# FIRST YEAR B.SC. MATHEMATICS PAPER - I 

SEMESTER - I
DIFFERENTIAL EQUATIONS
MODEL QUESTION PAPER (THEORY)
Time: 3 Hours
Max. Marks: 75

## *This Paper Consists of Two parts. Follow the Instructions Carefully

$$
\text { PART }-\mathrm{A}(5 \times 5 \mathrm{M}=25 \mathrm{M})
$$

## Answer any FIVE Questions, each question carries FIVE marks

1. Obtain the equation of the curve whose differential equation is $\left(1+x^{2}\right) \frac{d y}{d x}+2 x y-4 x^{2}=0$ and passing through the origin.
2. Solve the differential equation $\left(1+e^{x / y}\right) \mathrm{dx}+e^{x / y}\left(1-\frac{x}{y}\right) \mathrm{dy}=0$
3. Solve $\frac{d x}{z(x+y)}=\frac{d y}{z(x-y)}=\frac{d z}{\mathrm{x}^{2}+\mathrm{y}^{2}}$ using the method of multipliers.
4. Solve $y^{2} \log y=x p y+p^{2}$.
5. Solve $\left(D^{2}-3 D+2\right) y=$ Coshx.
6. Solve $y^{\prime \prime}+4 y=\cos x \cdot \cos 3 x$.
7. Solve the system $\frac{d x}{d t}=3 e^{-t}, \frac{d y}{d t}=x+y$.
8. Form the differential equation by eliminating $a$ and $b$ from $z=\left(x^{2}+a\right)\left(y^{2}+b\right)$.

$$
\text { PART - B }(5 \times 10 \mathrm{M}=50 \mathrm{M})
$$

## Answer All the FIVE questions, each question carries TEN marks

9. a) Define orthogonal trajectory and show that the system of confocal conics $\frac{x^{2}}{a^{2}+\lambda}+\frac{y^{2}}{b^{2}+\lambda}=1$ is self- orthogonal where $\mathrm{a}, \mathrm{b}$ are arbitrary constants.
b) Define Integrating Factor. Solve $\left(y^{4}+2 y\right) d x+\left(x y^{3}+2 y^{4}-4 x\right) d y=0$
10. a) Solve $p^{2}+2 p y \cot x=y^{2}$
(or)
b) Define clairaut's differential equation. Solve $y=2 p x+p^{4} x^{2}$
11. a) Define complementary function of the differential equations $F(D) y=b(x)$.

Solve $\left(D^{2}+3 D+2\right) y=x e^{x} \operatorname{Sin} x$
b) Define Auxilary equation of the differential equation $\mathrm{F}(\mathrm{D}) \mathrm{y}=\mathrm{b}(\mathrm{x})$.

Solve $\left[D^{2}-(a+b) D+a b\right] y=e^{a x}+e^{b x}$
12. a) Apply the method of variation of parameters to solve $\frac{d^{2} y}{d x^{2}}+4 y=4 \tan 2 x$
(or)
b) Solve $\mathrm{x}^{2} \frac{d^{2} y}{d x^{2}}-\mathrm{x} \frac{d y}{d x}-3 \mathrm{y}=\mathrm{x}^{2} \log \mathrm{x}$
13. a) Solve $p \tan x+q \tan y=\tan z$
b) Find a complete integral of $\mathrm{z}=\mathrm{px}+\mathrm{qy}+\mathrm{p}^{2}+\mathrm{q}^{2}$

# FIRST YEAR B.SC. MATHEMATICS PAPER - I <br> SEMESTER - I <br> DIFFERENTIAL EQUATIONS <br> MODEL QUESTION PAPER (THEORY) 

Time: 3 Hours
Max. Marks: 75

## *This Paper Consists of Two parts. Follow the Instructions Carefully

$$
\text { PART }-\mathbf{A}(5 \times 5 M=25 M)
$$

## Answer any FIVE Questions, each question carries FIVE marks

1. Obtain the equation of the curve whose differential equation is $\left(1+\mathrm{x}^{2}\right) \frac{d y}{d x}+2 \mathrm{xy}-4 \mathrm{x}^{2}=0$ and passing through the origin.
2. Solve the differential equation $\left(1+e^{x / y}\right) \mathrm{dx}+e^{x / y}\left(1-\frac{x}{y}\right) \mathrm{dy}=0$
3. Solve $\frac{d x}{z(x+y)}=\frac{d y}{z(x-y)}=\frac{d z}{\mathrm{x}^{2}+\mathrm{y}^{2}}$ using the method of multipliers.
4. Solve $y^{2} \log y=x p y+p^{2}$.
5. Solve $\left(D^{2}-3 D+2\right) y=\operatorname{Cosh} x$.
6. Solve $y^{\prime \prime}+4 \mathrm{y}=\cos \mathrm{x} \cdot \cos 3 \mathrm{x}$.
7. Solve the system $\frac{d x}{d t}=3 e^{-t}, \frac{d y}{d t}=x+y$.
8. Form the differential equation by eliminating $a$ and $b$ from $z=\left(x^{2}+a\right)\left(y^{2}+b\right)$.

$$
\text { PART }- \text { B }(5 \times 10 M=50 \mathrm{M})
$$

## Answer All the FIVE questions, each question carries TEN marks

9. a) Define orthogonal trajectory and show that the system of confocal conics $\frac{x^{2}}{a^{2}+\lambda}+\frac{y^{2}}{b^{2}+\lambda}=1$ is self- orthogonal where $a, b$ are arbitrary constants.
(or)
b) Define Integrating Factor. Solve $\left(y^{4}+2 y\right) d x+\left(x y^{3}+2 y^{4}-4 x\right) d y=0$
10. a) Solve $p^{2}+2 p y \cot x=y^{2}$
(or)
b) Define clairaut's differential equation. Solve $y=2 p x+p^{4} x^{2}$
11. a) Define complementary function of the differential equations $F(D) y=b(x)$.

Solve $\left(D^{2}+3 D+2\right) y=x e^{x} \operatorname{Sin} x$
(or)
b) Define Auxilary equation of the differential equation $F(D) y=b(x)$.

Solve $\left[D^{2}-(a+b) D+a b\right] y=e^{a x}+e^{b x}$
12. a) Apply the method of variation of parameters to solve $\frac{d^{2} y}{d x^{2}}+4 y=4 \tan 2 x$
(or)
b) Solve $\mathrm{x}^{2} \frac{d^{2} y}{d x^{2}}-\mathrm{x} \frac{d y}{d x}-3 \mathrm{y}=\mathrm{x}^{2} \log \mathrm{x}$
13. a) Solve $p \tan x+q \tan y=\tan z$
b) Find a complete integral of $z=p x+q y+p^{2}+q^{2}$

## FIRST YEAR B.SC. MATHEMATICS PAPER - I <br> SEMESTER -I <br> DIFFERENTIAL EQUATIONS <br> Practical -1 <br> on

Differential Equations of first order and first degree

1. Solve the differential equation $\left(1+\mathrm{y}^{2}\right)+\left(\mathrm{x}-e^{\tan ^{-1} \mathrm{y}}\right) \frac{d y}{d x}=0$
2. Solve the differential equation $\left(1+e^{x / y}\right) \mathrm{dx}+e^{x / y}\left(1-\frac{x}{y}\right) \mathrm{dy}=0$
3. Solve the simultaneous equation $\frac{d x}{x\left(y^{2}-\mathrm{z}^{2}\right)}=\frac{d y}{y\left(\mathrm{z}^{2}-\mathrm{x}^{2}\right)}=\frac{d z}{z\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)}$ using the method of multipliers.
4. Find the orthogonal trajectories of cardioids $r=a(1-\cos \theta)$, where ' $a$ ' is a parameter.
5. Solve the total differential equation $3 x^{2} d x+3 y^{2} d y-\left(x^{3}+y^{3}+e^{2 z}\right) d z=0$
6. Bacteria in a certain culture increase at a rate proportional to the number present. If the number doubles in one hour, how long does it take for the number to triple?

## Practical -2 <br> on

## Differential Equations of the first order, but not of the first degree

1. Solve the differential equation $p^{2}+2 p y \cot x=y^{2}$
2. Solve the differential equation $y=2 p x+p^{4} x^{2}$
3. Solve the differential equation $\mathrm{x}^{2} \mathrm{p}^{2}+\mathrm{yp}(2 \mathrm{x}+\mathrm{y})+\mathrm{y}^{2}=0$ where $\mathrm{p}=\frac{d y}{d x}$ by reducing it to Clairaut's form by using the substitution $y=u$ and $x y=v$
4. Solve the differential equation $y p^{2}-2 x p+y=0$
5. Solve the differential equation $y^{2} \log y=x p y+p^{2}$
6. Solve the differential equation $\mathrm{x}^{2}\left(\frac{d y}{d x}\right)^{2}-2 \mathrm{xy} \frac{d y}{d x}+2 \mathrm{y}^{2}-\mathrm{x}^{2}=0$

## Practical -3 <br> on <br> Higher Order Linear Differential Equations

1. Solve $y^{\prime \prime}-4 y=x^{2}+3 e^{x}$, given that $\mathrm{y}(0)=0$ and $y^{\prime}(0)=2$
2. Solve $y^{\prime \prime}+4 y=\cos x \cdot \cos 3 x$
3. Solve $y^{\prime \prime}+2 y^{\prime}+5 y=x \sin x+x^{2} \mathrm{e}^{2 x}$
4. Solve $\left(D^{2}+4\right) y=\tan 2 x$
5. Solve $\left(D^{2}+2\right) y=x^{2} e^{3 x}+e^{x} \cos 2 x$
6. Solve $\left(D^{2}-4 D+4\right) y=x^{2}+e^{x}+\cos 2 x$

## Practical -4 <br> on

## Higher Order Linear Differential Equations

1. Solve $y^{\prime \prime}+3 y^{\prime}+2 y=\frac{1}{e^{x}+1}$ by using the method of variation of parameters.
2. Solve $\left(x^{2} D^{2}-x D+2\right) y=x \log x$.
3. Solve $x^{2} y^{\prime \prime}+x y^{\prime}-y=0$, given that $x+\frac{1}{x}$ is one integral.
4. Solve $y^{\prime \prime}-\frac{2}{x} y^{\prime}+\left(1+\frac{2}{x^{2}}\right) y=x e^{x}$
5. Solve the system $\frac{d x}{d t}=3 e^{-t}, \frac{d y}{d t}=x+y$
6. Solve the system $\frac{d^{2} y}{d t^{2}}=x, \frac{d^{2} x}{d t^{2}}=y$

## Practical -5 on Partial Differential Equations

1. Find a partial differential equations by eliminating ' $a$ ' and ' $b$ ' from $z=a x+b y+a^{2}+b^{2}$.
2. Solve $\mathrm{p} \tan \mathrm{x}+\mathrm{q} \tan \mathrm{y}=\tan \mathrm{z}$.
3. Solve $y^{2} p-x y=x(z-2 y)$.
4. Solve $p y+q x=x y z^{2}\left(x^{2}-y^{2}\right)$.
5. Show that the equations $x p=y q$ and $Z(x p+y q)=2 x y$ are compatible and solve them.
6. Find the complete integral of $\mathrm{zpq}=\mathrm{p}+\mathrm{q}$.

## Practical-6 <br> on <br> 5 Units

1. Solve the differential equation $\mathrm{xy}-\frac{d y}{d x}=\mathrm{y}^{3} e^{-x^{2}}$
2. Bacteria in a certain culture increase at a rate proportional to the number present. If the number N increases from 1000 to 2000 in 1 hour, how many are present at the end of 1.5 hours?
3. Solve $(p x-y)(p y+x)=2 p$
4. Solve $\left(D^{2}-2 D\right) y=e^{x}$. sinx using the method of variation of parameters.
5. Solve $\left[x^{3} D^{3}+3 x^{2} D^{2}+x D+8\right] y=65 \cos (\log x)$
6. Solve $(y-z) p+(z-x) q=x-y$

# FIRST YEAR B.Sc. MATHEMATICS PAPER-I <br> SOLID GEOMETRY-SEMESTER-II <br> MODEL QUESTION PAPER (THEORY) 

Time: 3 Hrs.
Max Marks: 75
This Paper Consists and Two parts. Follow the Instructions Carefully

## PART-A ( $5 \times 5=25 \mathrm{M}$ )

Answer any FIVE questions, each question carries FIVE marks.

1. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2 x+3 y-z=-4$ and is parallel to $x$-axis.
2. Find the point of intersection of the lines $\frac{x-1}{-3}=\frac{y-2}{2}=\frac{z-3}{2}$ and $\frac{x-1}{3}=\frac{y-5}{1}=\frac{z}{-5}$
3. Find the equation of the line through the point $(1,2,3)$ and parallel to the line $x-y+2 z=5,3 x+y+z=6$.
4. Find the centre and the radius of the circle $x^{2}+y^{2}+z^{2}-2 y-4 z=11, x+2 y+2 z=15$
5. Find the equation of the sphere which touches the sphere $x^{2}+y^{2}+z^{2}-x+3 y+2 z-3=0$ at $(1,1,-1)$ and passes through the origin.
6. Find the enveloping cone of a sphere $x^{2}+y^{2}+z^{2}-2 x+4 z=1$ with its vertex at $(1,1,1)$.
7. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and $\frac{x}{3}=\frac{y}{-1}=\frac{z}{1}$
8. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and guiding curve is ellipse $x^{2}+2 y^{2}=1, z=3$

## PART-B ( $5 \times 10 \mathrm{M}=50 \mathrm{M}$ )

Answer ALL the questions, each question carries TEN marks.
9. a) A variable plane is at a constant distance $P$ from the origin and meets the axes in $A, B, C$. Show that the locus of the centroid of the tetrahedron OABC is $\mathrm{x}^{-2}+\mathrm{y}^{-2}+\mathrm{z}^{-2}=16 \mathrm{P}^{-2}$.
(Or)
b) Find the bisecting plane of the acute angle between the planes $3 x-2 y-6 z+2=0,-2 x+y-2 z-2=0$
10. a) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} ; \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.
(Or)
b) Find the Shortest distance and equations of the line of Shortest distance between the lines 3x$9 y+5 z=0=x+y-z$ and $6 x+8 y+3 z-10=0=x+2 y+z-3$.
11. a) Show that the two circles $x^{2}+y^{2}+z^{2}-y+2 z=0 ; x-y+z-2=0$ and $x^{2}+y^{2}+z^{2}+x-3 y+z-5=0,2 x-y+4 z-1=0$ lie on the same sphere and find its equation
(Or)
b) Find the limiting points of the coaxial system defined by the spheres $x^{2}+y^{2}+z^{2}+2 x-4 y-2 z+6=0$ and $x^{2}+y^{2}+z^{2}+2 x-4 y-2 z+6=0$
12. a) Find the equation to the right circular cone whose vertex $(2,-3,5)$, axes $P Q$ which makes equal angles with the axes and semi-vertical angle if $30^{\circ}$.
(Or)
b) If $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ represents one of a set of three mutually perpendicular generators of a cone $11 y z+6 z x-14 x y=0$, find the equations of the other two.
13. a) Find the equation of the right circular cylinder of radius 2 and whose axis passes through the point $(1,2,3)$ and has directions cosines proportional to $(2,-3,6)$.
(Or)
b) Find the equation of the enveloping cylinder of the sphere $x^{2}+y^{2}+z^{2}-2 x+4 y-1=0$, having its generators parallel to the line $x=y=z$.

# FIRST B.Sc MATHEMATICS PAPER - I <br> SEMESTER - II <br> SOLID GEOMETRY <br> PRACTICAL-1 <br> on <br> The Plane 

1. Find the bisecting plane of the acute angle between the planes $3 x-2 y+6 z+2=0$, $-2 \mathrm{x}+\mathrm{y}-2 \mathrm{z}-2=0$
2. Find the equation of planes bisecting the angles between the planes $3 x-6 y+2 z+5=0$, $4 x-12 y+3 z-3=0$ and specify the one, which bisects the obtuse angle.
3. Find the equation of the plane through the line of intersection of the planes $x-3 y+2 z=0$ and $3 x-y-2 z-5=0$ and passing through ( $1,1,1$ ).
4. Find the equation of the plan through the line of intersection of the planes $x+y+z=1$ and $2 x+3 y-z=-4$ and is parallel to $x-$ axis.
5. Show that the equation $x^{2}+4 y^{2}+9 z^{2}-12 y z-6 z x+4 x y+5 x+10 y-15 z+6=0$ represents a pair of parallel planes and find the distance between them.
6. Show that the equation $2 x^{2}-3 y^{2}+4 z^{2}+x y+6 z x-y z=0$ represents a pair of parallel planes and find the angle between them

## PRACTICAL - 2

## on

## The Line

1. Find the length and equations of Shortest distance. between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$; $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ and also find the points in which the Shortest distance. meets the given lines.
2. Find the length and eqn. of Shortest distance between the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{1} ; x+y+2 z-3=0$ $=2 x+3 y+3 z-4$
3. Find the length and equation of Shortest distance between the lines $5 x-y-z=0=x-2 y+z+3$ and $7 x-4 y-2 z=0=x-y+z-3$
4. Find the image of the point $(2,-1,3)$ in the plane $3 x-2 y+z=9$
5. Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} ; \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.
6. Examine the nature of intersection of the planes $x+2 y+3 z+1=0, x-y-z-2=0, x+2 y+3 z+4=0$

## PRACTICAL - 3

on

## SPHERE

1. Find the equation of the sphere which passes through the circle $x^{2}+y^{2}+z^{2}=5, x+2 y+3 z=3$ and touching the plane $4 x+3 y=15$
2. Find the equation of the sphere which has the circle $x^{2}+y^{2}+z^{2}-x+z-2=0, x+2 y-z-4=0$ as the great circle.
3. Show that the two circles $x^{2}+y^{2}+z^{2}-y+2 z=0, x-y+z-2=0$ and $x^{2}+y^{2}+z^{2}+x-3 y+z-5=0,2 x-$ $y+4 z-1=0$ lines on the same sphere and find its equation.
4. Find the equation of the sphere intersecting the spheres $x^{2}+y^{2}+z^{2}+x-3 z-2=0, x^{2}+y^{2}+z^{2}$ $+\frac{1}{2} x+\frac{3}{2} y+2=0$ orthogonally and passing through the points $(0,3,0),(-2,-1,-4)$
5. Find the limiting points of the coaxal defined by spheres $x^{2}+y^{2}+z^{2}+4 x-2 y+2 z+6=0$ and $x^{2}+y^{2}+z^{2}+2 x-4 y-2 z+6=0$
6. Find the limiting points of the coaxal system of spheres given by $x^{2}+y^{2}+z^{2}+3 x-3 y+6=0$, $x^{2}+y^{2}+z^{2}-6 y-6 z+6=0$

## PRACTICAL - 4 <br> on <br> CONES

1. Find the equation of the right circular cone whose vertex at $\mathrm{p}(2,-3,5)$, axis PQ that makes equal angle with axes and Semi-vertical angle is $30^{\circ}$.
2. Prove that the cones $\mathrm{ax}^{2}+\mathrm{by}^{2}+\mathrm{cz}^{2}=0, \frac{x^{2}}{a}+\frac{y^{2}}{b}+\frac{z^{2}}{c}=0$ are reciprocal.
3. Find the vertex of the cone $7 x^{2}+2 y^{2}+2 z^{2}-10 x z+10 x y+26 x-2 y+2 z-17=0$
4. Find the equation of the cone with vertex $(1,1,2)$ and guiding curve $x^{2}+y^{2}=4, z=0$
5. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$; represents one of a set of three mutually perpendicular generators of the cone $5 y z-8 z x-3 x y=0$ then find the other two generators
6. Find the equation of the right circular cone whose vertex at $\mathrm{p}(2,-3,5)$, axis PQ which makes equal angles with axes and passes through $\mathrm{A}(1,-2,3)$

## PRACTICAL -5

on

## CYLINDERS AND CONICOIDS

1. Find the equation of the enveloping cylinder of the sphere $x^{2}+y^{2}+z^{2}-2 x+4 y-1=0$, having its generators parallel to the line $\mathrm{x}=\mathrm{y}=\mathrm{z}$
2. Find the equation of the right circular cylinder of radius 2 whose axis passes through $(1,2,3)$ and has direction cosines. proportional to $(2,-3,6)$
3. Find the equation to the cylinder whose generators are parallel to the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$; and the guiding curve is the ellipse $x^{2}+2 y^{2}=1, z=3$
4. Find the equation to the right circular cylinder whose guiding curve is a circle
$x^{2}+y^{2}+z^{2}=9, x-y+z=3$
5. Show that the plane $3 x+12 y-6 z-17=0$ touches the conicoid $3 x^{2}-6 y^{2}+9 z 2+17=0$ and final the point of contact.
6. Prove that locus of poles of the tangent planes to $a^{2} x^{2}+b^{2} y^{2}-c^{2} z^{2}=1$ w.r.t $\alpha^{2} x^{2}+\beta^{2} y^{2}+\gamma^{2} z^{2}=1$ is the hyperboloid of one sheet.

## PRACTICAL -6

on
ALL UNITS

1. Prove that the equation $2 x^{2}-6 y^{2}-12 z^{2}+18 y z+2 z x+x y=0$ represents a pair of planes. Also find the angle between them.
2. Show that the lines $2 x+3 y-4 z=0=3 x-4 y+z-7$; and $5 x-y-3 z+12=0=x-7 y+5 z-6$ are parallel
3. Show that the four points $(-8,5,2),(-5,2,2),(-7,6,6),(-4,3,6)$ are concyclic
4. Find the enveloping cone of the sphere $x^{2}+y^{2}+z^{2}-2 x+4 z=1$ with its vertex at $(1,1,1)$
5. Find the equation of the cylinder whose generators are parallel to $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which passes through the curve $x^{2}+y^{2}=16, z=0$
6. Show that the plane $2 x-2 y+z+12=0$ touches the sphere $x^{2}+y^{2}+z^{2}-2 x-4 y+2 z-3=0$ and find the point of contact.
