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Determining the buckling resistance of steel and composite bridge structures

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FOREWORD

This guide was prepared in response to the identification by the Steel Bridge Group of the need for guidance on the determination of the buckling resistance of steel plate girders in composite bridges, both during construction and in service.

The guidance was prepared by David Iles (SCI), with significant input from members of the Steel Bridge Group, in particular Chris Hendy (Atkins), Chris Murphy (Flint & Neill) and Ian Palmer (Mott MacDonald). SCI is grateful to BCSA for financial support during the preparation of the guidance.

Contents

Page No

SUMMARY

This document provides an overview of the means to determine the buckling resistance of steel plate girders in composite bridges, both during construction (the bare steel stage) and in service (when the deck slab acts as the top flange).

The document notes that Eurocode 3 gives principles and general application rules but to apply the rules for composite bridges the designer needs to understand how buckling behaviour can be represented both in manual calculation and through finite element analysis.

The forms of buckling, for members in compression and in bending, for web panels in shear and compression, and local buckling of flange outstands is described and the means to determine 'non-dimensional slenderness' for each form is presented. It is noted that lateral bending of flanges occurs both during construction and in service and that the verification process must therefore take account of the interaction between the effects of vertical and lateral bending. Suggestions are made for modifications of verification criteria in Eurocode 3 (which do not fully cover the effects experienced in bridges).

Recommendations are summarized and examples of buckling modes found in FE analysis are given. Finally, the interaction between major axis and minor axis effects during the construction stage of the worked example presented in SCI publication P357 is considered. The lateral bending effects are greatest on the outer girder (which is not the most heavily loaded girder and the effects were thus not considered in detail in P357) and the calculations show how the combined effects can be verified.

1 INTRODUCTION

The design verification for bridge structures almost invariably involves at some stage the determination of the buckling resistance of members or elements of the structure. The design resistances are verified against design effects (internal forces, moments etc. due to the actions for the design situation) determined by a structural analysis.

For the design of steel structures to Eurocode 3[1], EN 1993-2 permits the internal forces and moments to be determined by first order analysis (not taking into account the deformation of the structure) if the structure is not sensitive (as defined in EN 1993-2) to elastic instability in a global mode (meaning in-plane instability of the members). The effects of out-of-plane instability of the members are then taken into account in determining the resistances of the members.

Beam type bridge structures are not sensitive to in-plane instability in a global mode and thus only the effects of out-of-plane stability need to be considered. (Arch structures may be sensitive to in-plane-instability - see further comment in Section 3.6, although arches are generally outside the scope of this report.) However, when several beams are connected by bracing, out-of-plane instability of one beam may lead to overall instability of the whole structure and the global behaviour of the structure does need to be considered; this is particularly the situation during construction when bare steel beams are paired together.

Eurocode 3 gives principles and general application rules for determining buckling resistance of members, including rules that can take account of global out-of-plane buckling, but to apply the rules the designer needs to understand buckling behaviour, how it can be represented in analysis and the limitations of analytical methods.

Primarily, this report provides an overview of buckling behaviour for steel plate girders (acting alone, without any composite action) and discusses the means to determine buckling resistance. Comments on the buckling resistance of composite slab-on-beam bridges is given in Section 7.

The report has been produced under the guidance of the Steel Bridge Group, in particular with the assistance of Chris Hendy (Atkins) and Chris Murphy (Flint & Neill). Thanks are expressed to both Flint & Neill, and Atkins for permission to include the examples of FE models in Section 11.

2 BUCKLING OF STRUCTURAL MEMBERS

Buckling is the phenomenon when a member or part of a member displaces laterally or out of its plane due to compressive forces or stresses. The displacements are associated with flexural stresses whose magnitude depends on the slenderness of the member. The resistance of the member (its buckling load) is limited by these flexural effects.

The various modes of buckling are described below. Expressions for evaluating elastic critical buckling loads are discussed in Section 3 and the determination of buckling load according to member slenderness and material strength are discussed in Section 4.

2.1 Buckling modes for members

Structural members without intermediate restraint along their length (or with only flexible intermediate restraint) can buckle when subject to either compression or bending. There are four potential modes of buckling for individual members:

Under axial compression:

- Flexural buckling
- Torsional buckling
- Flexural-torsional buckling

Under bending

- Lateral torsional buckling

In addition, where torsional restraint (about a longitudinal axis) is provided at a specific position in the cross section, distortional buckling can occur (the shape of cross section becomes distorted) under axial compression or bending.

2.2 Flexural buckling

Flexural buckling is perhaps the most easily recognized mode of buckling for members. If one considers a pin-ended member that is nominally straight but which has a small (out-of-straight) imperfection, the out-of-straightness increases under axial compression. The limiting resistance (buckling load) is reached, for a steel member, when the combined axial and flexural stresses reach yield stress at some point in the cross section (for a Class 3 cross section) or the bending moment reaches the plastic bending resistance as modified by the axial force (for a Class 1 or 2 cross section). For an I-section member, unrestrained against displacement in any transverse direction, the displacements and flexural stresses due to bending about the minor axis of the cross section are greater than about the major axis and thus the member is said to buckle about its weak axis, as shown in Figure 2.1.

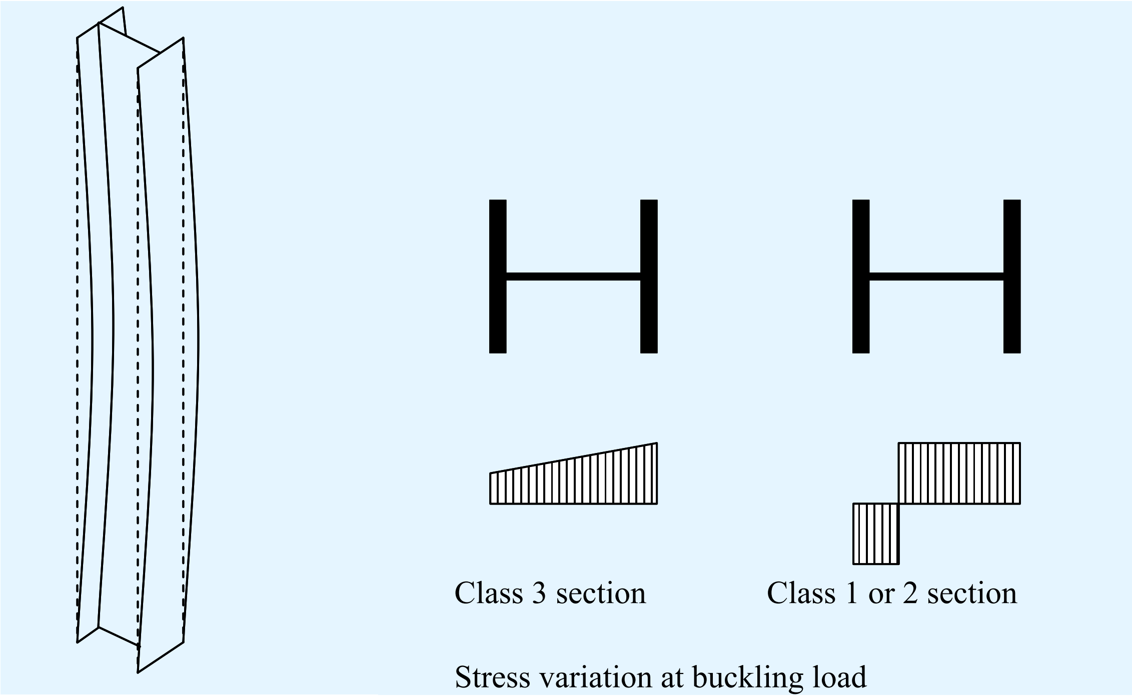


Figure 2.1 Flexural buckling

In steel bridges, flexural buckling is usually only of concern for trusses (where chords and diagonals are in compression) and for bracing members that are subject to compression.

In composite bridges, flexural buckling is normally only of concern for bracing members. (In an integral bridge, the composite deck will be subject to a modest axial force; flexural buckling will not be the governing mode but the axial stresses, and the interaction of buckling resistances, will need to be included when considering the buckling of the bottom flange.)

2.3 Torsional and flexural-torsional buckling

Doubly symmetric sections can buckle in a torsional buckling mode, involving only twist about their longitudinal axis: for sections such as a cruciform section the buckling load may be less than that for flexural buckling, if the member is short, Torsional buckling is shown in Figure 2.2. Such cross sections are rarely used in bridges.

For monosymmetric sections and asymmetric sections, torsional and flexural buckling modes are linked and in some cases may occur at a lower load than flexural buckling about the minor axis. In bridges, these modes of buckling are only relevant to angle and channel bracing members.

For a detailed discussion of torsional and flexural torsional modes, see Hendy and Murphy[2}.

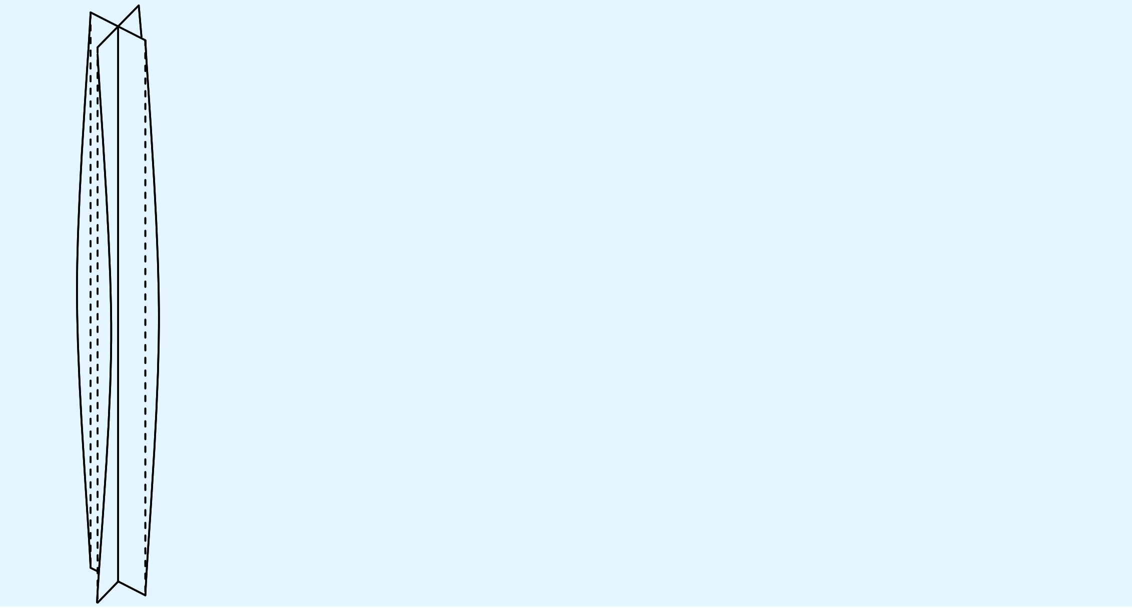


Figure 2.2 Torsional buckling of a doubly symmetric cruciform section

2.4 Lateral torsional buckling

Lateral-torsional buckling is a frequent consideration for the design of steel I-section members without intermediate restraint. In a member that is bent about its major axis, the compression flange will tend to buckle laterally: the flange is effectively a compression member (with a small initial imperfection) that is only free to buckle in one direction. But because the flange is connected to the web, it can only displace by twisting the cross section and by imposing a smaller lateral displacement of the tension flange. The shape of the buckling mode is shown in Figure 2.3.

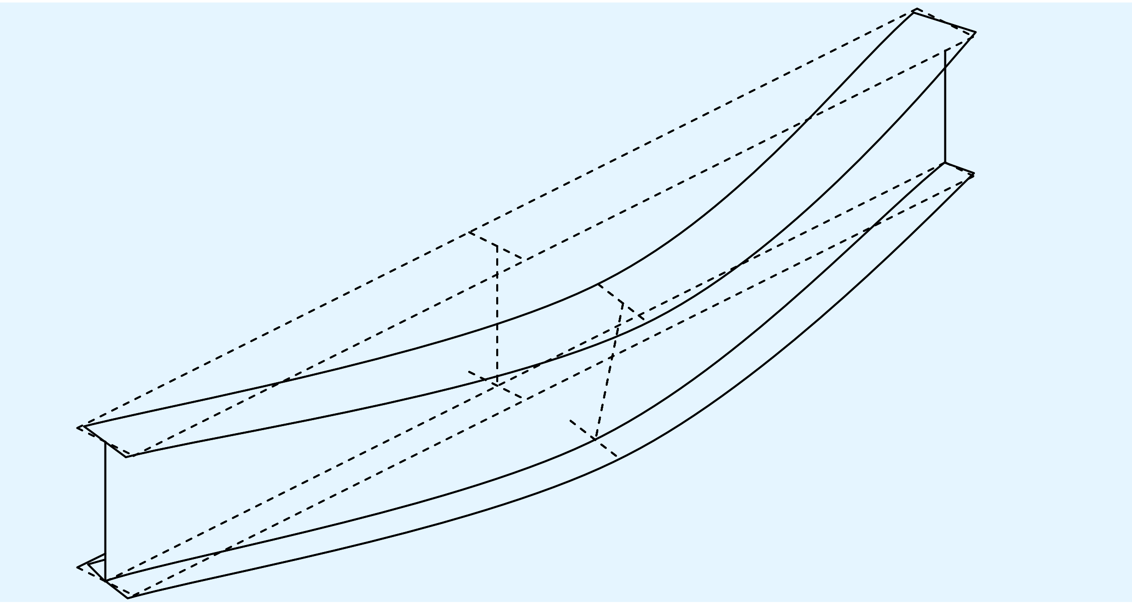


Figure 2.3 Lateral torsional buckling

The flanges of I section steel girders are usually very slender in transverse bending over their span and consequently the slenderness of individual unbraced girders against lateral torsional buckling is high; the buckling resistance of the member in bending (about its major axis) is therefore much less than the bending resistance of the cross section.

2.5 Distortional buckling

Where a bridge deck is formed by a reinforced concrete slab on top of steel I-section girders and acting compositely with them, lateral buckling of the bottom flanges can occur when they are subject to compressive stresses due to bending (or due to compression, although in practical situations axial force alone is too small to lead to buckling). Whilst it is possible to consider this situation as a series of Tee sections, side by side, that could buckle in a lateral torsional buckling mode, the continuity of the slab prevents all lateral displacement of the slab and, more significantly, provides a flexible torsional restraint (about a longitudinal axis) at the top of the web. The mode of buckling then becomes a lateral distortional mode, as shown in Figure 2.4.

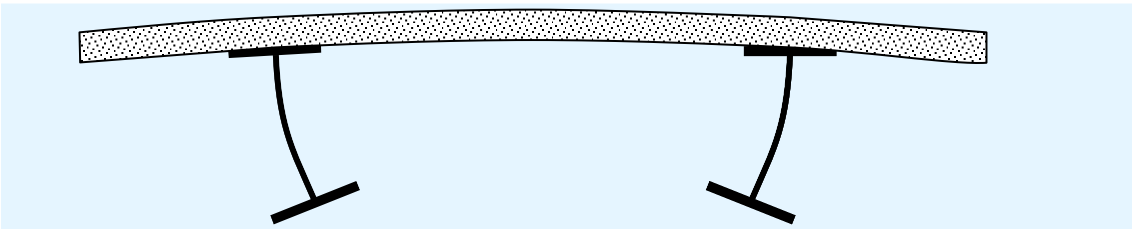


Figure 2.4 Distortional buckling (hogging region of continuous composite deck)

The buckling mode for a half-through U-frame bridge is effectively the inverse of the above mode; the stiffness provided by frequent vertical web stiffeners and cross girders is fundamental to the U-frame stiffness and thus the restraint to the (top) compression flange.

2.6 Local buckling

In addition to the buckling of the member as a whole, slender flanges can buckle locally under compression and slender webs can buckle under compression or shear.

Flange outstands tend to buckle in a single half wave over the length between out-of plane restraints, each flange displacing in opposite directions. (Only if there were significant rotational restraint provided by a thick web to a thin flange would shorter wavelengths occur.) This is effectively a torsional buckling mode and this local buckling is generally avoided by limiting the outstand to thickness ratio, which ensures that the elastic critical bucking stress is sufficiently above the yield stress that yield strength can be developed.

The compression zones of thin web panels in bending tend to buckle as a series of waves along the beam, as shown in Figure 2.5. Where there are closely spaced transverse stiffeners, the half wavelength is restricted to the distance between transverse stiffeners.

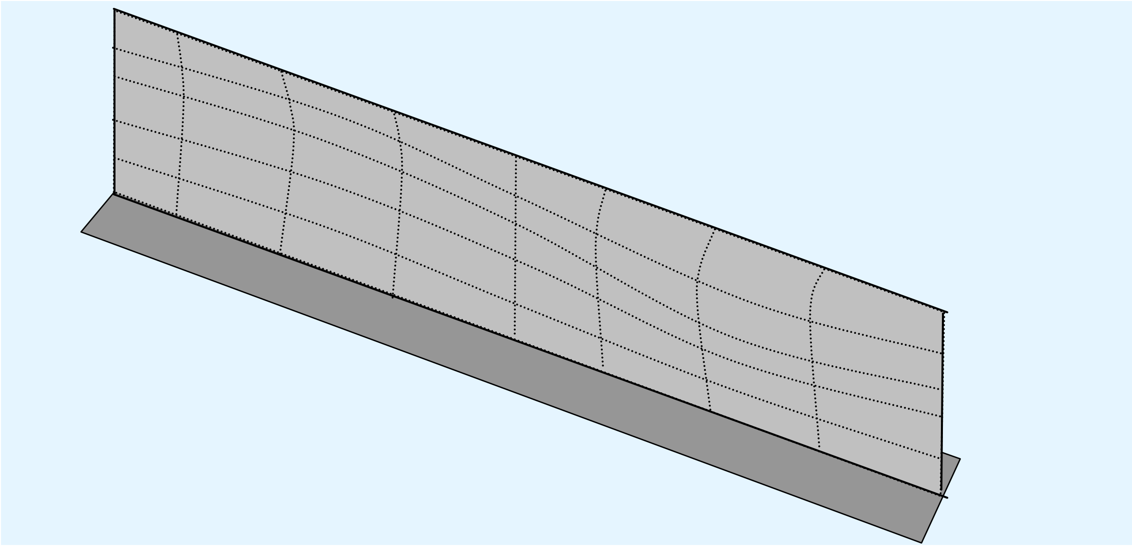


Figure 2.5 Compression buckling of a web in bending (top flange omitted for clarity)

Web panels in shear tend to buckle with waves extending across the diagonal of the web panel, effectively creating 'ripples' that align with the principal tensile stress. With the thin webs that are typically used in bridges, the elastic buckling load in shear is usually significantly less than the plastic shear resistance of the web. Consequently, the design strength of the web often relies on post-buckling behaviour and the shear resistance will exceed the elastic critical load.

3 ELASTIC CRITICAL BUCKLING

3.1 General

To determine the buckling resistance of a real, imperfectly straight, member of limited material strength it is necessary to consider first the elastic critical buckling force (or moment) for a perfectly straight member. The need for this value is best illustrated with reference to flexural buckling of a non-straight compression member.

Under the action of a compressive axial force, the cross sections along the length of the member are subject to both the compression force and a bending moment that is equal to the product of the offset of the cross section centroid from the axis of the member and the compression force. The bending moments result in increased out-of-straightness which in turn increases the bending moments, and so on. (This is sometimes referred to as the P-δ effect.) This is illustrated in Figure 3.1, in which the initial out-of straight is e_0 and the out-of-straight under compression is δ_N.

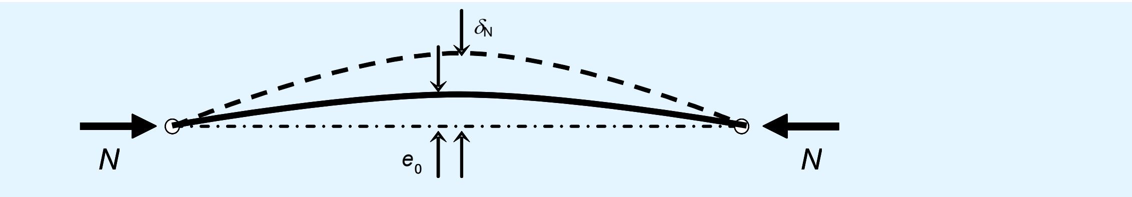


Figure 3.1 Flexural buckling

For small values of out-of straightness (i.e. such that the basic geometry is not significantly altered), the deflection when under compression is given by:

$$\delta_N = \frac{e_0}{(1 - N/N_{cr})}$$

Where N_{cr} is the elastic critical force for flexural buckling (sometimes referred to as the Euler load).

The elastic limit is reached at an extreme fibre when the sum of the axial stress and the bending stress reaches yield, i.e. when:

$$\frac{N}{A} + \frac{N\delta_N}{W} = f_y$$

The value of the elastic critical force is thus essential to determining the buckling resistance of the member. Its value is dependent on the member geometry and material stiffness (modulus of elasticity) but the strength of the material does not affect this theoretical critical value.

Sections 3.2 and 3.3 describe the theoretical derivation of elastic critical buckling forces and moments for flexural buckling and lateral-torsional buckling. Elastic critical forces and moments can also be determined by first order elastic finite element analysis, as discussed in Section 3.5.

3.2 Flexural buckling

The derivation of the 'buckling load' for a uniform pin-ended compression member is a straightforward task that is explained in detail in many texts.

The buckling load is referred to in Eurocode 3 as the *elastic critical buckling force* and, for a pin ended strut, is the axial force at which, for any lateral displacement, the curvature at any point due to that displacement equals the curvature due to a moment that is the product of the axial force and the displacement at that point. (It represents the situation when the stiffness due to any lateral force falls to zero.) For the model shown in Figure 3.2, this situation is expressed by the differential equation:

$$\frac{d^2 y}{dx^2} + \frac{Ny}{EI} = 0$$

The lowest solution to this expression, for a simple sinusoidal flexural displacement is:

$$N_{cr} = \frac{\pi^2 EI}{L^2}$$

Where E is the modulus of elasticity and I is the second moment of area of the member about the minor axis.

The elastic critical buckling force for flexural buckling N_{cr} is often referred to as the Euler load.

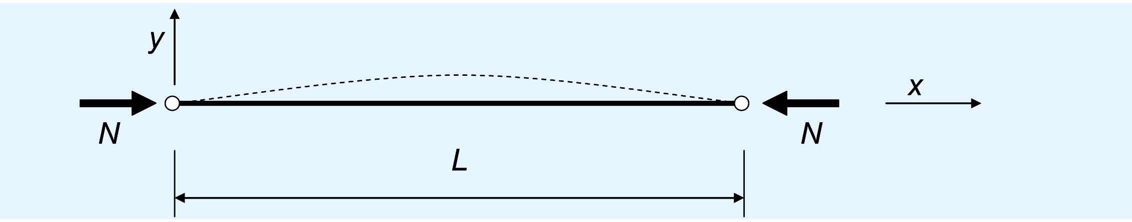


Figure 3.2 Flexural buckling

It may be noted that there are other solutions to the differential equation, for harmonics of the buckling shape. For example, the first harmonic has two sine waves and the elastic critical buckling force for that mode is four times that for the fundamental mode. Such harmonics will be revealed by the FE analysis discussed in Section 5.3.

Also of interest are the solutions for one or both of the ends fixed against rotation. These solutions may be expressed as:

$$N_{cr} = \frac{\pi^2 EI}{(kL)^2}$$

where $k = 1$ for both ends pinned, $k = 0.5$ for both ends fixed and $k = 0.7$ for one end pinned, one end fixed.

The product kL is often referred to as the effective length for buckling. The Eurocode uses the symbol L_{cr} for this length, associated with N_{cr}.

Solutions can also be found for flexible end restraints and for cantilevers, all of which can be expressed in terms of an effective length kL.

3.3 Lateral torsional buckling

For lateral torsional buckling, a similar approach can be adopted to determine an expression for the *elastic critical buckling moment*, although the algebra is more complex.

For a uniform doubly symmetric beam (such as an I-beam) and for a constant bending moment along its length, the elastic critical buckling moment can be expressed as:

$$M_{cr} = \frac{\pi^2 E I_z}{(kL)^2} \sqrt{\left(\frac{k}{k_w}\right) \frac{I_w}{I_z} + \frac{(kL)^2 G I_T}{\pi^2 E I_z}}$$

where

E is the modulus of elasticity ($E = 210000$ N/mm^2)
G is the shear modulus ($G = 80770$ N/mm^2)
I_z is the second moment of area about the minor axis
I_T is the St Venant torsional constant
I_w is the warping constant
L is the beam length between points which have lateral restraint
k and k_w are factors allowing for end restraint

Where the moment is not uniform, the value is increased by multiplying by a factor C_1. Values for C_1 are given in numerous sources, though the values have typically been determined by some process of iterative analysis and are not exact (different sources may give slightly different values).

Other algebraic expressions are available for sections that are not doubly symmetric, although they are more complex. No algebraic solutions are available that would take account of flexible end or intermediate restraints or non-uniform members.

Where algebraic solutions are not available, recourse can be made to finite element modelling (see Section 3.5) or possibly to empirical rules based on an equivalent strut or an effective length concept (see Section 5.2).

3.4 Local buckling of plate elements

The flanges and webs of members are susceptible to local buckling due to compressive effects and these elements have an elastic critical buckling stress that is given by established theory of plate behaviour.

3.4.1 Plate elements in compression

The elastic critical buckling stress of a plate element in compression is given by:

$$\sigma_{cr} = \frac{K\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

In which K is a parameter that depends on the boundary conditions and the variation of stress across the width of the panel. Values for a wide range of situations are given in Bulson[3] and values for some specific cases are given in EN 1993-1-5.

For long plates supported on both edges and subject to uniform compression, $K = 4$. This situation corresponds to web panels with widely spaced transverse stiffeners when the member is subject to axial force only.

For long plates supported on both edges and subject to pure bending (i.e. stress varies from compression at one edge to equal tension at the other), $K = 23.9$. This situation corresponds to web panels in symmetric beams subject to bending only.

For long plates supported only at one edge, with no rotational restraint at that edge, $K = 0.43$. This situation corresponds to flange outstands connected to thin deep web panels.

3.4.2 Plate elements in shear (web panels)

The theoretical approach to shear buckling of rectangular web panels generally considers the buckling of a panel between transverse web stiffeners. The solution is expressed similarly to that for panels in compression:

$$\tau_{\mathrm{cr}} = \frac{K\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

In which

$K = 5.34 + 4/\phi^2$ for $\phi \geq 1$ or

$K = 5.34\phi^2 + 4$ for $\phi < 1$

$\phi = a/b$ (the aspect ratio); a and b are the panel height and length

3.5 Solution by Finite Element Analysis

Where it is not possible to isolate uniform structural components, the loading on the components is complex or the interaction between components makes it difficult to determine boundary conditions for the critical components, then recourse can be made to finite element analysis software that can determine elastic critical buckling loads using matrix analysis.

First order analysis software will determine buckling loads by considering a particular loading situation and evaluating the eigenvalues for the stiffness matrix.

Each eigenvalue has a corresponding eigenvector that defines the particular buckling mode associated with that value. The eigenvalues thus represent the critical buckling loads for each possible mode of buckling. Each eigenvalue gives the multiple of the applied loading at which the structure buckles in that particular mode and thus it is only the lowest values that are of relevance.

To carry out a meaningful FE analysis, shell elements must be used, preferably for all the structural elements but at least for those elements that are susceptible to buckling. The mesh size must be sufficiently small that the buckling mode can be modelled. For I-section plate girders, four 4-noded shell elements across the flange (or two 8-noded elements), with an aspect ratio of not more than about 2 (or 4 for 8- nodded elements) will usually give sufficient accuracy for overall buckling. For local buckling at least 8 nodes per half wavelength will be needed (a coarse mesh will overestimate the local buckling load but this will not matter if local buckling is not of concern). Experience in analysis of similar structural configurations will aid the choice of an appropriate mesh and a sensitivity analysis can be used to confirm the accuracy of the chosen mesh size or to show what refinement is needed to improve accuracy.

First order buckling analysis will be adequate for determining elastic critical buckling loads for most situations, which means that material non-linearity and geometric deformation are not taken into account. Note, however, that since the software for determining elastic critical loads generally uses stress stiffness matrices, which are based on initial linear stress and displacements, the destabilizing effect of any loads applied above the member centroid is automatically taken into account.

The effects of initial imperfections are not considered in first order analysis. These effects are taken into account by the use of buckling curves, as discussed in Section 4.

3.6 Second order analysis

A full second order analysis takes account of material non-linearity and geometric deformation. To carry out such an analysis to determine failure load in accordance with Eurocodes requires complex software. It can determine failure loads directly, without reference to buckling curves, but the model does need to incorporate initial imperfections that are equivalent to those assumed in the Eurocode design rules; it should be noted that the design imperfections exceed the geometrical limits given in EN 1090-2 because the former also include the effects of residual stresses through additional equivalent geometric imperfection. Evaluation of appropriate imperfections for the analysis requires a thorough understanding of the design basis in Eurocode 3.

Second order analysis is essential when the buckling behaviour is influenced by the modified geometry of the structure under load. This effect is most easily illustrated by considering an arch and is shown schematically in Figure 3.3. First order buckling analysis would only give eigenvectors for buckling modes related to the original geometry. However, the axial strain in the arch members will cause the arch to flatten, which increases the axial forces and strains. (In a sufficiently flat arch, the arch will snap through.) The true buckling load is thus only given by a second order analysis.

Second order analysis and the design of arches is outside the scope of this report.

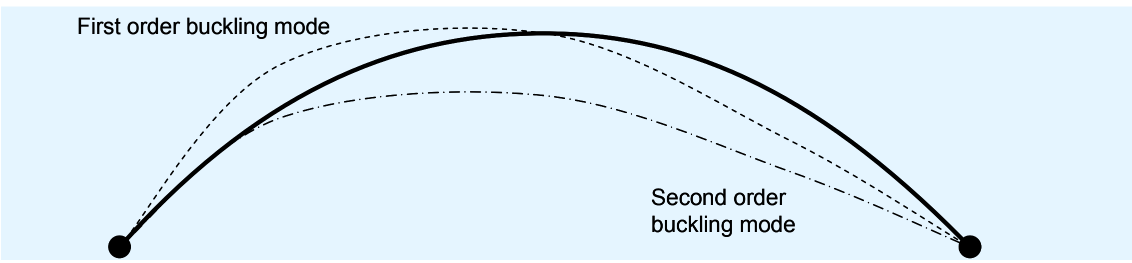


Figure 3.3 Buckling of an arch

4 BUCKLING RESISTANCE OF REAL MEMBERS

The theoretical solutions in Sections 3.2 to 3.5 take no account of material strength, nor of any out-of-straightness of the structural components. Real members are not perfectly straight and, as noted in Section 3.1, as compressive load is applied, the members will deflect, increasing the out-of-straightness and causing additional bending stresses. Yield will therefore be first attained at an extreme fibre at a lower load than if there were no imperfection. Once yield is reached, the bending stiffness decreases and the rate of deflection with load increases; yield at an extreme fibre thus represents at least an initial limit to the resistance of the member, although in some cases plastic straining can allow a higher resistance to develop.

4.1 Members in compression

For a compression member with a sinusoidal initial imperfection, the relationship between deflection and stress given in Section 3.1 can alternatively be expressed as the well-known Perry-Robertson formula.

$$(\sigma_a - f_y)(\sigma_a - \sigma_{cr}) = \eta \sigma_a \sigma_{cr}$$

Where

σ_a is the axial stress when yield is reached at an extreme fibre (= *N/A)*

f_y is the yield stress

σ_{cr} is the axial stress at the Euler load ($= \pi^2 EI / L^2 A$ = N_{cr} / *A*)

η is an imperfection parameter (related to the peak value of the imperfection and to the cross section properties)

This quadratic equation can be re-expressed by making the stress terms 'non-dimensional', dividing them by the yield stress f_y thus:

$$\left(\frac{\sigma_a}{f_y} - 1\right)\left(\frac{\sigma_a}{f_y} - \frac{\sigma_{cr}}{f_y}\right) = \eta \frac{\sigma_a}{f_y} \cdot \frac{\sigma_{cr}}{f_y}$$

The ratio σ_a/f_y may be expressed as a reduction factor for buckling χ and the ratio σ_{cr}/f_y may be expressed in terms of a 'non-dimensional slenderness' $\overline{\lambda}$, such that $\sigma_{cr}/f_y = 1/\overline{\lambda}^2$. See discussion of Eurocode definitions in Section 4.2.

The solution of the quadratic equation can then be expressed, after some algebraic manipulation as:

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \overline{\lambda}^2}}$$

In which $\phi = 0.5\left[1 + \eta + \overline{\lambda}^2\right]$

The greater the imperfection parameter, the smaller the value of the reduction factor.

If a fixed value of imperfection is presumed, independent of non-dimensional slenderness, the above expression gives a single buckling curve, as shown in Figure 4.1.

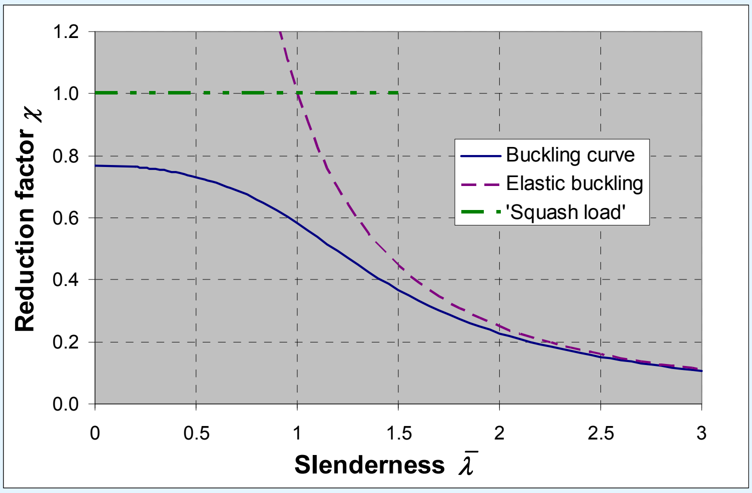


Figure 4.1 Simple buckling curve

The above curve represents the limiting situation when yield is reached at an extreme fibre. For very slender members, this represents failure but, at low slenderness, plastification and strain hardening will increase the load that can be sustained and this has been demonstrated in tests. To take advantage of this improved performance, it is accepted practice to equate the member resistance to the resistance of the cross section up to a certain limiting slenderness and thereafter to apply a reduction that is related to the slenderness.

In Eurocode 3, the imperfection parameter is generally taken as $\alpha(\bar{\lambda} - 0.2)$ where α is a parameter that has been determined by test. The resulting buckling curve thus has a 'plateau' up to a slenderness of 0.2. Curve-fitting to test results over very many years has established a range of values of α that represent the state of residual stress (due to rolling and welding) and out-of-straightness found in structural steel members, depending on cross section geometry and whether it is a rolled section or one fabricated by welding. Five different values of imperfection factor, and thus five buckling curves, are defined for compression members, as shown in Figure 4.2.

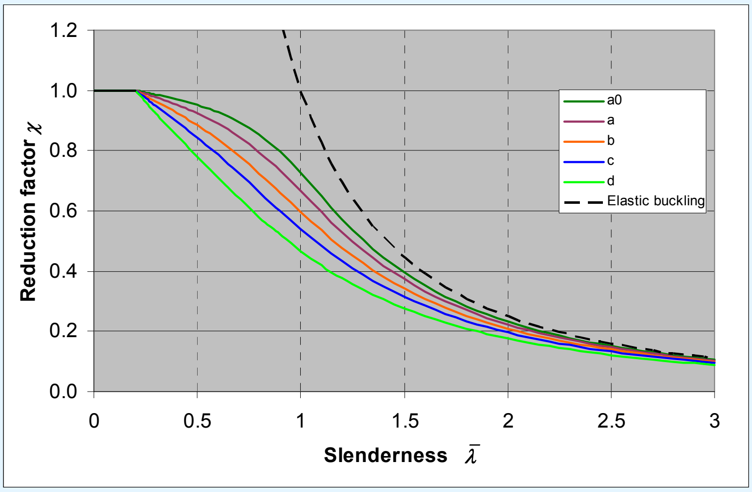


Figure 4.2 Compression buckling curves in Eurocode 3

4.2 Non-dimensional slenderness

For compression members, Eurocode 3 defines the non-dimensional slenderness as:

$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}}$ for Class 1, 2 and 3 members and

$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}}$ for Class 4 members (EN 1993-1-1, 6.3.1.3).

Where N_{cr} is the elastic critical buckling load for the member, determined taking account of end conditions, intermediate restraints etc.

In general terms this means:

$$\bar{\lambda} = \sqrt{\frac{\text{Resistance of the cross section}}{\text{Elastic critical load}}}$$

This general definition can be extended to the slenderness of members in bending (in relation to LTB), to plate panels in compression and to web panels in shear. Thus:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W f_y}{M_{cr}}} \quad \text{(EN 1993-1-1, 6.3.2.2)}$$

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \quad \text{(EN 1993-1-5, 4.4)}$$

$$\bar{\lambda}_w = \sqrt{\frac{f_y / \sqrt{3}}{\tau_{cr}}} \quad \text{(EN 1993-1-5, 5.3)}$$

A similar approach can be taken for a whole structure, taking the complete system of girders and deck, to determine a global non-dimensional slenderness at which buckling occurs. The ‘global slenderness’ is then given by:

$$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}}$$

Where

$\alpha_{ult,k}$ is the minimum load amplifier to reach characteristic resistance in the most critical cross section, considering only in-plane behaviour

$\alpha_{cr,op}$ is the minimum load amplifier to reach elastic critical load due to lateral or lateral torsional buckling.

4.3 Members in bending

Like members in compression, the resistance of members in bending to lateral torsional buckling depends on the non-dimensional slenderness and an allowance for initial imperfections. Although it is not as easy to model the effects of notional imperfections, such as the sine wave out-of-straightness in the Perry Robertson model, a similar approach can be adopted.

For the magnitude of the imperfection parameter, there is no simple out-of-straightness dimension equivalent to that for compression members but curve fitting to test results has led to the adoption in Eurocode 3 of the same values of α as for compression members. However, Eurocode 3 chooses a different relationship between the imperfection parameter and non-dimensional slenderness for rolled sections and welded sections. No curves are actually shown in EN 1993-1-1 for LTB but the relationships are defined.

For doubly symmetric I and H rolled sections, tests have shown that the effective plateau is longer and that the reduction factor is slightly better (than with the flexural buckling curve) at higher slenderness. As implemented by the UK NA, the three curves for LTB are as shown in Figure 4.3.

For welded sections, the LTB curves are the same as the two lowest curves for flexural buckling, as shown in Figure 4.4.

For bridges, the members are normally welded sections and thus only the second of these two sets of curves is used.

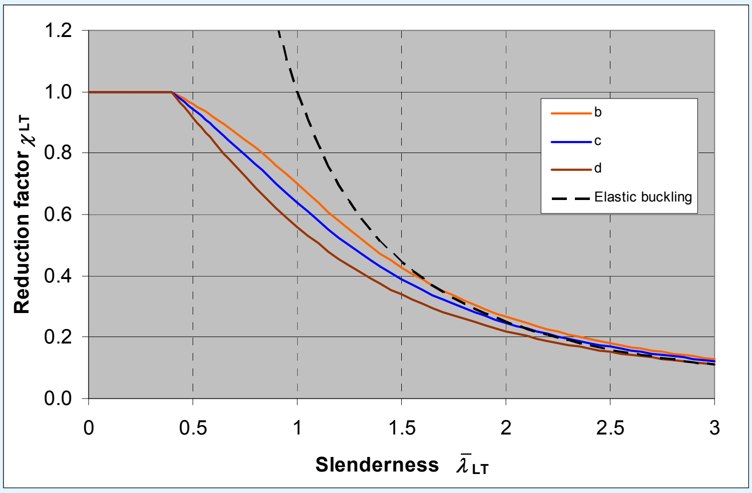


Figure 4.3 LTB buckling curves for rolled sections (EN 1993-1-1 and the UK NA)

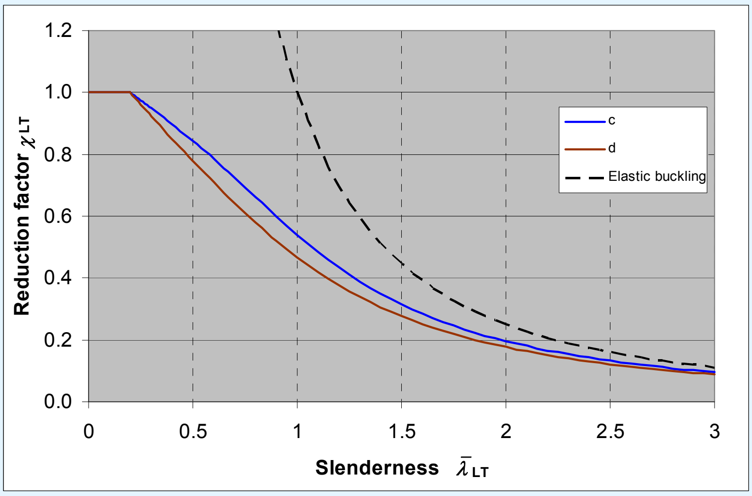


Figure 4.4 LTB buckling curves for welded sections (EN 1993-1-1 and the UK NA)

4.4 Plate elements

4.4.1 Elements in compression

Generally, slender plate elements have a considerable post-buckling reserve of strength. This reserve can be appreciated as a redistribution of stress toward the supported boundaries - for internal elements the stresses are higher at the edges than at the middle, for flange outstands, the stress is higher at the web than at the tip.

Although Eurocode 3 does not provide 'buckling curves' for plates in compression, EN 1993-1-5 does give values of 'effective areas' based on a reduction factor ρ that is equivalent to the reduction factor for member resistance. Values of ρ for internal and outstand elements are shown in Figure 4.5. The Eurocode values have been validated by reference to test results.

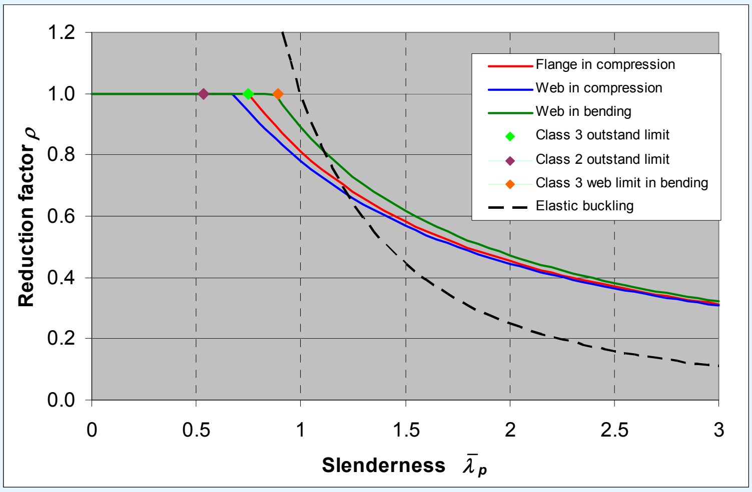


Figure 4.5 Reduction factor for plates (EN 1993-1-5)

Also shown on the Figure are the limiting plate width/thickness ratios for Class 3 elements (i.e. those where the full area may be used in determining elastic resistance of the member). The Figure shows that at slendernesses above about 1.1 the plate elements can resist a load in excess of their elastic critical buckling load.

4.4.2 Webs in shear

Imperfections in web panels have a different effect upon the design shear resistance (the Perry Robertson model is not applicable) and instead the Hőglund model, with rotation of principal stresses post-buckling, enables, for thin web panels, the mobilisation of a design resistance that is significantly greater than the elastic critical buckling load. EN 1993-1-5 expresses the reduction factor on plastic shear resistance in relation to the non-dimensional slenderness, as shown in Figure 4.6.

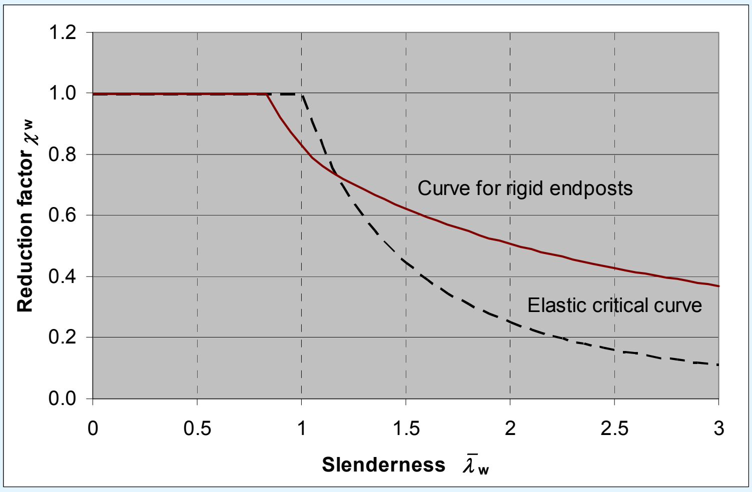


Figure 4.6 Reduction factor for shear resistance of webs

It should be noted that the buckling curves for shear are based on test results that were influenced by the boundary restraint provided by the flanges and by rigid end-post systems (where these exist and are sufficient to enhance the restraint of the web panel). If a fine mesh analysis of a web in shear were to be attempted, no rotational restraint should be provided if the Eurocode 3 buckling curve is then to be applied to the elastic buckling load.

5 DETERMINING NON-DIMENSIONAL SLENDERNESS

5.1 Accounting for instability

In almost all design situations, the design buckling resistance of members in bridges will be verified according to clause 6.3 of EN 1993-2. The calculation of buckling resistance then depends on the determination of a reduction factor (to be applied to the resistance of the cross section), which depends on the non-dimensional slenderness of the member. As explained earlier, the non-dimensional slenderness depends on the elastic critical buckling force or moment. In some cases the slenderness can be readily calculated manually but, for LTB slenderness in particular, 'simple' manual approximations may be too conservative and it will be worthwhile to undertake a FE analysis for a more accurate evaluation.

5.2 Manual calculation of slenderness

5.2.1 Flexural buckling

Clause 6.3.1.3 of EN 1993-1-1 gives a simple expression for non-dimensional slenderness for flexural buckling that avoids the explicit calculation of the elastic critical force:

$$\bar{\lambda} = \frac{L_{cr}}{i}\frac{1}{\lambda_1}$$

where L_{cr} is the critical buckling length, i is the radius of gyration about the relevant axis and $\lambda_1 = \pi\sqrt{E/f_y}$.

This expression presumes that the member has a constant cross section, with all the axial forces applied at the ends and the end conditions known with such confidence that the buckling length L_{cr} can be reliably assumed. (L_{cr} is effectively the same as kL in Section 3.2.) Where such presumptions do not apply, conservative assumptions can be made (such as ignoring beneficial influences that cannot be properly quantified) but it may be better to carry out a FE analysis if conditions are significantly non-uniform or the effective stiffness of the end restraints can only be determined by modelling.

5.2.2 Lateral torsional buckling

No simple expression is given for non-dimensional slenderness for lateral torsional buckling. Its value is to be derived from the elastic critical buckling moment for the member: as mentioned above and is given by:

$$\bar{\lambda}_{LT} = \sqrt{\frac{Wf_y}{M_{cr}}}$$

There are two methods for determining LTB slenderness manually, without the need to determine elastic critical moment. Both are based on empirical simplifications that give conservative values of slenderness.

Equivalent strut method

EN 1993-2, 6.3.4.2 builds on the simplified assessment method that is introduced (for buildings) in EN 1993-1-1, 6.3.2.4. The method considers the length of the compression flange between effective restraints (to that flange) and treats it as a compression member to derive slenderness. The compression member is taken to comprise the flange and one third of the adjacent depth of web in compression. Account is taken of variation of axial force along that notional member and of intermediate flexible restraint provided by U-frame action. The method is relatively easy to apply and not overly conservative (at least for the composite stage, in the regions adjacent to internal supports).

Effective length method

The other alternative is available through the NA to BS EN 1993-2, which refers to PD 6695-2[4] for complementary information. That document gives the slenderness for a number of restraint situations in the form:

$$\bar{\lambda}_{\mathrm{LT}} = \frac{1}{\sqrt{C_1}} UVD \frac{\lambda_z}{\lambda_1} \sqrt{\beta_{\mathrm{w}}}$$

The parameters in this expression (which are all defined in the PD) take account of torsional and warping properties of the cross section and an ‘effective length’ that depends on the stiffness of the end and intermediate restraints, as well as variation of bending moment and bending resistance of the cross section. The expressions for the various parameters have been taken from the rules in BS 5400-3, re-expressed in Eurocode terminology and symbols. It is generally accepted that the values of slenderness given by these expressions are conservative.

For the general case of a non-uniform member, there is no explicit expression that will give the value of M_{cr} and thus the LTB slenderness. The most effective means to derive its value is through a finite element elastic critical buckling analysis of the member or structure.

5.3 Determining slenderness through FE elastic buckling analysis (first order)

The primary result of an FE buckling analysis is a series of eigenvalues representing the load factors (multipliers on the magnitude of the given loading) at which the various buckling modes are critical (such as the higher harmonic modes referred to for flexural buckling in Section 3.2). The results are normally presented in ascending order and only the lowest modes are of interest. However, the effective design resistance is not necessarily given by the lowest eigenvalue. To determine the design resistance, the designer must consider not just the eigenvalues but also the associated eigenvectors (which reveal the mode shape): when there are slender plate elements, local buckling can occur at a lower load than member buckling but the local buckling does not represent failure and does not determine the slenderness that is needed in the evaluation of design resistance of the member.

5.3.1 Compression members

Where it is necessary to determine N_{cr} by FE analysis (for example when the member is non-uniform) representing the member as a series of beam elements will usually give a sufficiently reliable result. Typically, the member should be divided into at least 20 elements over the buckling length. The use of shell elements will give similar accuracy for overall buckling and may also show local buckling effects. (Local buckling can generally be ignored when the elements of the cross section comply with the limitations of EN 1993.)

5.3.2 Members in bending

To analyse members in bending, shell elements should be used for the webs and the flanges. Generally, the FE mesh size should be sufficiently fine that the model is able to represent torsional effects in the elements and the overall buckling modes (see comment in Section 3.5 on choice of mesh). The mesh will also be able to model the local buckling of the compression flange and the webs in bending, although not with accuracy unless the mesh is fine. Shear buckling of the web will not normally be modelled as it would require a much finer mesh than is appropriate for determining member buckling.

The top flange in a midspan region may well be proportioned such that it is close to the outstand limit for Class 3, when its slenderness is about 0.75. If the slenderness for LTB were the same as this value (and thus the eigenvalues for the two buckling modes would be the same), then the reduction factor for LTB (assuming a welded section and buckling curve d) would be about 0.6 (see Figure 4.4). In practice, economic design would probably require a 'better' (higher) reduction factor and thus a lower slenderness and a greater elastic critical buckling load. In such a situation, the eigenvalue for LTB (the ratio of elastic critical load to load applied to the model) would be higher than that for flange buckling. The eigenvalue of interest to the designer is therefore not the lowest value but the one relevant to the first global LTB mode.

Similarly, the web slenderness in midspan regions may well be close to the class 3 limit (at a corresponding slenderness of about 0.9, as can be seen in Figure 4.5) or even class 4 (with a slenderness of 1.0 or greater). The elastic critical buckling load of the web panel in bending may thus be well below that for LTB but will not be the limiting criterion.

The appearance of non-limiting buckling modes is illustrated in Figure 5.1 for a typical FE model. It can be seen that a web buckling mode has developed (the buckling has led to some associated out-of-plane flange displacement, although this is not a flange buckling mode) but there is no LTB at this stage. In a multi-beam model with varying geometry in each beam, there may well be very many similar non-limiting modes. Close examination of all the lowest eigenvalues and eigenvectors is needed to identify the mode that will correspond to failure of the structure. Separate analyses will be needed for each design situation. (Note that the eigenvalue found for this local buckling mode in the model illustrated will exceed the 'true' value because the mesh is coarse in relation to the half wavelength for local buckling. See example in Figure 11.4 for a better modelling of local buckling, using a finer mesh.)

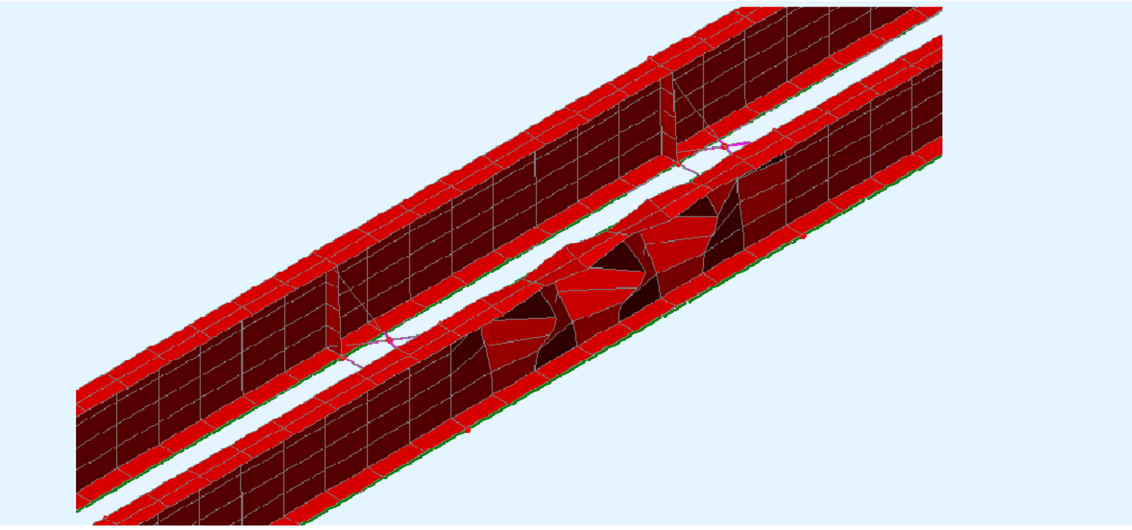


Figure 5.1 Local web buckling in a FE model

5.3.3 Derivation of slenderness from load factors

Since the result of the FE analysis is a load factor that represents a multiple of the applied loading at which elastic critical buckling occurs, it is a relatively straightforward matter when the member is subject to compression alone or bending alone to multiply the design effect at the critical location by the load factor to give either N_{cr} or M_{cr}, as appropriate, and thus to calculate the non-dimensional slenderness (as discussed in Section 4.2).

Note however, that Network Rail requires[5] that, where finite element modelling is used to determine buckling effects, a value of $\gamma_{Sd} = 1.1$ must be applied in the calculation of the effects of actions (this is in addition to any allowance for modelling that is included in the partial factors γ_G and γ_Q – see BS EN 1990, clause 6.3.2, which refers to them generally as $\gamma_{F,I}$).

Where the member is subject to combined axial force and bending, the general approach of EN 1993-1-1, 6.3.4 should be used. This determines a 'global' non-dimensional slenderness $\bar{\lambda}_{op}$, depending on the load factor and the utilization of the cross section under combined loading. From that global slenderness reduction factors for compression and bending may either be determined and applied separately (since the buckling curves may be different for each effect, the reduction factors may be different), or be taken as the lower value and applied to both effects.

6 OTHER CONSIDERATIONS

6.1 Design of lateral and torsional restraints

Neither the manual methods nor the FE analysis give design forces for lateral and torsional restraints. The rules in EN 1993-2, 6.3.4.2 should be used to determine the requirements for such restraints.

6.2 Lateral bending and warping

6.2.1 Individual steel beams

When there are lateral forces, such as wind forces at the bare steel stage, interaction between major and minor axis bending must be considered. There is no interaction criterion in EN 1993-1-1 for combined bending without axial force but with $N_{\mathrm{Ed}} = 0$, the criteria in 6.3.3 reduce to:

$$\frac{M_{y,Ed}}{\chi_{\mathrm{LT}} M_{\mathrm{y,Rk}} / \gamma_{\mathrm{M1}}} + \frac{M_{z,Ed}}{M_{\mathrm{z,Rk}} / \gamma_{\mathrm{M1}}} \leq 1$$

The expression indicates a linear interaction and demonstrates that minor axis bending is not destabilizing and does not need to be amplified. The interaction criteria in EN 1993-1-1 were derived for doubly symmetric I-sections but bridge girders are typically only monosymmetric and hence lateral loads applied other than at the shear centre will cause a twist as well as a moment. Because this twist from transverse loads is not usually calculated, it has therefore been suggested[6] that a more conservative criterion should be used for monosymmetric sections when the torsion is not explicitly considered:

$$\frac{M_{y,Ed}}{\chi_{\mathrm{LT}} M_{\mathrm{y,Rk}} / \gamma_{\mathrm{M1}}} + \frac{1}{1 - M_{\mathrm{y,Ed}} / M_{\mathrm{cr}}} \times \frac{M_{z,Ed}}{M_{\mathrm{z,Rk}} / \gamma_{\mathrm{M1}}} \leq 1.$$

When there are torsional effects, such as due to the support of cantilever falsework or in girders curved in plan, these also need to be included in the interaction criteria. EN 1993-1-1 does not provide such criteria but EN 1993-6 does provide the following criterion (re-expressed in terms of warping moment, rather than bi-moment):

$$\frac{M_{\mathrm{y,Ed}}}{\chi_{\mathrm{LT}} M_{\mathrm{y,Rk}} / \gamma_{\mathrm{M1}}} + \frac{C_{\mathrm{mz}} M_{\mathrm{z,Ed}}}{M_{\mathrm{z,Rk}} / \gamma_{\mathrm{M1}}} + \frac{k_{\mathrm{w}} k_{\mathrm{zw}} k_{\alpha} M_{\mathrm{w,Ed}}}{M_{\mathrm{w,Rk}} / \gamma_{\mathrm{M1}}} \leq 1$$

in which:

C_{mz} is the equivalent uniform moment factor for bending about the z-axis according to EN 1993-1-1 Table B.3. (For a simply supported beam with a parabolic bending moment diagram due to UDL $C_{\mathrm{mz}} = 0.95$; for a triangular bending moment diagram due to a single point load $C_{\mathrm{mz}} = 0.9$.)

k_{w} $= 0.7 - 0.2\, M_{\mathrm{w,Ed}}/M_{\mathrm{w,Rd}}$

k_{zw} $= 1 - M_{\mathrm{z,Ed}}/M_{\mathrm{z,Rd}}$

k_{α} $= 1/[1 - M_{\mathrm{y,Ed}}/M_{\mathrm{cr}}]$

M_{cr} is the elastic critical moment about the y-axis.

$M_{w,Ed}$ is the warping moment in one flange

$M_{w,Rk}$ is the characteristic bending resistance of the (weaker) flange.

k_w can conservatively be taken as 0.7;. C_{mz} and k_{zw} can conservatively be taken as 1; but k_α does need to be evaluated.

However, this expression has again been derived for doubly symmetric sections and it is again suggested that the k_α factor should be applied to the M_z term as well, which would result in:

$$\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk} / \gamma_{M1}} + \frac{k_\alpha C_{mz} M_{z,Ed}}{M_{z,Rk} / \gamma_{M1}} + \frac{k_w k_{zw} k_\alpha M_{w,Ed}}{M_{w,Rk} / \gamma_{M1}} \leq 1$$

6.2.2 Paired steel beams

The above criteria are expressed for single members but can be used for paired girders (without plan bracing) if χ_{LT} has been determined for that configuration.

Alternatively, the lengths of compression flange between bracing positions may be treated as 'simple struts', in a similar manner to the general method in EN 1993-2, 6.3.4. The effective area of the simple strut is that of the compression flange and one third of the depth of web in compression. The strut is considered to be subject only to axial force and lateral bending and the interaction criteria are thus expressed in the same manner as for flexural buckling with bending, as follows:

$$\frac{N_{Ed}}{\chi N_{Rd}} + \frac{1}{1-(N_{Ed} / N_{cr,z})} \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1.0$$

Where

N_{Ed} is the design compression force in the effective strut section

N_{Rd} is the compression resistance of the effective strut section

$N_{cr,z}$ is the elastic critical force for the effective strut, given by $N_{cr,z} = M_{cr} \times A_e / W_y$ ($= A_e f_y / \bar{\lambda}^2$)

χ = χ_{LT} determined for the paired girders

M_{cr} is the elastic critical moment for the member, determined for the paired girders (alternatively, may be expressed in terms of $\bar{\lambda}$)

W_y is the elastic section modulus of the member

A_e is the area of the effective strut

M_z is the maximum value of bending of the flange about the z-z axis (in its plane) within the length of the effective strut

$M_{z,Rd} = W_z f_y / \gamma_{M1}$ (the elastic bending resistance of the flange, using the γ_{M1} factor)

6.2.3 Composite beams

The strut model of the bottom flange is ideal for verifying the resistance of the bottom flange in hogging regions. Rules are given in EN 1993-2. 6.3.4 for determining the slenderness of the effective strut that represents the bottom flange and part of the web of the composite section. However, the rules need to be supplemented when there is lateral bending of the flange (in its plane) and the bending effects need to be amplified. The following interaction criterion is suggested:

$$\frac{N_{Ed}}{\chi_z N_{Rd}} + \frac{1}{1-(N_{Ed}/N_{cr,z})}\frac{M_{z,Ed}}{M_{z,Rd}} \leq 1.0$$

Where

N_{ED} is the design compression force in the effective strut section (determined from the stresses due to summation of effects from all stages)

N_{Rd} is the compression resistance of the effective strut section

$N_{cr,z}$ is the elastic critical force for the effective strut, as given by the rules in EN 1993-2, 6.3.4

$M_{z,Ed}$ is the maximum value of bending of the flange about the z-z axis (in its plane) within the length of the effective strut

$M_{z,Rd} = W_z f_y / \gamma_{M1}$ (the design value of elastic bending resistance of the flange, calculated using the γ_{M1} factor)

Note that clause 6.3.4 allows for situations where N_{Ed} is not constant over the length of the effective strut and permits the verification to be carried out at $0.25L_k$ from the end with the larger force, provided that the resistance of the cross section is also verified at the end. The above criterion conservatively excludes the C_{mz} factor applied to $M_{Z,Ed}$ (an appropriate value is difficult to determine).

6.3 Interaction criteria when buckling load has been determined by FE analysis

When the buckling load has been determined by FE analysis, the reduction factor χ needed in the expressions in Section 6.2 is given by the 'global slenderness', as defined in Section 4.2. However, that value of slenderness depends only on in-plane behaviour. If lateral effects have been induced in the critical member, the reduction factor $\alpha_{ult,k}$, due to in-plane effects alone, is more difficult to define and the expression for global slenderness is not valid (although it has been shown by others[7] to give satisfactory results in some cases). For such situations, it is suggested that the elastic critical force or moment in the critical member is determined from the FE analysis and used directly to determine χ or χ_{LT}. This has been found to be satisfactory by others[7].

7 COMPOSITE MEMBERS

At the composite stage, the only regions of a slab-on-beam composite bridge that will buckle are the lower flanges adjacent to intermediate supports and, possibly, the lower flanges adjacent to the abutments in integral bridges (where there is axial compression).

As discussed in Section 2.5, the form of buckling is then a distortional mode.

Although the manual effective length method can be applied to such regions, treating each main beam as a separate T section, this is known to be conservative.

The equivalent strut method is useful for this situation, since the notional strut, as set out in EN 1993-2, 6.3.4.2, gives a good representation of the behaviour of the lower flanges. It is particularly useful when the bridge is constructed in stages (as is the usual case), as the stresses in the bottom flange are determined by summation and the buckling resistance is determined without the need to consider the stages.

A FE analysis can determine a load factor (for the appropriate buckling mode) for the entire loading applied to a single structure but it will be very time consuming compared to the equivalent strut method (and will probably give a similar answer after much more design time!).

8 BUCKLING RESISTANCE FOR HALF-THROUGH BRIDGES

Certain structures, such as half through plate girder bridges, have a deck at tension flange level and rely on discrete U-frames to restrain the compression flange against buckling. The U-frames are not rigid but offer flexible intermediate restraint to the compression flange: the non-dimensional slenderness will be between that for an unrestrained length over the full main span and that over the length between U-frames, depending on the stiffness of the U-frames.

The form of buckling is a distortional mode.

The equivalent strut method referred to in Section 5.2.2 is the most straightforward and generally the most appropriate method of calculating the non-dimensional slenderness and therefore resistance to buckling. However, calculation of the slenderness needs to take account of the flexibility of the end restraints (it is usually impractical to make them so stiff that they are effectively rigid) and a method for doing so is given in PD 6695-2, Section 9. The effects on the design of the U-frames due to the restraint they provide to the compression flange must also be considered; this can be determined from BS EN 1993-2, 6.3.4.2(5).

The equivalent strut method is conservative in relation to the buckling resistance of the main girders, as it does not take into account the St Venant torsional stiffness of the webs and flanges. However, the lateral force representing the restraint required to the compression flange (as given by clause 6.3.4.2(5)) can be much more onerous than previous rules (in BS 5400-3), resulting in greater design forces on the U-frame connections, and this conservatism can be problematic where there are geometrical constraints due to limited construction depth; a second order FE analysis may be justified in order to minimize the connection design forces.

The complicated and time consuming approach of a second order FE analysis should not be considered lightly, as the benefits sought may not necessarily materialise: Some clients, including Network Rail[5], require an increased value for γ_{Sd} or an increased value for γ_{Rd} when using FE analysis in the design of any structure on their network. (The factor γ_{Sd} is the partial factor taking into account uncertainties in modelling the effect of actions, see BS EN 1990, 6.3.2, and the factor γ_{Rd} is the partial factor taking into account uncertainties in the resistance model, which forms part of $\gamma_{M,}$ see BS EN 1990, 6.3.5.) Network Rail's requirement is understood to be to allow quick structural assessments without the need for complex analysis, e.g. in the event of a bridge strike, and avoid the need to adopt sophisticated analytical methods to aid their on-going management.

9 SECOND ORDER ANALYSIS

The alternative to eigenvalue analysis to derive buckling reduction factors is to account for buckling directly in the analysis. An analysis allowing for the additional buckling deflections is called a second order analysis or a geometrically non-linear analysis. Where the effects of yielding are to be modelled as well, the analysis is geometrically and materially non-linear. For local plate buckling problems, the true resistance will usually be significantly underestimated if material non-linearity is not included as well. In all types of second order analysis, it is important to model all imperfections if the failure load is to be determined directly.

Second order analysis of frames and members is covered in EN 1993-1-1 clause 5.2.1 and imperfections for these are covered in section 5.3. For local buckling, second order analysis and imperfections are covered in Annex C of EN 1993-1-5.

Second order analysis can be a very time-consuming process that offers little advantage in simple flexural and lateral buckling situations, but can offer greater advantage in more complex global buckling modes (such as combined buckling of paired girders) and in some local buckling cases. It may also be advantageous where bridge geometry makes the design rules difficult to apply, such as for curved girders.

No detailed guidance is offered here. A broad description on how such an analysis would be carried out to check the stability of a pair of plate girders is given in Hendy and Jones[8].

10 RECOMMENDATIONS FOR DETERMINING BUCKLING RESISTANCE

10.1 Members in axial compression

For uniform members under uniform axial force, non-dimensional slenderness can be easily calculated and the resistance determined using the appropriate reduction factor for flexural buckling from EN 1993-1-1.

For non-uniform members and non-uniform loading, it may be feasible to calculate a 'lower bound' value of buckling resistance by considering an equivalent uniform member and loading but more accurate values require finite element modelling. Beam elements will usually suffice for this, unless there are local effects that need to be taken into account.

10.2 Members in bending

For all members in bending, the determination of non-dimensional slenderness can be either by means of empirical rules or by use of finite element modelling.

Empirical rules offer a quick solution but they can be quite conservative and become difficult to apply in non-uniform situations.

Finite element modelling, using shell elements, will give a much more accurate prediction of elastic critical buckling loads, but the output from the analysis needs to be interpreted carefully. Local buckling modes may well occur before member buckling modes but if the plate elements have been verified according to the Eurocode criteria (thickness-to-width ratios, etc.) those local buckling modes are not of concern.

10.3 Local buckling of elements

In most situations there is no need to determine local buckling loads for plate elements.

As noted earlier, although local buckling may appear in a FE model, it is not normally a governing criterion. If local buckling modes need to be modelled, then a suitably fine mesh will be needed - at least six elements per half wave length (this can be confirmed by a sensitivity analysis). It is impractical in normal design situations to model shear buckling; design for shear should be based on the rules in EN 1993-1-5.

10.4 Second order analysis

Second order analysis, due to its complexity and the lack of applicability of the principle of superposition for load cases, will seldom be used for the design of common bridge types, with the possible exception of those with complex geometry where it is difficult to apply the Eurocode formulae without adaptation. It may sometimes however be used to retrospectively justify minor non-compliances with the intended design rules or construction requirements.

11 EXAMPLES

The following examples of buckling modes derived from FE analyses were produced by Flint & Neill. SCI is grateful to Flint & Neill for permission to reproduce these images and the results of the buckling analysis for a model of the bridge example in P357.

11.1 Flexural buckling

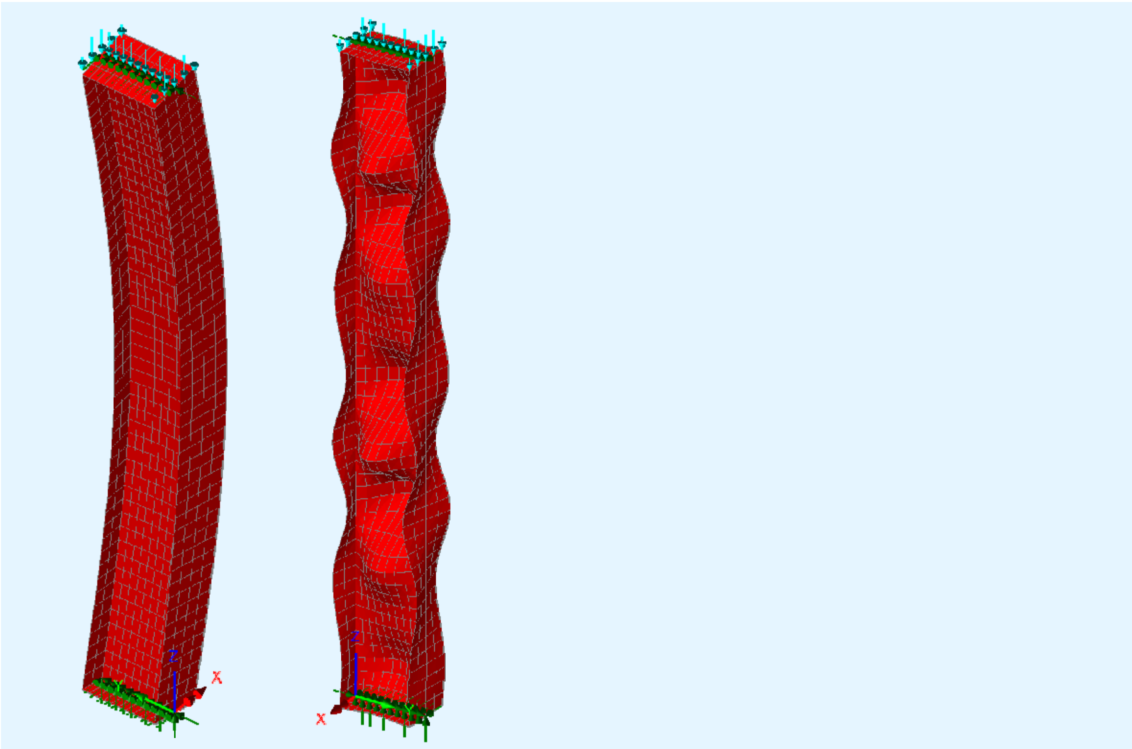


Figure 11.1 Compression strut model showing overall flexural buckling and a higher mode showing local buckling of flanges and web

11.2 Lateral torsional buckling

The example below shows a model of paired girders, at the wet concrete stage, based on Example 1 in SCI publication P357. The loading was applied at top flange level and the buckling mode shown was the lowest global mode (although there were numerous lower eigenvalues, all related to local buckling of web panels).

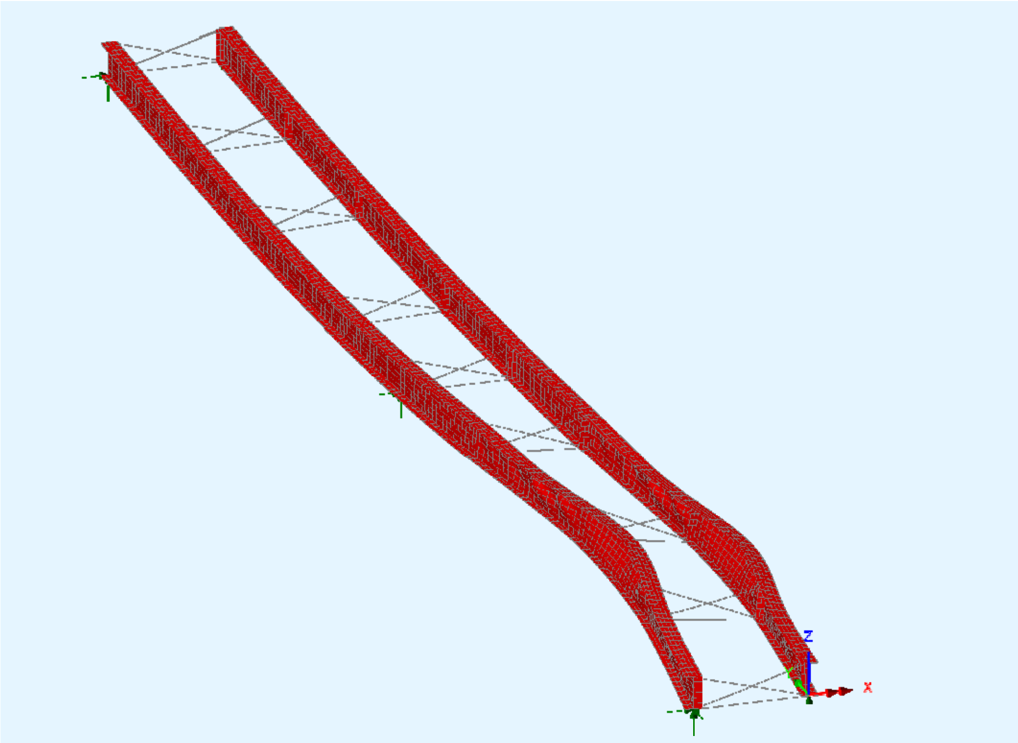


Figure 11.2 Buckling of paired beams in Example 1 of SCI P357

The load factor from the buckling analysis was 6.38, leading to a non-dimensional slenderness of 0.636. This may be compared with the non-dimensional slenderness derived in P357, using the rules from PD 6695-2, which was 0.89, based on values of $1/\sqrt{C_1}$ = 1.0 (appropriate for uniform bending moment – with a udl and with hogging at one end, a value of 0.91 could have been justified) and D = 1.2 (for the destabilizing effect of load applied on the top flange).

11.3 Distortional buckling in a half-through bridge

The example below shows the buckling mode for a half-through bridge. More details of this example can be found in Hendy and Jones[8].

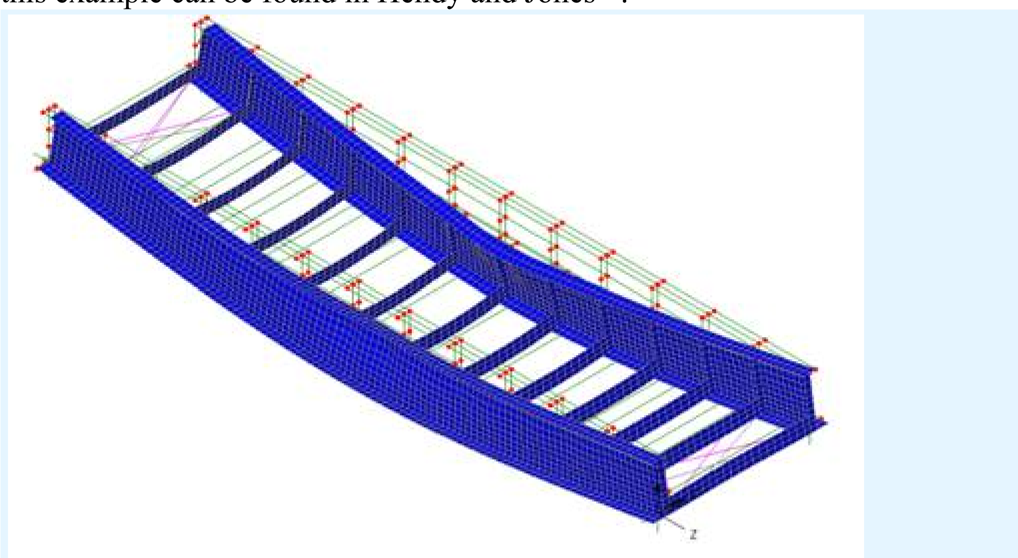


Figure 11.3 Buckling of girders in a half through bridge

11.4 Local buckling

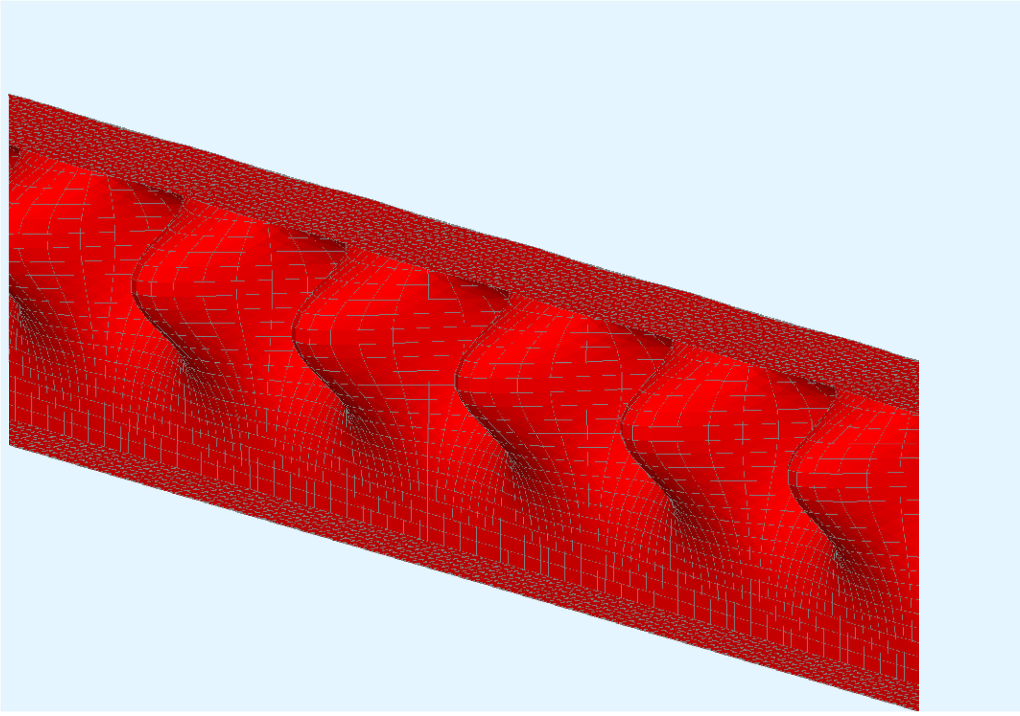


Figure 11.4 Local buckling of web in bending, shown with a fine FE mesh

12 REFERENCES

1 BS EN BS EN 1993-1-1:2005. *Eurocode 3: Design of steel structures General rules and rules for buildings*. BSI.

BS EN 1993-1-5:2006. Eurocode 3: *Design of steel structures Plated structural elements*. BSI.

BS EN 1993-2:2005. Eurocode *3: Design of steel structures. Steel bridges*. BSI.

(*Each with its UK National Annex*)

2 Hendy, C.R. and Murphy, C.J., *Designers' guide to EN 1993-2 Eurocode 3: design of steel structures. Part 2 Steel bridges*, Thomas Telford, 2007.

3 Bulson, P.S.,*The stability of flat plates*, Chatto and Windus, 1970.

4 PD 6695-2:2008. *Recommendations for the design of bridges to BS EN 1993*. BSI.

5 NR/L2/CIV/002/F1993, Technical requirements for BS EN 1993 and NR/L2/CIV/002/F1994, Technical requirements for BS EN 1993, Network Rail

6 Hendy, C.R. and Murphy, C.J., private communication.

7 Baird D, Hendy C, Wong P, Jones R, Sollis A, Nuttall H, *Design of the Olympic Park Bridges H01 and L01*, IABSE Structural Engineering International, February 2011.

8 Hendy, C.R. and Jones, R.P., *Lateral buckling of plate girders with flexible restraints*. Paper 800027, Bridge Engineering 162, Proc. Inst. Civil Engineers, 2009.

See also

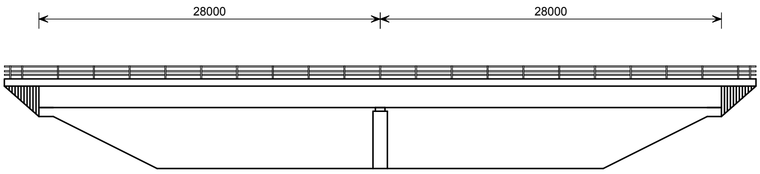
Timoshenko, S.P. and Gere, J.M., *Theory of elastic stability*, McGraw-Hill, 1961.

APPENDIX A. WORKED EXAMPLE

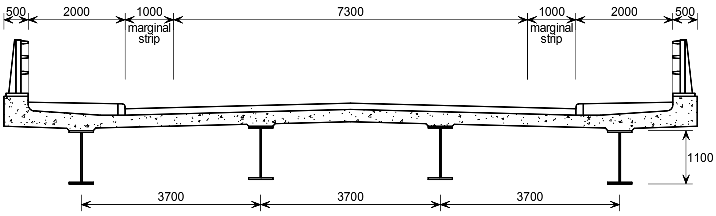
The following example illustrates the interaction between major and minor axis bending and warping torsion during the construction stage of the multi-girder bridge example in publication P357.

A.1 Geometry and analysis model

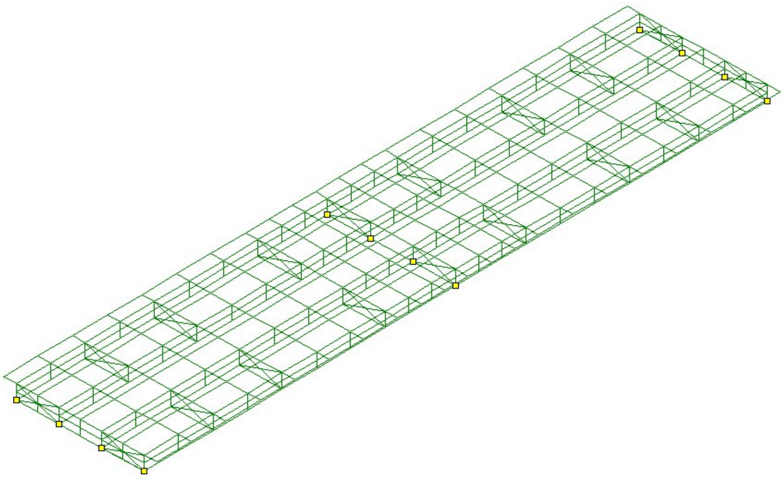
The structural arrangement given in P357 is shown below.



Elevation



Cross section



FE model, showing disposition of bracing

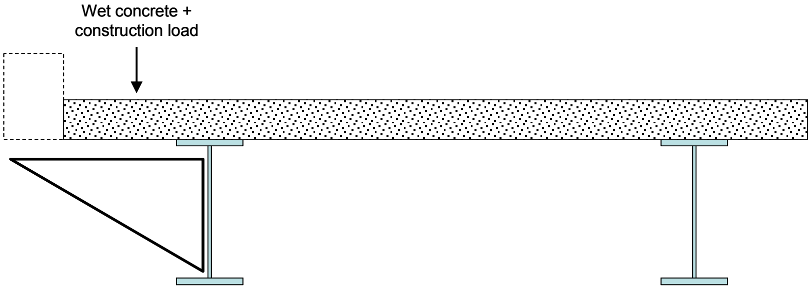
A.2 Design values of effects

In the construction stage (Stage 1 in P357), the weight of wet concrete, including the cantilevers but not the edge beams, is carried by the bare steelwork. The vertical bending moments and shears in the main girders (ULS values) are given by the analysis that was carried out for P357. The values of vertical bending moment and shear for the inner girder are given in P357 are presented below, together with values for the outer girders.

Distance from pier (m)	Inner girder		Outer girder	
	M_y (kNm)	F_z (kN)	M_y (kNm)	F_z (kN)
0	–2573	689	–2068	538
6.3	1024	415	738	325
15.6	3132	43	2381	28
28	25	–521	23	–404

The effects of supporting cantilever formwork from the outer girder (i.e. the horizontal forces that applies to the outer girder) were not modelled in the analysis, nor was the effect of wind loads during construction. Both actions lead to in-plane bending of the flanges of the beams.

A.2.1 Actions on cantilever



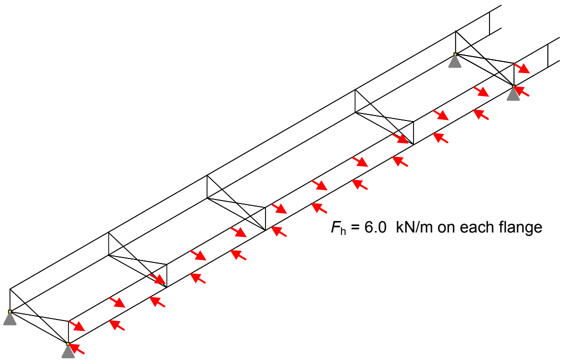
The design value of the load due to wet concrete and construction load is 10.5 kN/m^2 and the cantilever width is 1.10 m. Hence the resultant torque on the outer girder is:

$M \quad = 10.5 \times 1.10^2/2 = 6.35$ kNm/m

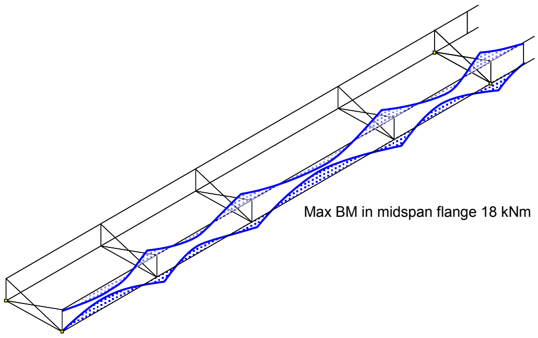
The horizontal force on each flange is thus:

$F_h \quad = 6.35/1.06 = 6.0$ kN/m

The application of this force is illustrated diagrammatically below. The flanges bend in plan as a continuous beam, for which the bending moment diagram is as shown. The maximum value of bending moment in the midspan region is approximately 18 kNm.



Horizontal forces from cantilever falsework



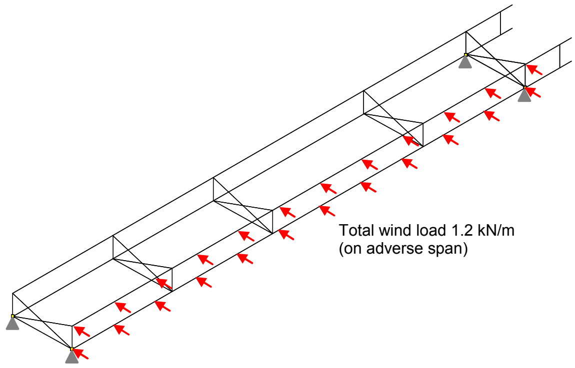
Bending moments in flanges due to cantilever loads

A.2.2 Wind actions

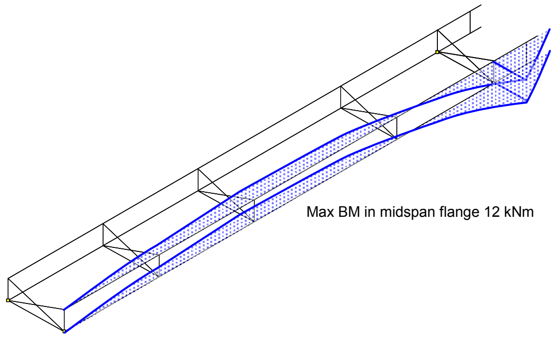
The wind load during construction depends on the location, the geometry of the structure and the return period appropriate to the construction period.

In this case, for a duration of this construction stage up to 3 months, the recommended return period is 5 years (see BS EN 1991-1-6); the basic wind speed (including adjustment for altitude) is 25 m/s; no seasonal or direction reduction is made; the pressure coefficient is 2.0; and the overall depth is 1.4 m. Treating wind as an accompanying action this leads to a total adverse wind load of 1.2 kN/m and a favourable wind load (on the other span) of 0.33 kN/m.

Considering the structure as a series of four uniform 2-span beams, bending in plan, and sharing the bending equally between all 8 flanges (the bracing and control bracing achieve this) the maximum bending moment in a flange at midspan is approximately 12 kNm.



Horizontal forces due to wind



(BMs shown only on outer girder)
Bending moments in flanges due to wind

A.3 Verification of design resistance

The interaction of lateral bending with vertical bending, and the verification of resistance to buckling of the top flange of the outer girder, is considered below.

The interaction is verified using the criterion in Section 6.2.2 of this document.

$$\frac{N_{Ed}}{\chi N_{Rd}} + \frac{1}{1-(N_{Ed}/N_{cr,z})}\frac{M_{z,Ed}}{M_{z,Rd}} \leq 1.0$$

The values of the parameters in this expression are as follows:

A.3.1 Compression force

Maximum moment in midspan = 2381 kNm

W_y = 22.87 x 10^6 mm^3 (from P357, page 18)

A_e = Area of flange plus 1/3 web below the centroid
= 20000 + 510 x 10/3 = 21700 mm^2

N_{Ed} = 2381 x 10^6 x 21700/22.87 x 10^6 = 2260 kN

A.3.2 Lateral bending moment

$M_{z,Ed}$ = 12 +18 = 30 kNm

A.3.3 Design values of resistances

N_{Rd} = 21700 x 345 /1.1 = 6810 kN

W_z = 500^2 x 40/6 = 1670 x 10^3 mm^3

$M_{z,Rd}$ = 1670 x 10^3 x 345 /1.1 = 523 kNm

A.3.4 Buckling parameters

χ = χ_{LT} = 0.525 (P357, page 35)

M_{cr} = $W_y f_y / \bar{\lambda}^2$ = 22.87 x 10^6 x 345 / 0.89^2 = 9960 kNm ($\bar{\lambda}$ from P357, page 34)

$N_{cr,z}$ = $M_{cr} \times A_e / W_y$ = 9960 x 21700 / 22.87 x 10^6 = 9450 kN

A.3.5 Verification of resistance to combined effects

$$\frac{N_{Ed}}{\chi N_{Rd}} + \frac{1}{1-(N_{Ed}/N_{cr,z})}\frac{M_{z,Ed}}{M_{z,Rd}} \leq 1.0$$

Using the above design values:

$$\frac{2260}{0.525 \times 6810} + \frac{1}{1-(2260/9450)}\frac{30}{523} = 0.63 + 1.31 \times 0.06 = 0.71 \leq 1.0$$

The resistance of the outer girder to the combined effects is satisfactory.