FIRST YEAR B.SC. MATHEMATICS PAPER - I SEMESTER - I **DIFFERENTIAL EQUATIONS MODEL OUESTION PAPER (THEORY)**

Time: 3 Hours Max. Marks: 75

*This Paper Consists of Two parts. Follow the Instructions Carefully

PART - A (5x5M = 25M)

Answer any FIVE Questions, each question carries FIVE marks

- 1. Obtain the equation of the curve whose differential equation is $(1 + x^2) \frac{dy}{dx} + 2xy 4x^2 = 0$ and passing through the origin.
- 2. Solve the differential equation $(1+e^{x/y}) dx + e^{x/y} (1-\frac{x}{y}) dy = 0$
- 3. Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$ using the method of multipliers.
- 4. Solve $y^2 \log y = xpy + p^2$.
- 5. Solve $(D^2-3D+2)y = Coshx$.
- 6. Solve $y'' + 4y = \cos x \cdot \cos 3x$.
- 7. Solve the system $\frac{dx}{dt} = 3e^{-t}$, $\frac{dy}{dt} = x + y$.
- **8.** Form the differential equation by eliminating a and b from $z = (x^2+a)(y^2+b)$.

PART - B (5x10M = 50M)

Answer All the FIVE questions, each question carries TEN marks

9. a) Define orthogonal trajectory and show that the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self- orthogonal where a,b are arbitrary constants.

- b) Define Integrating Factor. Solve $(y^4+2y)dx + (xy^3 + 2y^4 4x)dy = 0$ 10. a) Solve $p^2+2pycotx = y^2$

(or)

- b) Define clairaut's differential equation. Solve $y = 2px+p^4x^2$
- 11. a) Define complementary function of the differential equations F(D)y = b(x). Solve $(D^2 + 3D+2)y = x e^x Sinx$

b) Define Auxilary equation of the differential equation F(D)y = b(x). Solve $[D^2 - (a+b)D+ab]y = e^{ax} + e^{bx}$

12. a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ (or)

b) Solve
$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x$$

13. a) Solve $p \tan x + q \tan y = \tan z$

(or)

b) Find a complete integral of $z = px + qy + p^2 + q^2$

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FIRST YEAR B.SC. MATHEMATICS PAPER – I SEMESTER – I DIFFERENTIAL EQUATIONS MODEL QUESTION PAPER (THEORY)

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- 3. Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$ using the method of multipliers.
- 4. Solve $y^2 \log y = xpy + p^2$.
- 5. Solve $(D^2-3D+2)y = Coshx$.
- 6. Solve $y'' + 4y = \cos x \cdot \cos 3x$.
- 7. Solve the system $\frac{dx}{dt} = 3e^{-t}$, $\frac{dy}{dt} = x + y$.
- 8. Form the differential equation by eliminating a and b from $z = (x^2+a)(y^2+b)$.

PART - B (5x10M = 50M)

Answer All the FIVE questions, each question carries TEN marks

9. a) Define orthogonal trajectory and show that the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self- orthogonal where a,b are arbitrary constants.

(or)

- b) Define Integrating Factor. Solve $(y^4+2y)dx + (xy^3 + 2y^4 4x)dy = 0$
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$$z = px + qy + p^2 + q^2$$

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FIRST YEAR B.SC. MATHEMATICS PAPER – I SEMESTER -I DIFFERENTIAL EQUATIONS Practical -1 on Differential Equations of first order and first degree

1. Solve the differential equation $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

- 2. Solve the differential equation $(1+e^{x/y}) dx + e^{x/y} (1-\frac{x}{y}) dy = 0$
- 3. Solve the simultaneous equation $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$ using the method of multipliers.
- 4. Find the orthogonal trajectories of cardioids $r = a (1 \cos\theta)$, where 'a' is a parameter.
- 5. Solve the total differential equation $3x^2 dx + 3y^2 dy (x^3 + y^3 + e^{2z}) dz = 0$
- 6. Bacteria in a certain culture increase at a rate proportional to the number present. If the number doubles in one hour, how long does it take for the number to triple?

Practical -2

on

Differential Equations of the first order, but not of the first degree

- 1. Solve the differential equation $p^2 + 2py \cot x = y^2$
- 2. Solve the differential equation $y = 2px + p^4 x^2$
- 3. Solve the differential equation $x^2 p^2 + yp (2x+y) + y^2 = 0$ where $p = \frac{dy}{dx}$ by reducing it to Clairaut's form by using the substitution y = u and xy = v
- 4. Solve the differential equation $yp^2-2xp+y=0$
- 5. Solve the differential equation $y^2 \log y = xpy + p^2$
- 6. Solve the differential equation $x^2 (\frac{dy}{dx})^2 2xy\frac{dy}{dx} + 2y^2 x^2 = 0$



Practical -3 on Higher Order Linear Differential Equations

- 1. Solve $y'' 4y = x^2 + 3e^x$, given that y(0) = 0 and y'(0) = 2
- 2. Solve $y'' + 4y = \cos x \cdot \cos 3x$
- 3. Solve $y'' + 2y' + 5y = x \sin x + x^2 e^{2x}$
- 4. Solve $(D^2+4)y = \tan 2x$
- 5. Solve $(D^2+2)y = x^2 e^{3x} + e^x \cos 2x$
- 6. Solve($D^2 4D + 4$)y = $x^2 + e^x + \cos 2x$

Practical -4 on Higher Order Linear Differential Equations

- 1. Solve $y'' + 3y' + 2y = \frac{1}{e^{x} + 1}$ by using the method of variation of parameters.
- 2. Solve $(x^2D^2 xD + 2)y = x \log x$.
- 3. Solve $x^2y'' + xy' y = 0$, given that $x + \frac{1}{x}$ is one integral.
- 4. Solve $y'' \frac{2}{x}y' + (1 + \frac{2}{x^2})y = xe^x$
- 5. Solve the system $\frac{dx}{dt} = 3e^{-t}$, $\frac{dy}{dt} = x + y$
- 6. Solve the system $\frac{d^2y}{dt^2} = x$, $\frac{d^2x}{dt^2} = y$

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Practical -5 on Partial Differential Equations

- 1. Find a partial differential equations by eliminating 'a' and 'b' from $z = ax+by+a^2+b^2$.
- 2. Solve $p \tan x + q \tan y = \tan z$.
- 3. Solve $y^2 p xy = x(z-2y)$.
- 4. Solve $py + qx = xyz^2 (x^2 y^2)$.
- 5. Show that the equations xp = yq and Z(xp + yq) = 2xy are compatible and solve them.
- 6. Find the complete integral of zpq = p+q.

Practical-6 on 5 Units

- 1. Solve the differential equation $xy \frac{dy}{dx} = y^3 e^{-x^2}$
- 2. Bacteria in a certain culture increase at a rate proportional to the number present. If the number N increases from 1000 to 2000 in 1 hour, how many are present at the end of 1.5 hours?
- 3. Solve (px-y)(py+x) = 2p
- 4. Solve $(D^2 2D)y = e^x$. sinx using the method of variation of parameters.
- 5. Solve $[x^{3}D^{3}+3x^{2}D^{2}+xD+8]y = 65\cos(\log x)$
- 6. Solve (y z)p + (z x)q = x y



FIRST YEAR B.Sc. MATHEMATICS PAPER-I SOLID GEOMETRY-SEMESTER-II MODEL QUESTION PAPER (THEORY)

Time: 3 Hrs. Max Marks: 75

This Paper Consists and Two parts. Follow the Instructions Carefully

$\underline{\mathbf{PART-A}} (5 \ge 5 = 25 \text{ M})$

Answer any <u>**FIVE**</u> questions, each question carries <u>**FIVE**</u> marks.

- 1. Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y-z=-4 and is parallel to x-axis.
- 2. Find the point of intersection of the lines $\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{2}$ and $\frac{x-1}{3} = \frac{y-5}{1} = \frac{z}{-5}$
- 3. Find the equation of the line through the point (1,2,3) and parallel to the line x-y+2z=5, 3x+y+z=6.
- 4. Find the centre and the radius of the circle $x^2 + y^2 + z^2 2y 4z = 11$, x + 2y + 2z = 15
- 5. Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 x + 3y + 2z 3 = 0$ at (1,1,-1) and passes through the origin.
- 6. Find the enveloping cone of a sphere $x^2 + y^2 + z^2 2x + 4z = 1$ with its vertex at (1,1,1).
- 7. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$

8. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and guiding curve is ellipse $x^2 + 2y^2 = 1, z = 3$

PART-B (5 x 10 M= 50 M)

Answer <u>ALL</u> the questions, each question carries <u>TEN</u> marks.

9. a) A variable plane is at a constant distance P from the origin and meets the axes in A,B,C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16 P^{-2}$.

(**O**r)

b) Find the bisecting plane of the acute angle between the planes 3x-2y-6z+2=0, -2x+y-2z-2=0

10. a) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

(**Or**)

b) Find the Shortest distance and equations of the line of Shortest distance between the lines 3x-9y+5z=0=x+y-z and 6x+8y+3z-10=0=x+2y+z-3.

11. a) Show that the two circles $x^2+y^2+z^2-y+2z=0$; x-y+z-2=0 and $x^2+y^2+z^2+x-3y+z-5=0$, 2x-y+4z-1=0 lie on the same sphere and find its equation

(**O**r)

b) Find the limiting points of the coaxial system defined by the spheres $x^2+y^2+z^2+2x-4y-2z+6=0$ and $x^2+y^2+z^2+2x-4y-2z+6=0$

12. a) Find the equation to the right circular cone whose vertex (2,-3,5), axes PQ which makes equal angles with the axes and semi-vertical angle if 30° .

(**O**r)

b) If $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ represents one of a set of three mutually perpendicular generators of a cone 11yz+6zx-14xy=0, find the equations of the other two.

13. a) Find the equation of the right circular cylinder of radius 2 and whose axis passes through the point (1,2,3) and has directions cosines proportional to (2,-3,6).

(**O**r)

b) Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y-1=0$, having its generators parallel to the line x=y=z.



FIRST B.Sc MATHEMATICS PAPER – I SEMESTER – II SOLID GEOMETRY PRACTICAL-1 on

The Plane

- 1. Find the bisecting plane of the acute angle between the planes 3x-2y+6z+2 = 0, -2x+y-2z-2 = 0
- 2. Find the equation of planes bisecting the angles between the planes 3x-6y+2z+5 = 0, 4x-12y+3z-3 = 0 and specify the one, which bisects the obtuse angle.
- 3. Find the equation of the plane through the line of intersection of the planes x-3y+2z = 0and 3x-y-2z-5 = 0 and passing through (1,1,1).
- 4. Find the equation of the plan through the line of intersection of the planes x+y+z=1 and 2x+3y-z=-4 and is parallel to x axis.
- 5. Show that the equation $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6 = 0$ represents a pair of parallel planes and find the distance between them.
- 6. Show that the equation $2x^2-3y^2+4z^2+xy+6zx-yz = 0$ represents a pair of parallel planes and find the angle between them

PRACTICAL – 2

on

The Line

- 1. Find the length and equations of Shortest distance. between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ and also find the points in which the Shortest distance. meets the given lines.
- 2. Find the length and eqn. of Shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$; x+y+2z-3 = 0 =2x+3y+3z-4
- Find the length and equation of Shortest distance between the lines 5x-y-z = 0= x-2y+z+3 and 7x-4y-2z = 0 = x-y+z-3
- 4. Find the image of the point (2,-1,3) in the plane 3x-2y+z = 9
- 5. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.
- 6. Examine the nature of intersection of the planes x+2y+3z+1 = 0, x-y-z-2=0, x+2y+3z+4=0



PRACTICAL - 3 on SPHERE

- 1. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2 = 5$, x+2y+3z = 3and touching the plane 4x+3y = 15
- 2. Find the equation of the sphere which has the circle $x^2+y^2+z^2-x+z-2 = 0$, x+2y-z-4 = 0 as the great circle.
- 3. Show that the two circles $x^2+y^2+z^2-y+2z = 0$, x-y+z-2=0 and $x^2+y^2+z^2+x-3y+z-5 = 0$, 2x-y+4z-1 = 0 lines on the same sphere and find its equation.
- 4. Find the equation of the sphere intersecting the spheres $x^2+y^2+z^2+x-3z-2 = 0$, $x^2+y^2+z^2$ $+\frac{1}{2}x + \frac{3}{2}y+2 = 0$ orthogonally and passing through the points (0,3,0), (-2,-1,-4)
- 5. Find the limiting points of the coaxal defined by spheres $x^2+y^2+z^2+4x-2y+2z+6=0$ and $x^2+y^2+z^2+2x-4y-2z+6=0$
- 6. Find the limiting points of the coaxal system of spheres given by $x^2+y^2+z^2+3x-3y+6=0$, $x^2+y^2+z^2-6y-6z+6=0$

PRACTICAL – 4 on CONES

- 1. Find the equation of the right circular cone whose vertex at p(2,-3,5), axis PQ that makes equal angle with axes and Semi-vertical angle is 30° .
- 2. Prove that the cones $ax^2+by^2+cz^2 = 0$, $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal.
- 3. Find the vertex of the cone $7x^2+2y^2+2z^2-10xz+10xy+26x-2y+2z-17 = 0$
- 4. Find the equation of the cone with vertex (1,1,2) and guiding curve $x^2+y^2=4$, z=0
- 5. If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$; represents one of a set of three mutually perpendicular generators of the cone 5yz-8zx-3xy = 0 then find the other two generators
- 6. Find the equation of the right circular cone whose vertex at p (2,-3,5), axis PQ which makes equal angles with axes and passes through A(1,-2,3)



PRACTICAL -5 on CYLINDERS AND CONICOIDS

- 1. Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y-1=0$, having its generators parallel to the line x=y=z
- 2. Find the equation of the right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines. proportional to (2,-3,6)
- 3. Find the equation to the cylinder whose generators are parallel to the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$; and the guiding curve is the ellipse $x^2+2y^2=1,z=3$
- 4. Find the equation to the right circular cylinder whose guiding curve is a circle $x^2+y^2+z^2=9$, x-y+z=3
- 5. Show that the plane 3x+12y-6z-17 = 0 touches the conicoid $3x^2-6y^2+9z^2+17 = 0$ and final the point of contact.
- 6. Prove that locus of poles of the tangent planes to $a^2 x^2+b^2 y^2-c^2 z^2=1$ w.r.t $\alpha^2 x^2+\beta^2 y^2+\gamma^2 z^2=1$ is the hyperboloid of one sheet.

PRACTICAL -6 on ALL UNITS

- 1. Prove that the equation $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents a pair of planes. Also find the angle between them.
- 2. Show that the lines 2x+3y-4z=0=3x-4y+z-7; and 5x-y-3z+12=0=x-7y+5z-6 are parallel
- 3. Show that the four points (-8,5,2),(-5,2,2),(-7,6,6), (-4,3,6) are concyclic
- 4. Find the enveloping cone of the sphere $x^2+y^2+z^2-2x+4z = 1$ with its vertex at (1,1,1)
- 5. Find the equation of the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which passes through the curve $x^2+y^2=16$, z=0
- 6. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

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