand and solve problems in their respective fields. A good foundation in mathematics and the ability to reason mathematically are therefore essential for students to be successful in their pursuit of various disciplines.

H2 Mathematics is designed to prepare students for a range of university courses, including mathematics, sciences, engineering and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

SYLLABUS AIMS

The aims of H2 Mathematics are to enable students to:

(a) acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences, engineering and other related disciplines

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving

(c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines

(d) experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

ASSESSMENT OBJECTIVES (AO)

There are three levels of assessment objectives for the examination.

The assessment will test candidates' abilities to:

AO1 Understand and apply mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.

AO2 Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.

AO3 Reason and communicate mathematically through making deductions and writing mathematical explanations, arguments and proofs.

USE OF A GRAPHING CALCULATOR (GC)

The use of an approved GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

LIST OF FORMULAE AND STATISTICAL TABLES

Candidates will be provided in the examination with a list of formulae and statistical tables.

INTEGRATION AND APPLICATION

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

Possible list of H2 Mathematics applications and contexts:

Applications and contexts	**Some possible topics involved**
Kinematics and dynamics (e.g. free fall, projectile motion, collisions)	Functions; Calculus; Vectors
Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits	Complex numbers; Calculus
Population growth, radioactive decay, heating and cooling problems	Differential equations
Financial maths (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression

The list illustrates some types of contexts in which the mathematics learnt in the syllabus may be applied, and is by no means exhaustive. While problems may be set based on these contexts, no assumptions will be made about the knowledge of these contexts. All information will be self-contained within the problem.

SCHEME OF EXAMINATION PAPERS

For the examination in H2 Mathematics, there will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

PAPER 1 (3 hours)
A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be at least two questions on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

PAPER 2 (3 hours)
A paper consisting of two sections, Sections A and B.

Section A (Pure Mathematics – 40 marks) will consist of 4 to 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

Section B (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

CONTENT OUTLINE

Knowledge of the content of the O-Level Mathematics syllabus and of some of the content of the O-Level Additional Mathematics syllabuses are assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics. The assumed knowledge for O-Level Additional Mathematics is appended after this section.

	Topic/Sub-topics	Content
SECTION A: PURE MATHEMATICS		
1	**Functions and graphs**	
1.1	Functions	Include: • concepts of function, domain and range • use of notations such as $\mathrm{f}(x) = x^2 + 5$, $\mathrm{f}: x \mapsto x^2 + 5$, $\mathrm{f}^{-1}(x)$, $\mathrm{fg}(x)$ and $\mathrm{f}^2(x)$ • finding inverse functions and composite functions • conditions for the existence of inverse functions and composite functions • domain restriction to obtain an inverse function • relationship between a function and its inverse Exclude the use of the relation $(\mathrm{fg})^{-1} = \mathrm{g}^{-1}\mathrm{f}^{-1}$, and restriction of domain to obtain a composite function.
1.2	Graphs and transformations	Include: • use of a graphing calculator to graph a given function • important characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes of the following: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ $y = \frac{ax+b}{cx+d}$ $y = \frac{ax^2+bx+c}{dx+e}$ • determining the equations of asymptotes, axes of symmetry, and restrictions on the possible values of x and/or y • effect of transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ and $y = \mathrm{f}(ax)$, and combinations of these transformations • relating the graphs of $y = \mathrm{f}^{-1}(x)$, $y = \lvert\mathrm{f}(x)\rvert$, $y = \mathrm{f}(\lvert x\rvert)$, and $y = \frac{1}{\mathrm{f}(x)}$ to the graph of $y = \mathrm{f}(x)$ • simple parametric equations and their graphs

	Topic/Sub-topics	Content
1.3	Equations and inequalities	Include: • formulating an equation, a system of linear equations, or inequalities from a problem situation • solving an equation exactly or approximately using a graphing calculator • solving a system of linear equations using a graphing calculator • solving inequalities of the form $\frac{\mathrm{f}(x)}{\mathrm{g}(x)} > 0$ where f(x) and g(x) are linear expressions or quadratic expressions that are either factorisable or always positive • concept of $\|x\|$, and use of relations $\|x-a\| < b \Leftrightarrow a-b < x < a+b$ and $\|x-a\| > b \Leftrightarrow x < a-b$ or $x > a+b$, in the course of solving inequalities • solving inequalities by graphical methods
2	**Sequences and series**	
2.1	Sequences and series	Include: • concepts of sequence and series for finite and infinite cases • sequence as function y = f(n) where n is a positive integer • relationship between u_n (the nth term) and S_n (the sum to n terms) • sequence given by a formula for the nth term • use of Σ notation • sum and difference of two series • summation of series by the method of differences • convergence of a series and the sum to infinity • formula for the nth term and the sum of a finite arithmetic series • formula for the nth term and the sum of a finite geometric series • condition for convergence of an infinite geometric series • formula for the sum to infinity of a convergent geometric series

	Topic/Sub-topics	Content
3	**Vectors**	
3.1	Basic properties of vectors in two and three dimensions	Include: • addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations • use of notations such as $\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i}+y\mathbf{j}$, $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$ • position vectors, displacement vectors and direction vectors • magnitude of a vector • unit vectors • distance between two points • collinearity • use of the ratio theorem in geometrical applications
3.2	Scalar and vector products in vectors	Include: • concepts of scalar product and vector product of vectors and their properties • angle between two vectors • geometrical meanings of $\lvert \mathbf{a} \cdot \hat{\mathbf{n}} \rvert$ and $\lvert \mathbf{a} \times \hat{\mathbf{n}} \rvert$, where $\hat{\mathbf{n}}$ is a unit vector Exclude triple products $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} \times \mathbf{c}$.
3.3	Three-dimensional vector geometry	Include: • vector and cartesian equations of lines and planes • finding the foot of the perpendicular and distance from a point to a line or to a plane • finding the angle between two lines, between a line and a plane, or between two planes • relationships between (i) two lines (coplanar or skew) (ii) a line and a plane (iii) two planes Exclude: • finding the shortest distance between two skew lines • finding an equation for the common perpendicular to two skew lines

	Topic/Sub-topics	Content
4	**Introduction to Complex numbers**	
4.1	Complex numbers expressed in cartesian form	Include: • extension of the number system from real numbers to complex numbers • complex roots of quadratic equations • conjugate of a complex number • four operations of complex numbers • equality of complex numbers • conjugate roots of a polynomial equation with real coefficients
4.2	Complex numbers expressed in polar form	Include: • representation of complex numbers in the Argand diagram • complex numbers expressed in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, or $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$ • calculation of modulus (r) and argument (θ) of a complex number • multiplication and division of two complex numbers expressed in polar form
5	**Calculus**	
5.1	Differentiation	Include: • graphical interpretation of (i) $\mathrm{f}'(x) > 0$, $\mathrm{f}'(x) = 0$ and $\mathrm{f}'(x) < 0$ (ii) $\mathrm{f}''(x) > 0$ and $\mathrm{f}''(x) < 0$ • relating the graph of $y = \mathrm{f}'(x)$ to the graph of $y = \mathrm{f}(x)$ • differentiation of simple functions defined implicitly or parametrically • determining the nature of the stationary points (local maximum and minimum points and points of inflexion) analytically, in simple cases, using the first derivative test or the second derivative test • locating maximum and minimum points using a graphing calculator • finding the approximate value of a derivative at a given point using a graphing calculator • finding equations of tangents and normals to curves, including cases where the curve is defined implicitly or parametrically • local maxima and minima problems • connected rates of change problems Exclude finding non-stationary points of inflexion and finding second derivatives of functions defined parametrically.

	Topic/Sub-topics	Content
5.2	Maclaurin series	Include: • standard series expansion of $(1 + x)^n$ for any rational n, e^x, $\sin x$, $\cos x$ and $\ln(1 + x)$ • derivation of the first few terms of the Maclaurin series by – repeated differentiation, e.g. $\sec x$ – repeated implicit differentiation, e.g. $y^3 + y^2 + y = x^2 - 2x$ – using standard series, e.g. $e^x \cos 2x$, $\ln\left(\frac{1+x}{1-x}\right)$ • range of values of x for which a standard series converges • concept of "approximation" • small angle approximations: $\sin x \approx x$, $\cos x \approx 1-\frac{1}{2}x^2$, $\tan x \approx x$ Exclude derivation of the general term of the series.
5.3	Integration techniques	Include: • integration of $f'(x)[f(x)]^n$ (including $n = -1$), $f'(x)e^{f(x)}$ $\sin^2 x$, $\cos^2 x$, $\tan^2 x$, $\sin mx \cos nx$, $\cos mx \cos nx$ and $\sin mx \sin nx$ $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$, $\frac{1}{a^2-x^2}$ and $\frac{1}{x^2-a^2}$ • integration by a given substitution • integration by parts
5.4	Definite integrals	Include: • concept of definite integral as a limit of sum • definite integral as the area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • area below the x-axis • finding the area under a curve defined parametrically • finding the volume of revolution about the x- or y-axis • finding the approximate value of a definite integral using a graphing calculator Exclude finding the volume of revolution about the x-axis or y-axis where the curve is defined parametrically.

	Topic/Sub-topics	Content
5.5	Differential equations	Include: • solving for the general solutions and particular solutions of differential equations of the forms (i) $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$ (ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(y)$ (iii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{f}(x)$ including those that can be reduced to (i) and (ii) by means of a given substitution • formulating a differential equation from a problem situation • interpreting a differential equation and its solution in terms of a problem situation

	Topic/Sub-topics	Content
SECTION B: PROBABILITY AND STATISTICS		
6	**Probability and Statistics**	
6.1	Probability	Include: • addition and multiplication principles for counting • concepts of permutation (${}^{n}P_{r}$) and combination (${}^{n}C_{r}$) • arrangements of objects in a line or in a circle, including cases involving repetition and restriction • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities • calculation of conditional probabilities in simple cases • use of: $\mathrm{P}(A') = 1 - \mathrm{P}(A)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
6.2	Discrete random variables	Include: • concept of discrete random variables, probability distributions, expectations and variances • concept of binomial distribution $\mathrm{B}(n, p)$ as an example of a discrete probability distribution and use of $\mathrm{B}(n, p)$ as a probability model, including conditions under which the binomial distribution is a suitable model • use of mean and variance of binomial distribution (without proof) Exclude finding cumulative distribution function of a discrete random variable.

	Topic/Sub-topics	Content
6.3	Normal distribution	Include: • concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $\mathrm{N}(\mu, \sigma^2)$ as a probability model • standard normal distribution • finding the value of $\mathrm{P}(X < x_1)$ or a related probability, given the values of x_1, μ, σ • symmetry of the normal curve and its properties • finding a relationship between x_1, μ, σ given the value of $\mathrm{P}(X < x_1)$, or a related probability • solving problems involving the use of $\mathrm{E}(aX + b)$ and $\mathrm{Var}\,(aX + b)$ • solving problems involving the use of $\mathrm{E}(aX + bY)$ and $\mathrm{Var}\,(aX + bY)$, where X and Y are independent Exclude normal approximation to binomial distribution.
6.4	Sampling	Include: • concepts of population and simple random sample • concept of the sample mean $\overline{X}$ as a random variable with $\mathrm{E}\left(\overline{X}\right) = \mu$ and $\mathrm{Var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$ • distribution of sample means from a normal population • use of the Central Limit Theorem to treat sample mean as having normal distribution when the sample size is sufficiently large (e.g. $n \geqslant 30$) • calculation and use of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form Σx and Σx^2, or $\Sigma(x - a)$ and $\Sigma(x - a)^2$

6.5	Hypothesis testing	Include:
		• concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance and p-value
		• formulation of hypotheses and testing for a population mean based on:
		  – a sample from a normal population of known variance
		  – a large sample from any population
		• 1-tail and 2-tail tests
		• interpretation of the results of a hypothesis test in the context of the problem
		
		Exclude the use of the term ‘Type I’ error, concept of Type II error and testing the difference between two population means.

	Topic/Sub-topics	Content
6.6	Correlation and Linear regression	Include: • use of scatter diagram to determine if there is a plausible linear relationship between the two variables • correlation coefficient as a measure of the fit of a linear model to the scatter diagram • finding and interpreting the product moment correlation coefficient (in particular, values close to –1, 0 and 1) • concepts of linear regression and method of least squares to find the equation of the regression line • concepts of interpolation and extrapolation • use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model • use of a square, reciprocal or logarithmic transformation to achieve linearity Exclude: • derivation of formulae • relationship $r^2 = b_1 b_2$, where b_1 and b_2 are regression coefficients • hypothesis tests

ASSUMED KNOWLEDGE

Content from O-Level Additional Mathematics	
ALGEBRA	
A1	Equations and inequalities • conditions for a quadratic equation to have: (i) two real roots (ii) two equal roots (iii) no real roots • conditions for $ax^2 + bx + c$ to be always positive (or always negative) • solving simultaneous equations with at least one linear equation, by substitution
A2	Indices and surds • four operations on indices and surds • rationalising the denominator
A3	Polynomials and partial fractions • multiplication and division of polynomials • use of remainder and factor theorems • partial fractions with cases where the denominator is not more complicated than: – $(ax + b)(cx + d)$ – $(ax + b)(cx + d)^2$ – $(ax + b)(x^2 + c^2)$
A4	Power, Exponential, Logarithmic, and Modulus functions • power functions $y = ax^n$, where n is a simple rational number, and their graphs • functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs • laws of logarithms • equivalence of $y = a^x$ and $x = \log_a y$ • change of base of logarithms • function $\|x\|$ and graph of $\|\mathrm{f}(x)\|$, where $\mathrm{f}(x)$ is linear, quadratic or trigonometric • solving simple equations involving exponential and logarithmic functions
GEOMETRY AND TRIGONOMETRY	
B5	Coordinate geometry in two dimensions • graphs of equations $y^2 = kx$ • coordinate geometry of the circle with the equation in the form $(x - a)^2 + (y - b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$
B6	Trigonometric functions, identities and equations • six trigonometric functions, and principal values of the inverses of sine, cosine and tangent • trigonometric equations and identities (see List of Formulae) • expression of $a \cos \theta + b \sin \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$

Content from O-Level Additional Mathematics	
CALCULUS	
C7	Differentiation and integration • derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point • derivative as rate of change • derivatives of x^n for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences • derivatives of composite functions • derivatives of products and quotients of functions • increasing and decreasing functions • stationary points (maximum and minimum turning points and points of inflexion) • use of second derivative test to discriminate between maxima and minima • connected rates of change • maxima and minima problems • integration as the reverse of differentiation • integration of x^n for any rational n, e^x, $\sin x$, $\cos x$, $\sec^2 x$ and their constant multiples, sums and differences • integration of $(ax + b)^n$ for any rational n, $\sin(ax + b)$, $\cos(ax + b)$ and $e^{ax + b}$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a+b$	a plus b
$a-b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\dfrac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum\limits_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\dbinom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$
	$\dfrac{n(n-1)\ldots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	$\}$ the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	$\}$ the inverse circular functions

7. *Complex Numbers*

i	the square root of –1
z	a complex number, $z = x + \mathrm{i}y$
	$= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$
	$= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\, z$	the real part of z, $\mathrm{Re}\,(x + \mathrm{i}y) = x$
$\mathrm{Im}\, z$	the imaginary part of z, $\mathrm{Im}\,(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a}\times\mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2, \ldots$occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binomial distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum(x - \bar{x})^2$
φ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample