Domain: Multiplication/Geometry
Distributive Property
Grade: 3
Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

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Revised 2019

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Mathematical Goals

This lesson is intended to help you assess how well students are able to:

- Compose and decompose models to show the distributive property.
- Use the distributive property to split a factor into benchmark numbers.
- Use area models, arrays, and equal group models to represent and solve multiplication problems.
- Justify their choice of how to apply the distributive property.

Kentucky Academic Standards

This lesson involves mathematical content in the standards within the grade, with emphasis on:

Grade 3 Operations and Algebraic Thinking

Cluster: Understand properties of multiplication and the relationship between multiplication and division.

KY.3.OA.5 Apply properties of operations as strategies to multiply and divide.

Grade 3 Measurement and Data

Cluster: Geometric measurement: understanding concepts of area and relate area to multiplication and to addition.

KY.3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it and show the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a x c. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of figures that can be decomposed into non-overlapping rectangles by adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

This lesson involves a range of Standards for Mathematical Practice, with emphasis on:

MP1. Make sense of problems and persevere in solving them.
MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Overall

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review/analyze their responses and create questions for students to consider/answer in order to improve their solutions.
- After a whole class introduction, students work in partners collaboratively on a card matching activity.
- Students work with a partner on collaborative discussion tasks. Throughout their work, students justify and explain their decisions to their peers.
- Toward the end of the lesson there is a whole class discussion.
- Students return to a similar task and try to improve their responses.

Materials Required

Each individual student will need:

- Pre-assessment
- Post assessment
- Dry Erase board/marker

Each partner group of students will need the following resources:

- Card set A
- Card set B ready, but do not pass out
- Card set C ready, but do not pass out
- Card set D ready, but do not pass out
- Card set E ready, but do not pass out
- Record Sheet for Extension activity ready, but do not pass out until a group works through all other sets successfully.
- Crayons, optional

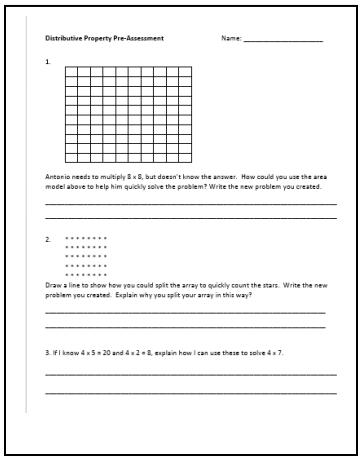
Time Needed

Approximately 15 minutes a day or two before the lesson for the individual assessment task, one 45 minute lesson, and then 25 minutes for a follow-up lesson for students to revisit individual assessment task. Timings given are approximated. All students may not complete all sets of cards activities and/or groups may only complete a partial section of each set. Exact timings will depend on the needs of the class.

Before the Lesson

Students should have knowledge on how to create multiplication models- area models, arrays, and equal groups (this would include physical activities such as building with tiles as well as drawing representations). Students should have been exposed to splitting the area models to solve multiplication problems using the distributive property. Students should be able to determine two addends that can replace a factor. Students should recognize and choose benchmark numbers to solve the problem the most efficient way. The assessment focuses on several of the *Standards for Mathematical Practice,* with emphasis on: MP.3. Construct viable arguments and critique the reasoning of others; MP.4. Model with mathematics; and MP.7. Look for and make use of structure.

Assessment Task: Distributive Property Pre-Assessment (15 minutes)


Distributive Property Pre-Assessment Name: _____

1.

Antonio needs to multiply 8 x 8, but doesn't know the answer. How could you use the area model above to help him quickly solve the problem? Write the new problem you created.

_____

_____

2.

Draw a line to show how you could split the array to quickly count the stars. Write the new problem you created. Explain why you split your array in this way?

_____

_____

3. If I know 4 x 5 = 20 and 4 x 2 = 8, explain how I can use these to solve 4 x 7.

_____

_____

Have students do this task individually in class a day or two before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will be able to target your help more effectively in the follow-up lesson. Depending on your class you can have them do it all at once or in small groups (they should still work individually.)

Give each student a copy of the assessment task *Distributive Property Pre-Assessment.* A sample dialogue:

> *"Today we are going to work on a task involving the distributive property. This task is to help me see ways that I can help you if you are having any problems with splitting area models and arrays to solve multiplication problems. If you are not sure about all of your answers, it is okay. We are going to do an activity that will help you improve. You will follow the directions for each question. Some ask you to draw and some ask you to explain. Some may even have more than one step."*

It is important that the students are allowed to answer the questions without your assistance. If students struggle to get started ask questions that help them understand what they are being asked to do, but do not do the problem for them. See the *Common Misconceptions* table.

Students should not worry if they do not understand or cannot do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently.

Assessing Students' Responses

Collect students' responses to the task. Make notes about what their work reveals about their current levels of understanding and their different problem solving approaches. Partner students with others who displayed similar errors/misconceptions on the pre-assessment task. For this activity, the orientation of the model does not matter (Commutative Property) and should not be considered as an error. On question #6 students may choose either answer, but it must be efficient and they must justify their reasoning.

We suggest that you do not score student's work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions in the *Common Misconceptions* table may serve as examples. These questions have been drawn from commonly identified student misconceptions.

We recommend you either:

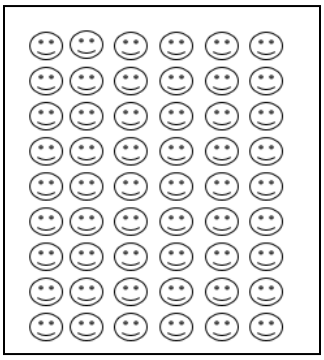
- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student or
- display a small list of questions on the board that will be of help to the majority of students or
- provide a few questions for each pair to answer together based on their misconceptions and need

Below is a list of common misconceptions and questions/prompts that may be written on individual tasks, on the board or asked during the collaborative activity to help students clarify and extend their thinking.

Common Misconceptions:	**Suggested Questions and Prompts:**
Student has trouble getting started.	• What information do you already know? • What do you need to find out?
Student has trouble using model correctly. **Student has trouble drawing and counting area models and arrays.**	• How can you use the tool to show your work? • How many rows and columns do you see? • How can you use the rows and columns to help you calculate the total? • What multiplication problem does the model show?
Student cannot identify the equation that matches a given model.	• What is the multiplication problem for each section of the area model or array?
Student does not choose two addends that add up to the factor.	• What two numbers can you add to make this factor? • What is the sum of your addends? • Does the sum match the factor?
Student splits both factors instead of just one.	• Shade the model to show your answer. Did you shade in the entire model?
Student does not use the benchmark numbers to split the factor.	• Why did you choose to split the factor in this way? • Is there a more efficient way to split the factor?
Student does not correctly write the equation that matches the model.	• Show me where the numbers are represented in the area model. • What happens inside the parenthesis? • How can you use the two area models to find the whole?
Student does not recognize how the equal groups have been split. (Card Sets C and D)	• What pattern do you recognize in the groups? What problem can help you count the marbles? • How are the groups different? • How many groups of striped marbles? How many groups of black marbles?
	•
	•

Suggested Lesson Outline

Whole Class Introduction (10 minutes)



To begin the activity display the 6 x 9 array of happy faces from page 12 to students.

Pass out dry erase boards/markers to each student. Ask students:

> *What do you notice about this display?*
> *Record possible ways to split the model using the distributive property on your dry erase boards.*

If students answer quickly, encourage them to come up with other ways to solve the same problem. After a few minutes allow students to compare and share their answers with a partner. Be sure students explain how they arrived at their choice.

Have students physically show how they split the array using different strategies on the displayed happy faces for all to see. You may chart student thinking quickly. Then discuss which may be valid and which strategy may be the most efficient method and why. It's important to not RETEACH content at this time, allow students to share and think and have non-judgmental responses to student answers. The collaborative activity to follow is intended to clarify misconceptions for students.

Collaborative Activity: (30 minutes)

Strategically partner students based on pre assessment data. Partner students with others who display similar errors/misconceptions on the preassessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

Explain to students how they are to work collaboratively. Sample dialogue:

> *"You are now going to work together, taking turns to find cards that match.*
> *Each time you find a match, justify your reasoning to your partner. If your partner disagrees with your placement, then challenge him or her to explain why. It is important that you both understand why each card is placed where it is.*
>
> *There is a lot of work to do today and you may not all finish. The important thing is to learn something new, so take your time."*

The purpose of this structured work is to encourage students to engage with each other's explanations and take responsibility for each other's understanding.

Make a note of student approaches to the task

Your task during the partner work is to listen and watch students work - make notes of student approaches to the task and ways you can support student problem solving. As you monitor the work, listen to the discussion and ask questions to help students understand concepts and clarify misconceptions. Make a note of the sequence in which you may want students to present their findings to reveal the learning objectives. Use the question chart on page 5.

You use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, as students complete Card Set A they may not understand how to combine the area models to create the larger model to represent the multiplication problem. For Card Set B, students may not be able to match the pictorial and symbolic representations for the distributive property. For Card Set C, students may not recognize the pattern within each group and how it can help them count efficiently. For Card Set D, students may not recognize the difference between the groups, and how this split represents the problem. For Card Set E, students may not choose benchmark numbers to split their area models after matching them. A lack of understanding in any of these

representations may reveal the struggle of students in using the distributive property as a strategy for solving multiplication problems.

Support Student Problem Solving

Try not to make suggestions that move students toward a particular approach to the task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning. Encourage students to explain their reasoning carefully. It is important that students feel comfortable talking with and critiquing each other. This lesson involves a range of *Standards for Mathematical Practice,* with emphasis on: MP.1. Make sense of problems and persevere in solving them; MP.3. Construct viable arguments and critique the reasoning of others; MP.4. Model with mathematics; and MP.7. Look for and make use of structure.

If one student has placed a particular card on the chart, challenge their partner to provide an explanation.

If students have difficulty articulating their decisions, then you may want to use the questions from the *Common Misconceptions* table to support your questioning.

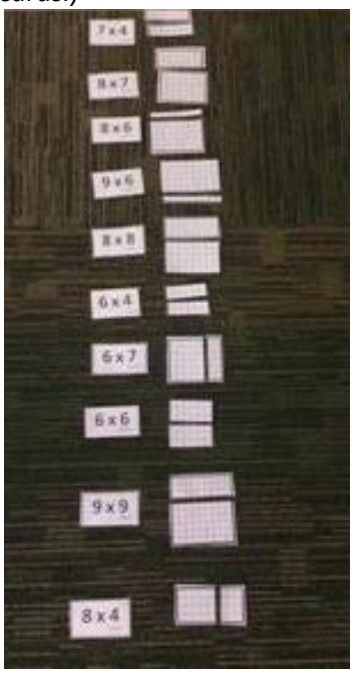
If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and engineer a quick whole class discussion. Only use this if most partners are frustrated. This may be a sign students are not ready for the formative assessment lesson.

*** Each card Set A-D should be cut and placed into a separate envelope, baggie, etc. for each group. Each group will need one sheet for Set E.
*** Students will keep each set of cards displayed and continue building matches. (Be careful not to confuse Sets A and E.)

Give each group Card Set A.

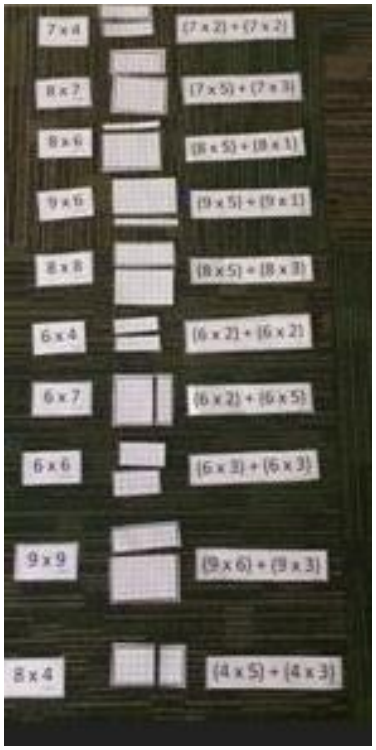
Card Set A

For Card Set A, students will be given cards with multiplication problems and area models. Students will then find two area model cards to compose the whole area model that matches the multiplication problem. *(Teacher Note: Be sure to cut the area models apart as you prepare the cards.)*



Card Set B

For Card Set B, students will be given cards that contain the original multiplication problem written as a distributive property expression to match the original model the students composed in Card Set A.



Card Set C

For Card Set C, students will be given cards with pictures of equal groups of marbles. Students should recognize the pattern within the groups *(dice pattern)* and use this as the way to split the marbles. This can help them count efficiently, but it may be tricky for students to notice the pattern at first. If students are struggling, ask questions from misconceptions chart. Card Set C will match one of the distributive property cards from Set B.

Card Set D

For Card Set D, students will be given cards with pictures of equal groups of marbles. Students should recognize the difference between the groups, and use this color difference to split the groups. Each card will match one of the cards from Set B.

Card Set E

For Card Set E, students will be given a sheet with blank area models. They cut out each area model. *(Teacher can do this ahead of time.)* Students match the model to the original multiplication problem from Set A. Then students should draw a line(s) to split their model into easier pieces to calculate. Students should be able to justify their reasoning. Tell students there may be more than one correct answer. Ask students to turn the shape over, write the distributive property expression they created, and then solve. *(May match the original split or be a new problem. Students may color the two sections with different colors if they wish.)*

Taking two class periods to complete all activities

If you have to divide the lesson into two class periods, you may want to have a way for students to save the work they have done with the place card sets. Select one group to tape their display to use as a whole group introduction to day two.

Sharing Work (10 minutes)

As students complete their work, allow partners to compare their matches to other groups. Students are permitted to ask questions and make changes to their original work. Each partner group does not have to complete all of the card matches.

> *If you are staying at your desk, be ready to explain the reasons for your group's work. If you are visiting another group, check to see which answers or explanations are different from your own.*
>
> *If their matches are different than your groups, ask for an explanation. If you don't agree, explain your own thinking. When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.*

Provide time for groups to discuss and make changes to their own work.

Extension Activity

Each group will need one record sheet. The record sheet will consist of three columns. The first one will be the multiplication problem, the second column will be the area model, and the third will be the distributive property equation. Students will fill in the missing boxes inside the three columns. *(Some may be missing the problem, model, or equation. The main focus of this activity is to have students use the benchmark numbers to split the factor and then defend that strategy with their partner.)*

Whole-Class Discussion (10 minutes)

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions students came up with. Allow partners to bring up some of their work samples and share their thinking. The purpose of this discussion is to explore the processes involved in a range of different approaches. The aim is to get students to understand and share their reasoning, not just checking that everyone found the correct matches. This lesson involves a range of *Standards for Mathematical Practice,* with emphasis on: MP.3. Construct viable arguments and critique the reasoning of others; and MP.7. Look for and make use of structure.

You may want to hold up several examples of possible matches and ask students to explain their thinking.

Ask students during the discussion:

1. *How did you know which two cards to put together to make the problem?*
2. *How did you match the distributive property problem to the model?*
3. *How did you match the equal groups picture to the problems and models?*
4. *Why did you choose to partition your model in that way?*
5. *Did everyone partition their model the same way?*
6. *Why do you think so many problems were split using 5's?*
7. *Did anyone have a problem they struggled with?*
8. *Which cards did you find easy to match?*
9. *Which cards did you find difficult to match?*

Extension activity questions:

1. *How did you partition your model and what equation did you create to match your model?*
2. *Does anyone else have a different way? (show as many ways as possible)*
3. *Which way is the most efficient and why?*

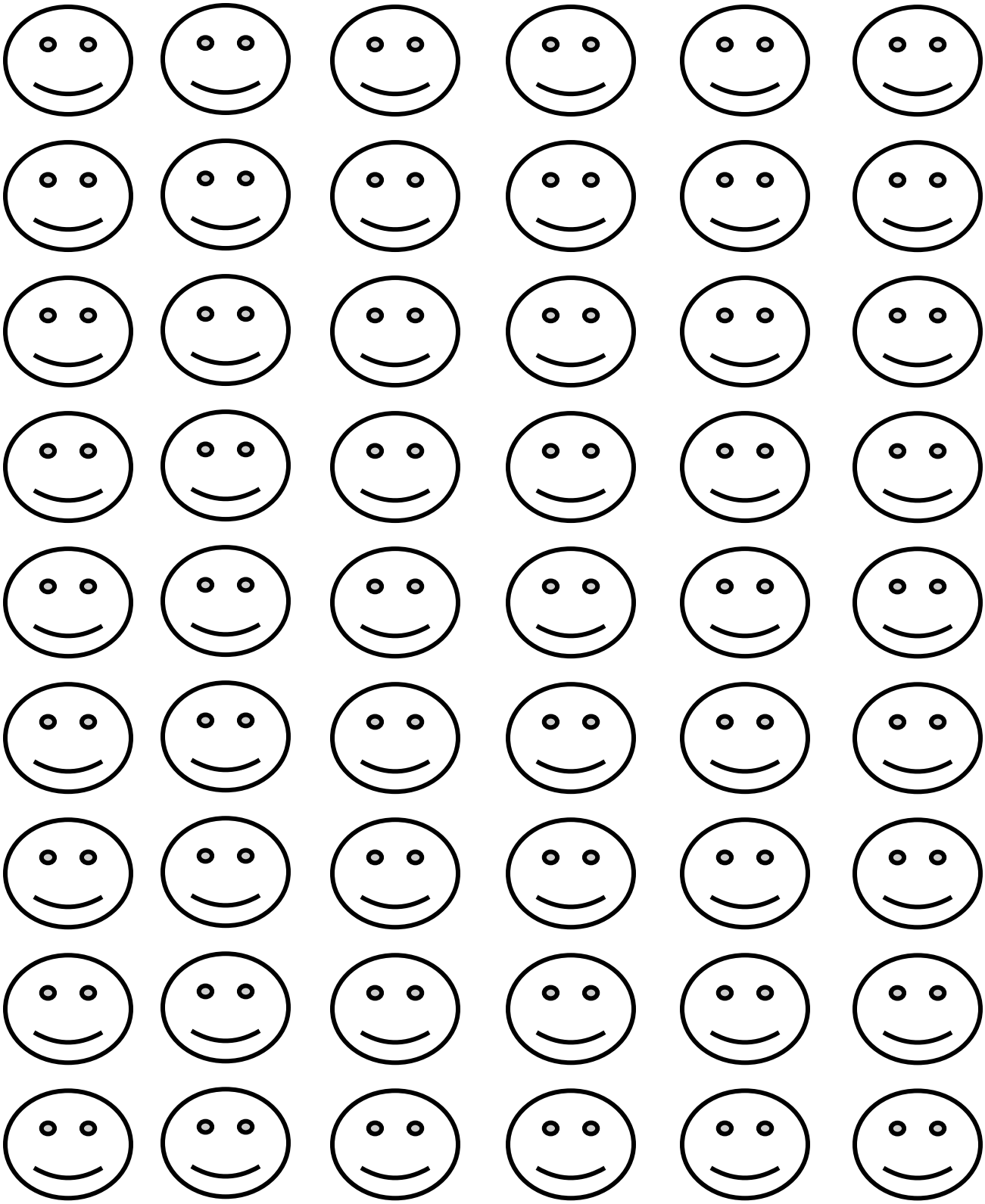
Improving Individual Solutions to the Assessment Task (20 minutes)

Give the students the Distributive Property Post Assessment and their Distributive Property Pre-assessment,

Think about what you have learned during this lesson. Using what you have learned try to improve your work.

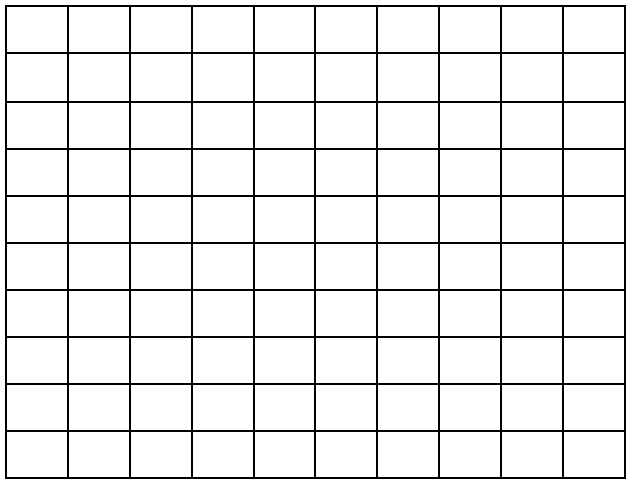
To focus your students, refer to the common misconceptions chart. Use the questions which reflect the greatest need(s) of your students.

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.



Distributive Property Pre-Assessment Name: _____

1.



Antonio needs to multiply 8 x 8, but doesn't know the answer. How could you use the area model above to help him quickly solve the problem? Write the new problem you created.

_____

_____

2.

* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *

Draw a line to show how you could split the array to quickly count the stars. Write the new problem you created. Explain why you split your array in this way?

_____

_____

3. If I know 4 x 5 = 20 and 4 x 2 = 8, explain how I can use these to solve 4 x 7.

_____

_____

4. If I want to solve 6 x 9, which of the following will NOT help me. Why not?

(6 x 4) + (6 x 5)

(6 x 6) + (6 x 3)

(6 x 4) + (6 x 4)

(6 x 7) + (6 x 2)

_____

_____

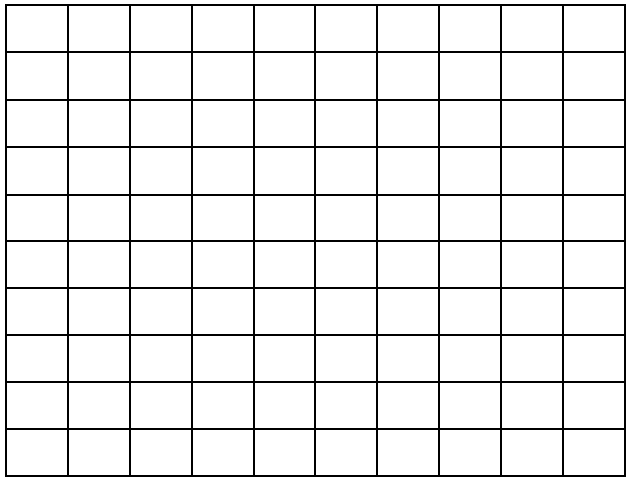
5. In the equation 3 x 9 = (3 x 5) + (3 x w), what should the **w** equal? Explain how you found your answer.

_____

_____

Distributive Property Post-Assessment Name: _____

1.



Antonio needs to multiply 4 x 9, but doesn't know the answer. How could you use the area model above to help him quickly solve the problem? Write the new problem you created.

_____

_____

2. * * * * * * *
   * * * * * * *
   * * * * * * *
   * * * * * * *
   * * * * * * *

Draw a line to show how you could split the array to quickly count the stars. Write the new problem you created. Explain why you split your array in this way?

_____

_____

3. If I know 4 x 5 = 20 and 4 x 3 = 12, explain how I can use these to solve 4 x 8.

_____

_____

4. If I want to solve 5 x 8, which of the following will NOT help me. Why not?

(5 x 4) + (5 x 5)

(5 x 6) + (5 x 2)

(5 x 4) + (5 x 4)

(5 x 7) + (5 x 1)

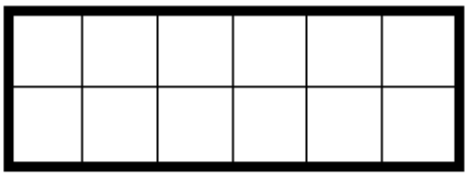
_____

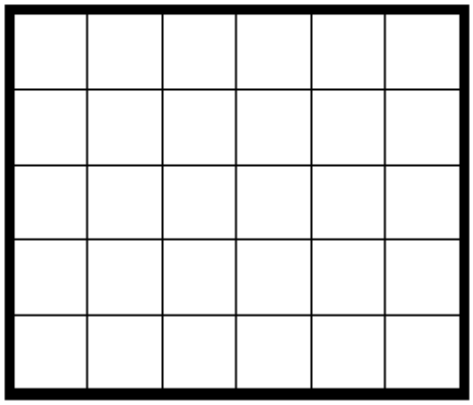
_____

5. In the equation 7 x 8 = (7 x k) + (7 x 3), what should the **k** equal? Explain how you found your answer.

_____

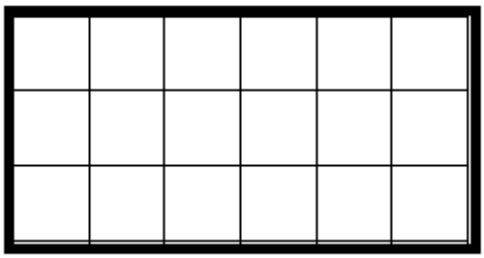
_____

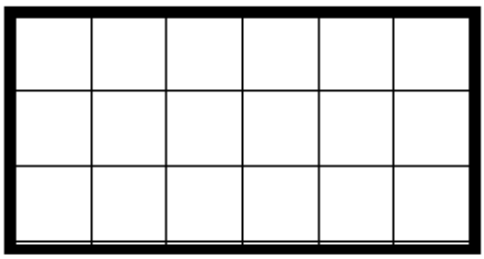




6 x 7

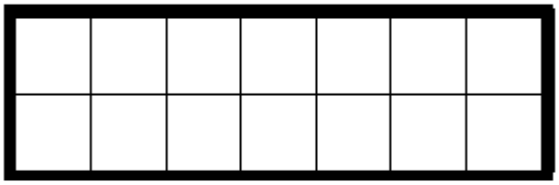
Card Set A

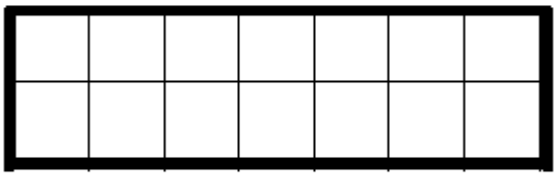




6 x 6

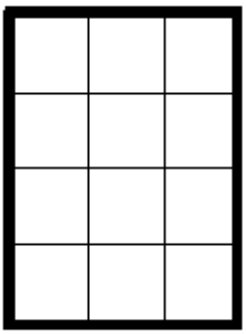
Card Set A

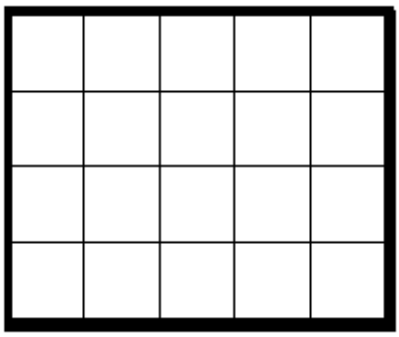




7 x 4

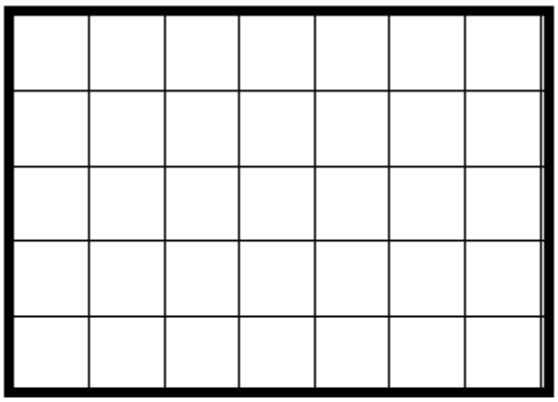
Card Set A

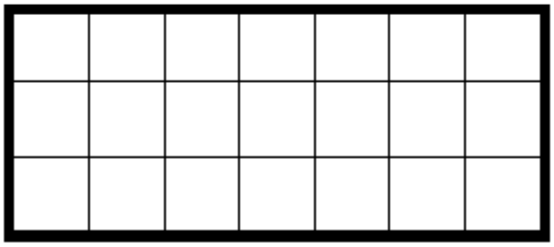




8 x 4

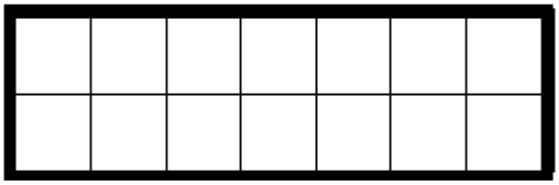
Card Set A

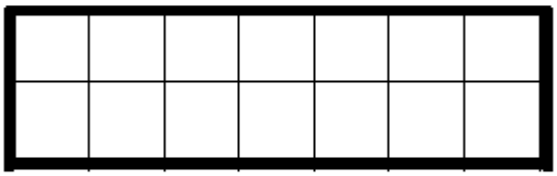




8 x 7

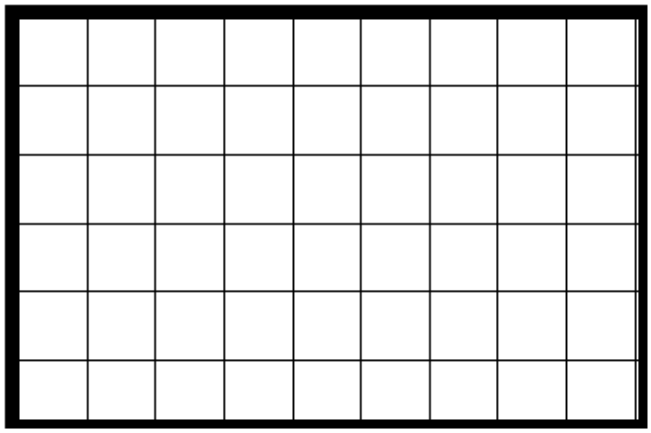
Card Set A

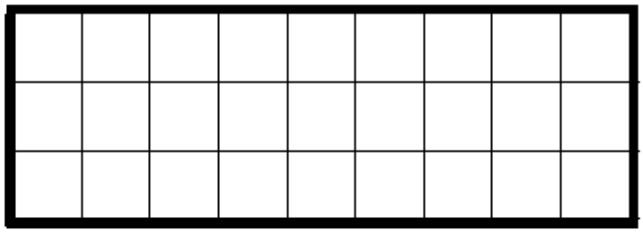




7 x 4

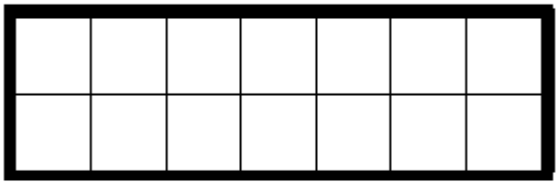
Card Set A

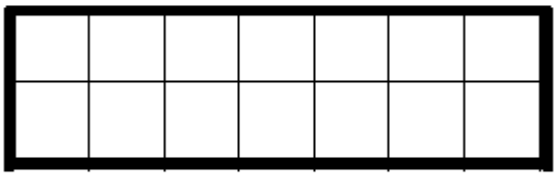




9 x 9

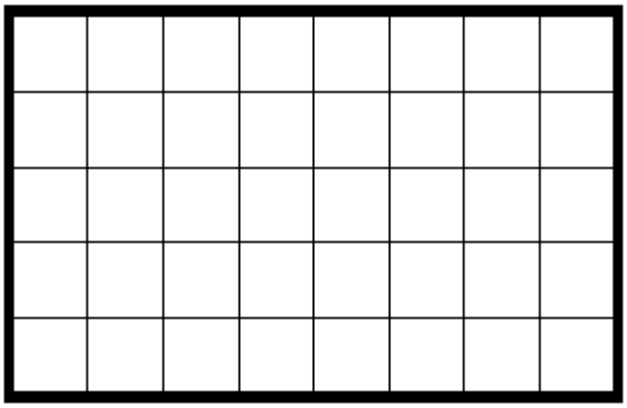
Card Set A

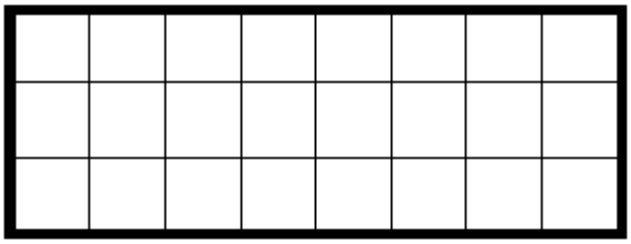




7 x 4

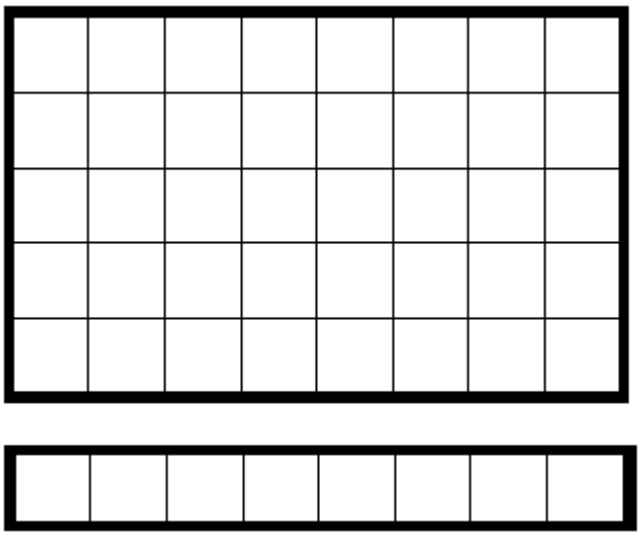
Card Set A





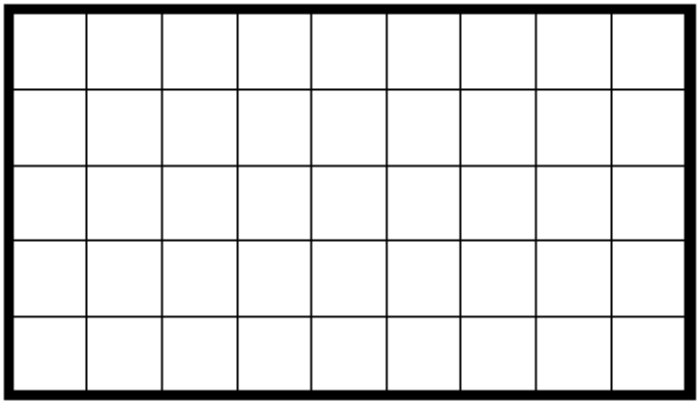
8 x 8

Card Set A



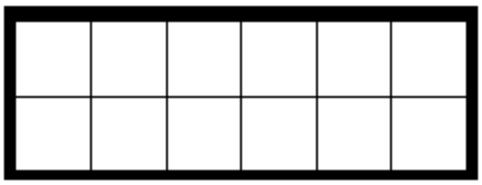
8 x 6

Card Set A



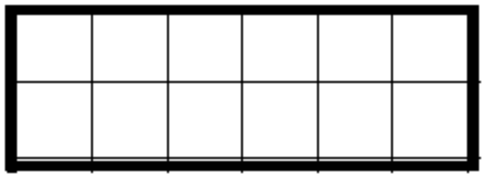
9 x 6

Card Set A



6 x 4

Card Set A



$(6 \times 2) + (6 \times 5)$

Card Set B

$(6 \times 3) + (6 \times 3)$

Card Set B

$(7 \times 2) + (7 \times 2)$

Card Set B

$(6 \times 2) + (6 \times 2)$

Card Set B

$(7 \times 5) + (7 \times 3)$

Card Set B

$(6 \times 2) + (6 \times 5)$

Card Set B

$(9 \times 6) + (9 \times 3)$

Card Set B

$(8 \times 5) + (8 \times 3)$

Card Set B

$(8 \times 5) + (8 \times 1)$

Card Set B

$(9 \times 5) + (9 \times 1)$

Card Set B

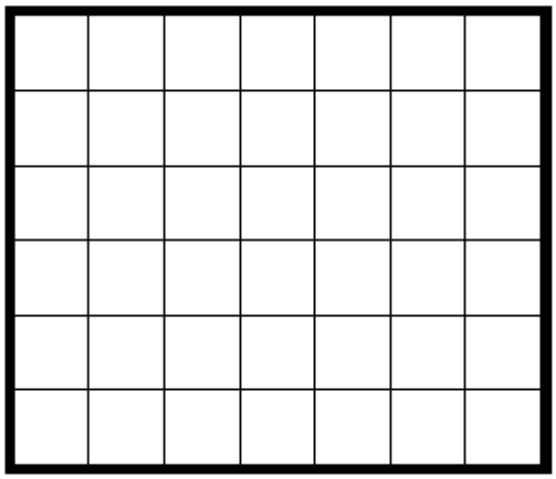
$$(6 \times 2) + (6 \times 5)$$

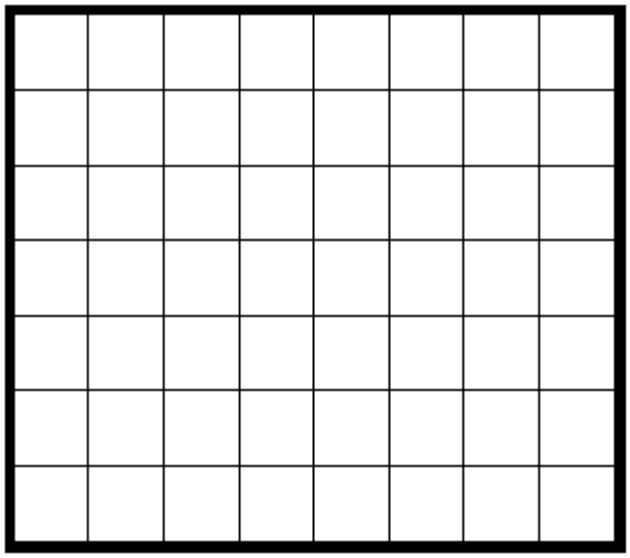
Card Set B

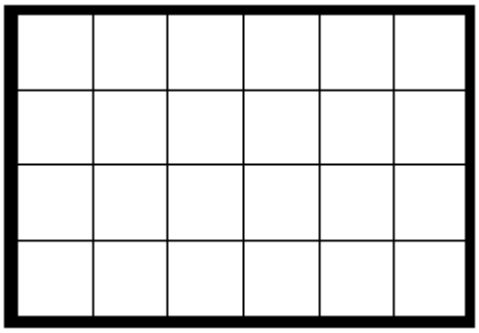
$$(4 \times 5) + (4 \times 3)$$

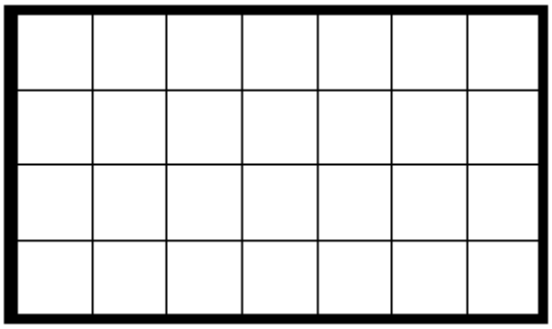
Card Set B

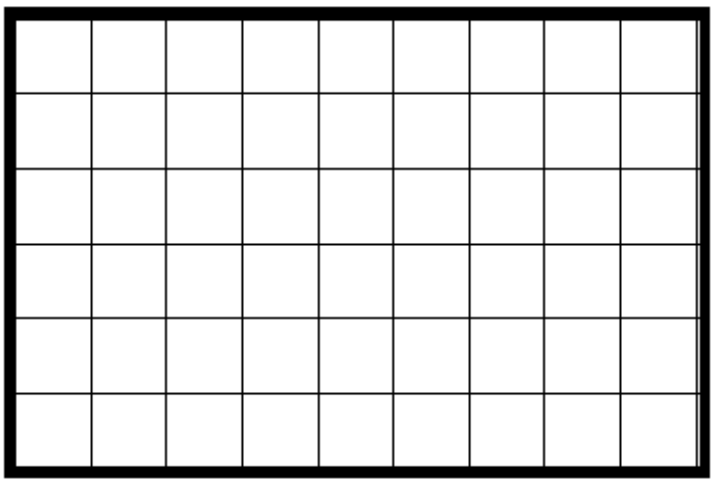
Card Set E

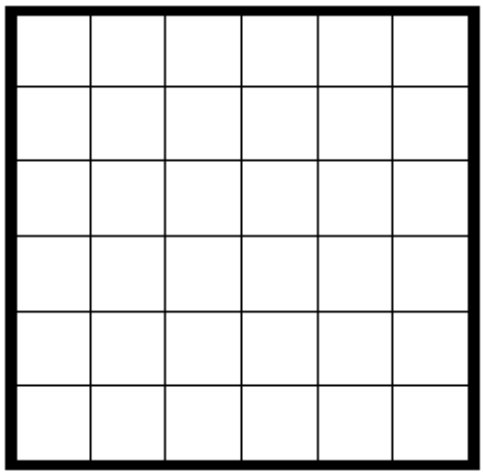




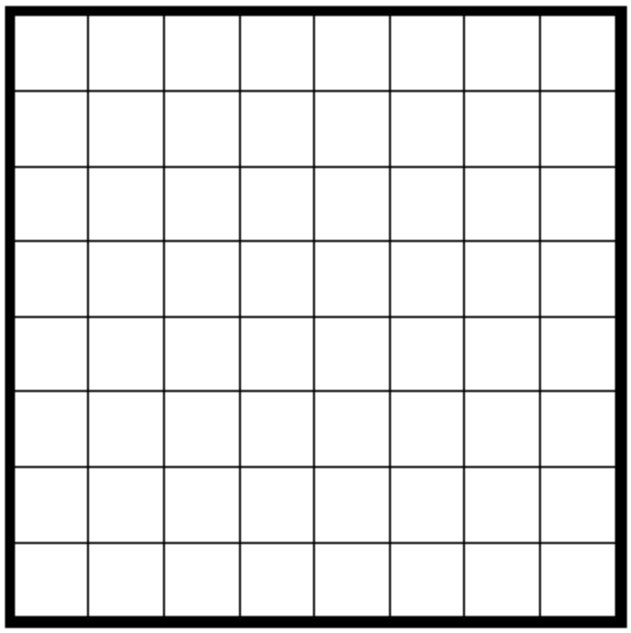


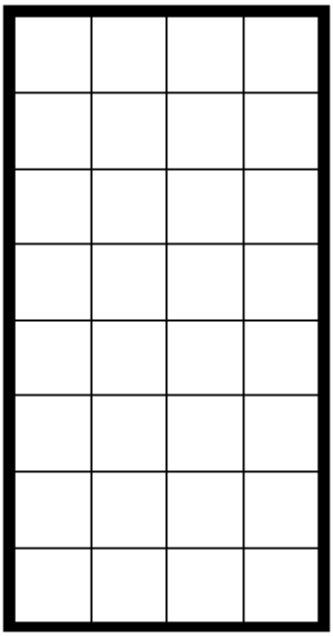


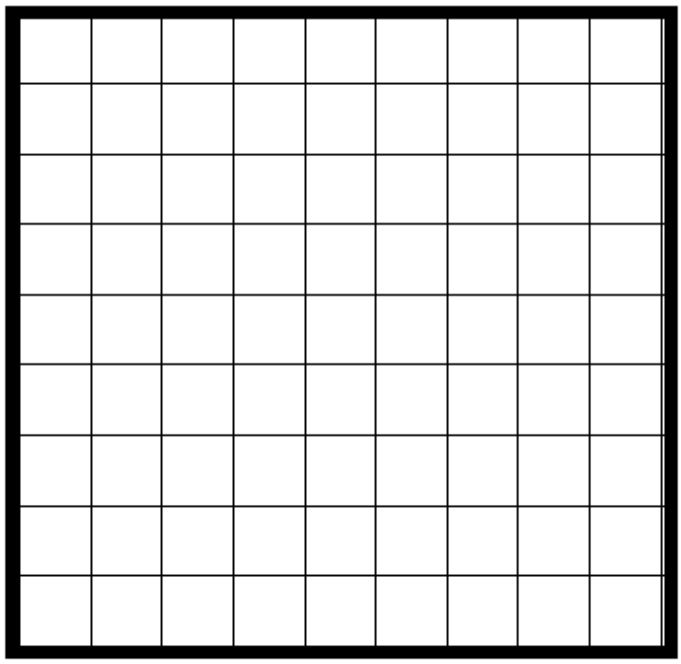


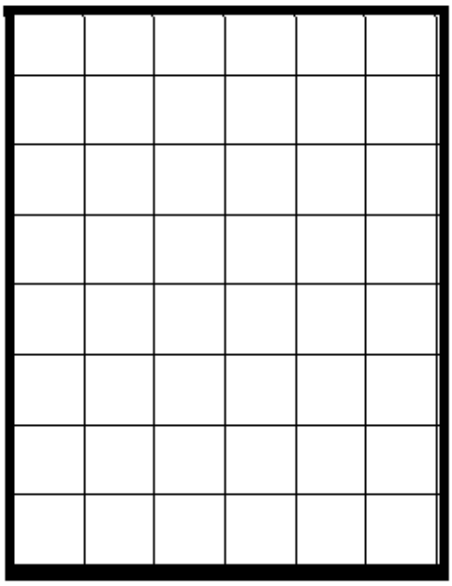


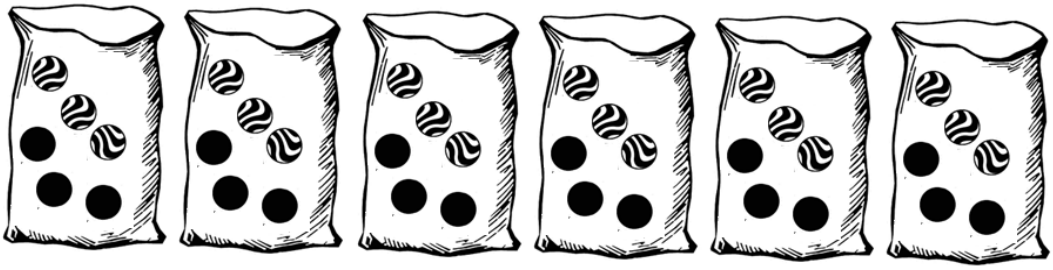
Card Set E











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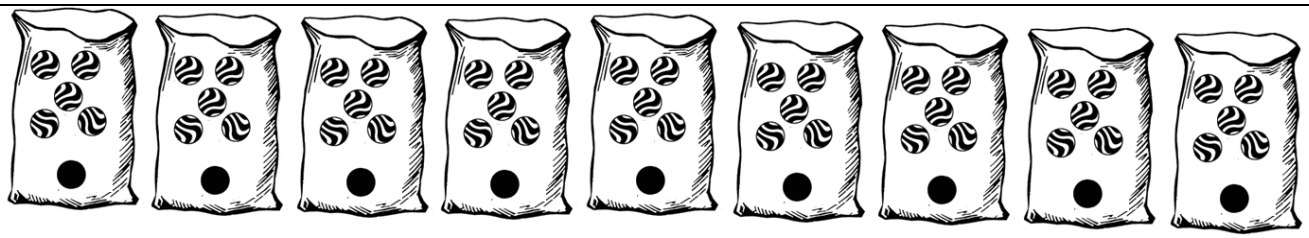

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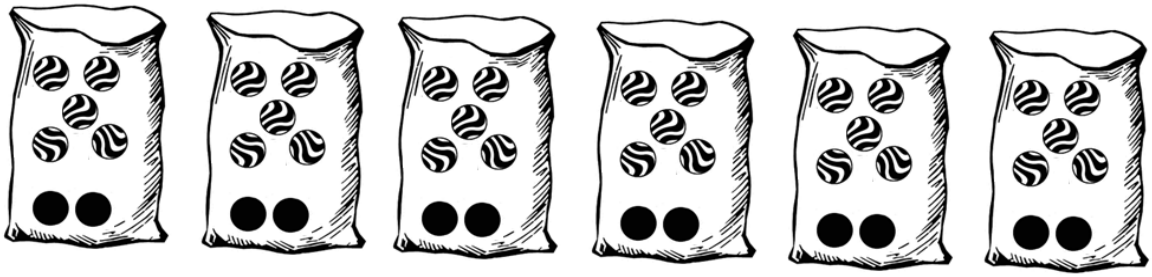

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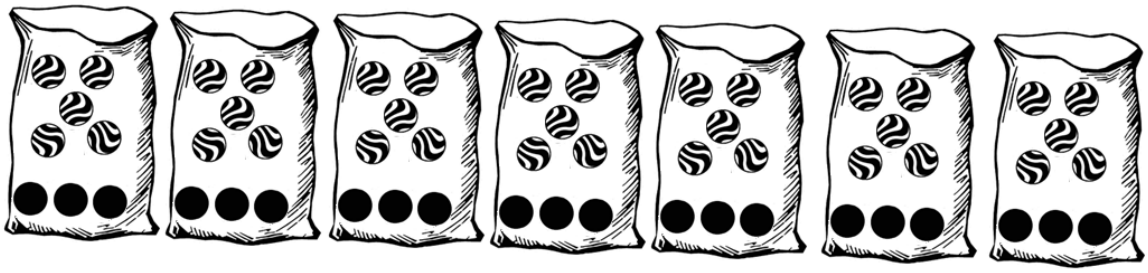

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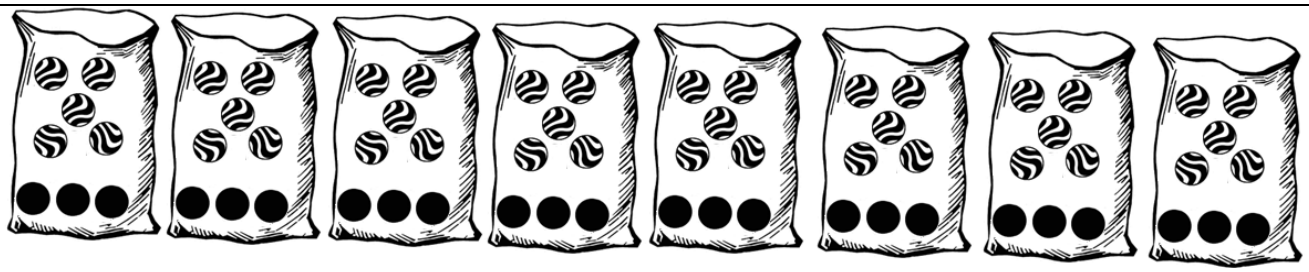

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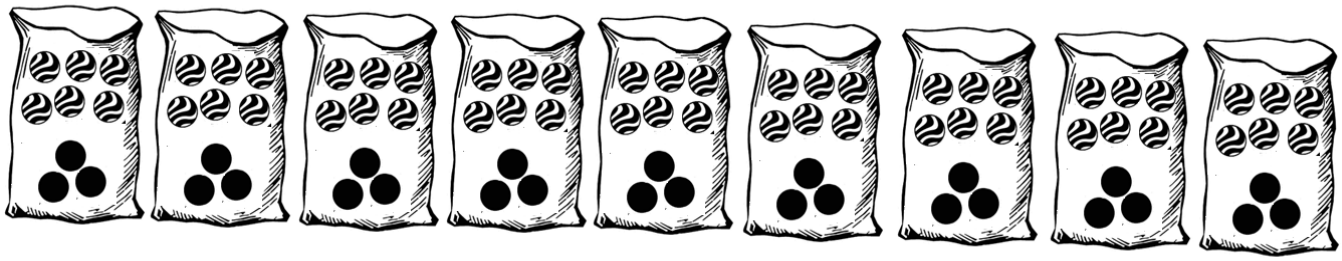

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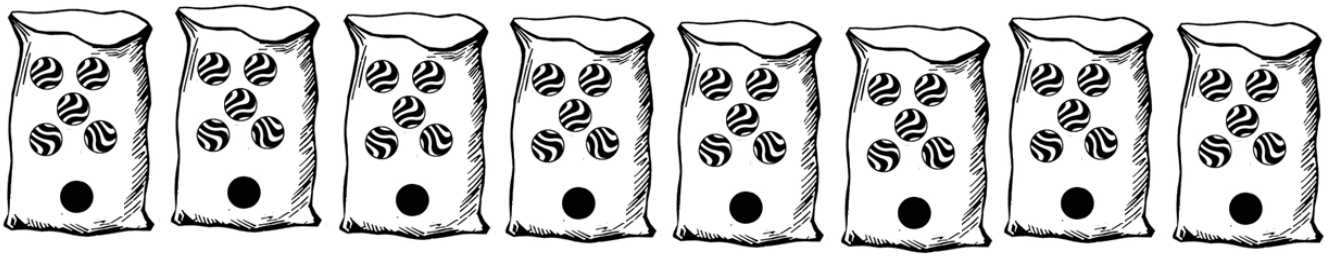

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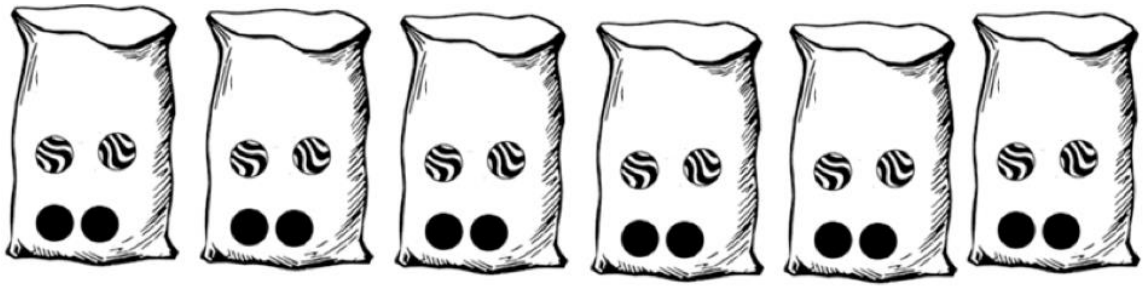

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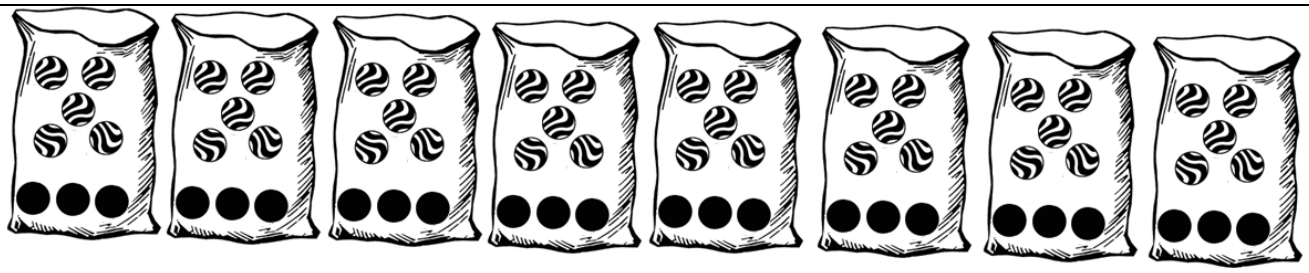

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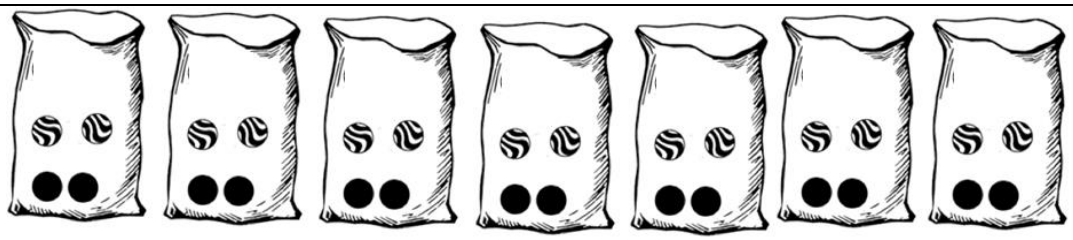

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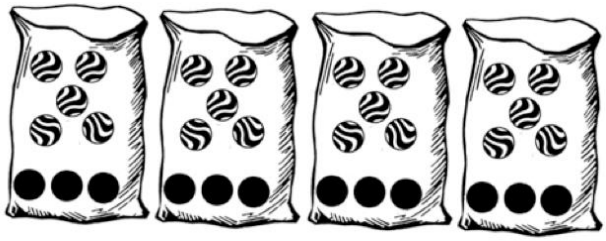

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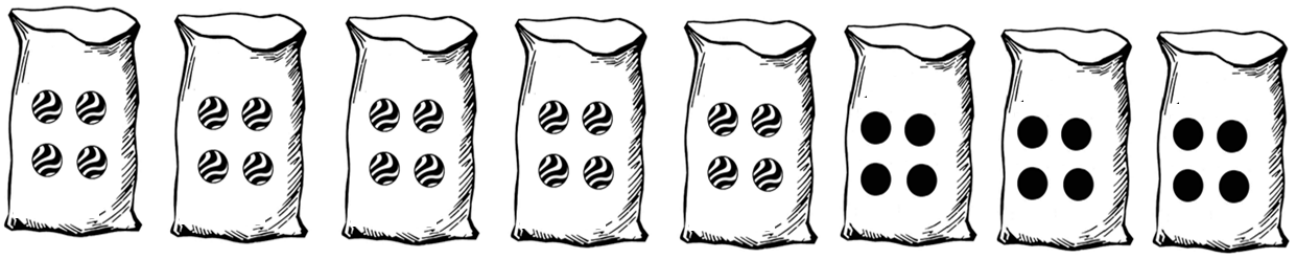

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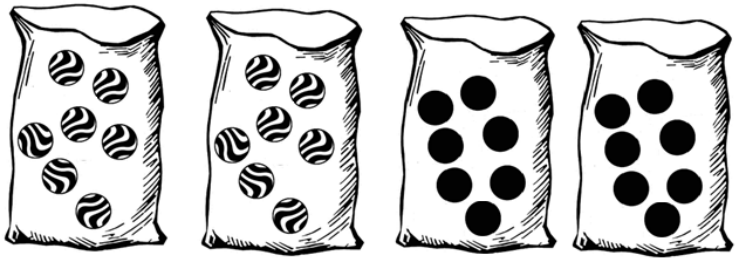

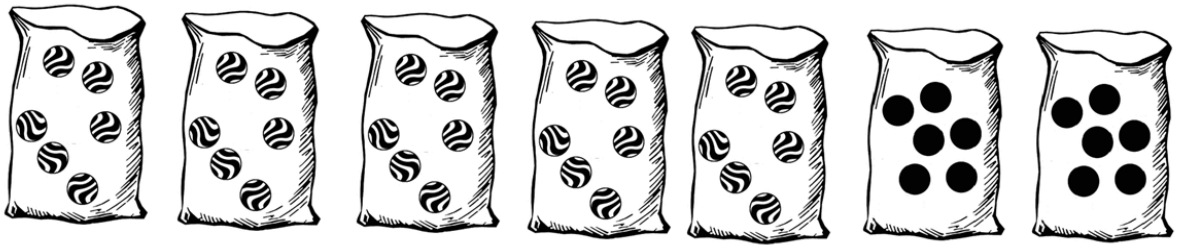

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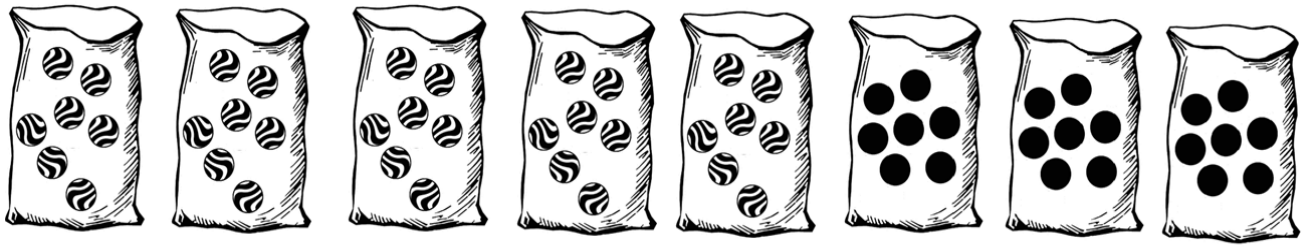

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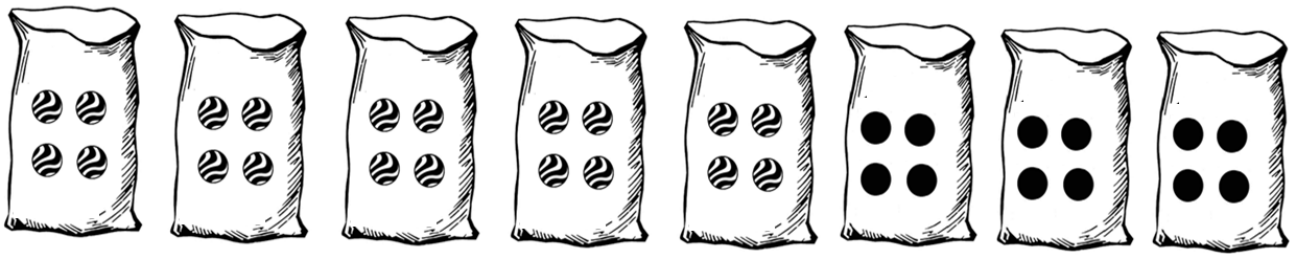
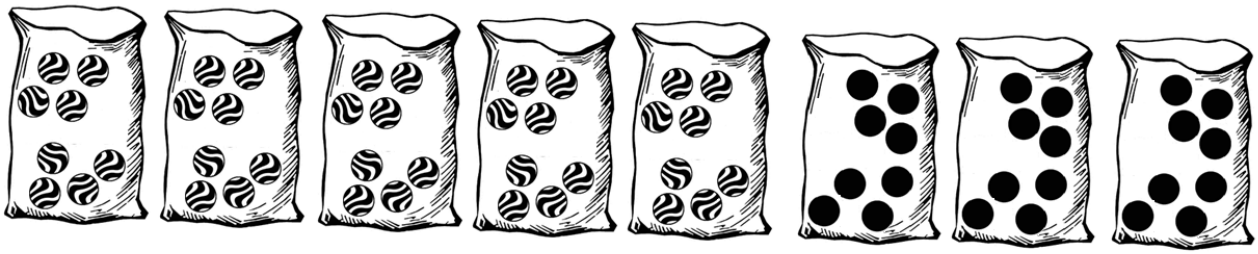
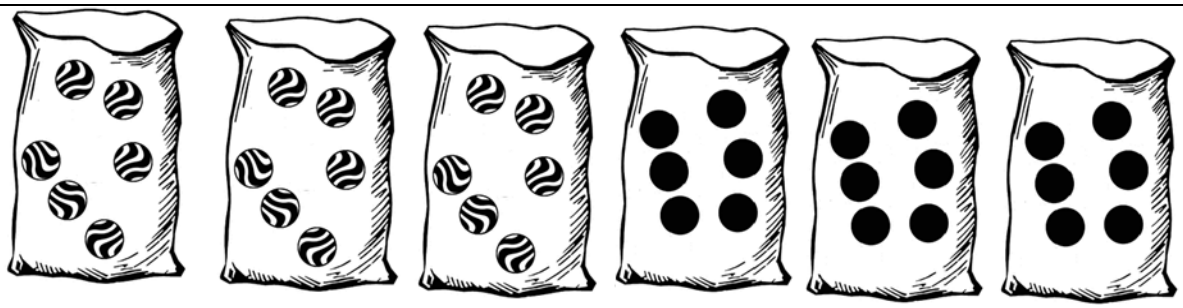




Card Set D

Card Set D



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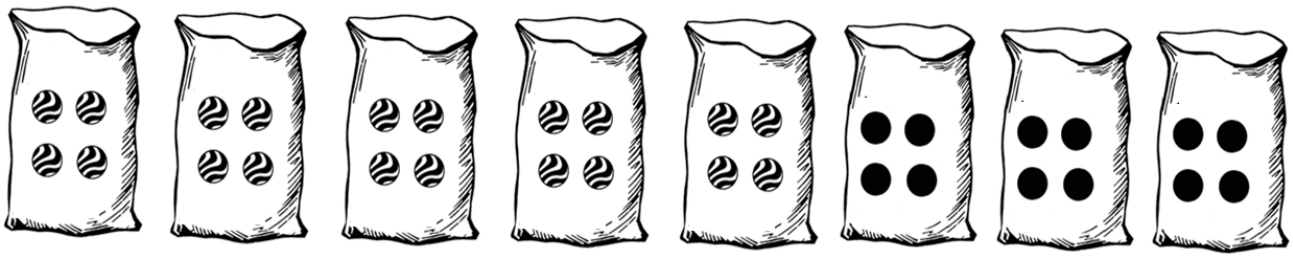

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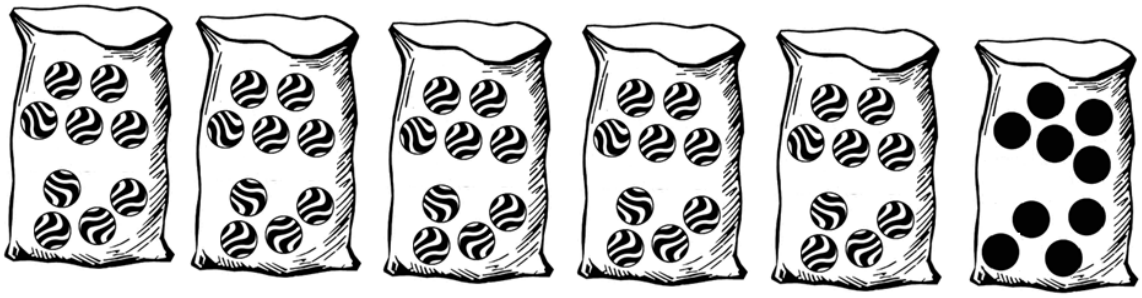

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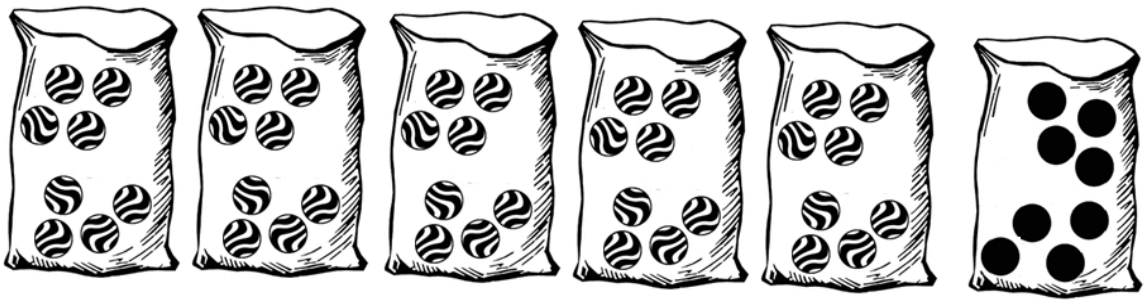
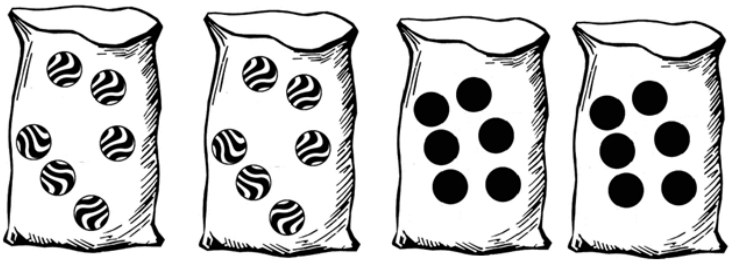
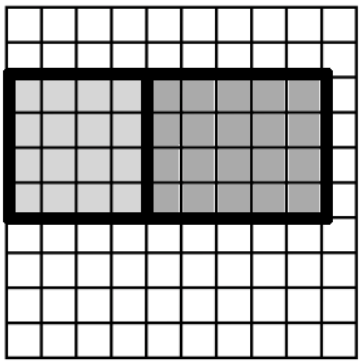
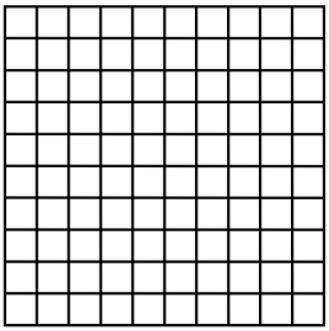
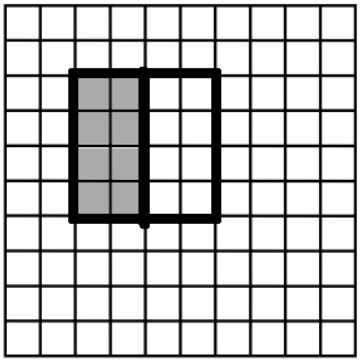
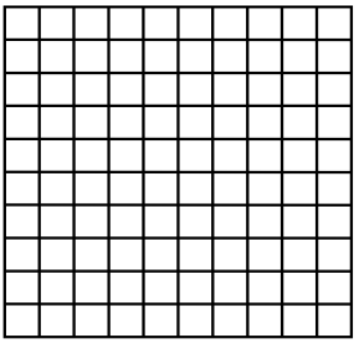


Card Set D



Name: _____ Date: _____

Distributive Property: Extention Activity

Multiplication Problem	Split Area Model	Distributive Property Equation
4 x 9		(4 x 4) + (4 x 5) = 16 + 20 = 36
7 x 7		
		
		(7 x 3) + (7 x 5) = _____