Pre-AP Algebra 2
Unit 9 – Lesson 1 – Rational Exponents

Objectives:

- Students will understand that a radical can be represented as a rational exponent
- Students will be able to convert between radicals and rational exponents

Materials: Do Now and answers overhead; note-taking templates; practice worksheet; homework #9-1

<table>
<tr><th>Time</th><th>Activity</th></tr>
<tr><td>15 min</td><td>DO NOW
- Student investigate rational exponents using their calculators</td></tr>
<tr><td>30 min</td><td>Direct Instruction

Mathematicians wanted a way to write radical expressions – those with the root symbol – using exponents, so that they could be worked with just like all other numbers. Here is a proof that shows why it works.

<table>
<tr><th>Step</th><th>Reason</th></tr>
<tr><td>$\sqrt{a} \cdot \sqrt{a} = a$</td><td>By the definition of a square root</td></tr>
<tr><td>$a^k \cdot a^k = a$</td><td>Define the radical as an unknown exponent</td></tr>
<tr><td>$a^{2k} = a$</td><td>When multiplying, add exponents</td></tr>
<tr><td>$a^{2k} = a^1$</td><td>Anything to the 1st power is itself</td></tr>
<tr><td>$2k = 1$</td><td>If the powers are equal, the exponents must be too</td></tr>
<tr><td>$k = \frac{1}{2}$</td><td>Solve for k. Thus, $\sqrt{a} = a^{\frac{1}{2}}$</td></tr>
</table>
This proof could be repeated with any other n-th roots. You try! Prove that $\sqrt[3]{a} = a^{\frac{1}{3}}$.
Examples
1. $\sqrt[7]{128} = 128^{\frac{1}{7}} = 2$
2. $\sqrt[4]{625} = 625^{\frac{1}{4}} = 5$
3. $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$
4. $49^{\frac{1}{2}} = \sqrt[2]{49} = 7$
What about more complex functions?
5. $8^{\frac{2}{3}}$
6. $25^{\frac{3}{2}}$
7. $16^{-\frac{5}{4}}$
8. $64^{\frac{7}{6}}$
9. $81^{-\frac{3}{4}}$
What about combinations?
10. $3^{\frac{4}{3}} \cdot 3^{\frac{5}{3}}$
11. $(7^3)^{\frac{2}{3}}$
12. $8^{-\frac{5}{3}} \cdot 8^{\frac{6}{3}}$
13. $\frac{32^{\frac{7}{5}}}{32^{\frac{9}{5}}}$
14. $\left(64^{\frac{3}{2}}\right)^{-\frac{1}{3}}$</td></tr>
<tr><td>20 min</td><td>Pair Work
Practice worksheet</td></tr>
<tr><td></td><td></td></tr>
</table>

Homework #9-1: Rational Exponents

Pre-AP Algebra 2 **Name: ___________________________**
DO NOW

DO NOW

Find the exact values for each of the following

$\sqrt[2]{4} =$	$(-125)^{\frac{1}{3}} =$
$\sqrt[3]{-8} =$	$4^{\frac{1}{2}} =$
$\sqrt[3]{27} =$	$(-8)^{\frac{1}{3}} =$
$\sqrt[3]{-125} =$	$(-16)^{\frac{1}{2}}$
$\sqrt[2]{-16} =$	$27^{\frac{1}{3}}$
$\sqrt[4]{81} =$	$81^{\frac{1}{4}}$

What pattern do you see?

How could you find $\sqrt[6]{729}$? Try it!

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9-1 Pair Work

Practice with Rational Exponents

1) **Rewrite** each radical using rational exponent notation.

a. $\sqrt[3]{7} =$ b. $\left(\sqrt{11}\right)^5 =$ c. $\sqrt[4]{x^8} =$

2) **Rewrite** each power using radical notation.

a. $43^{1/5} =$ b. $8^{-3/4} =$ c. $x^{5/2} =$

3) **Find** the exact, simplified value of each expression **without a calculator**. *If you are stuck, try converting between radical and rational exponential notation first, and then simplify.* Sometimes, simplifying the exponent (or changing a decimal to a fraction) is very helpful.

a. $8^{2/3} =$ b. $(-27)^{2/3} =$ c. $25^{-3/2} =$

d. $\left(\frac{8}{27}\right)^{-2/3} =$ e. $4^{1.5} =$ f. $\left(\frac{1}{4}\right)^{-1.5} =$

g. $\left(\sqrt[3]{64}\right)^4 =$ h. $\left(\sqrt{3}\right)^6 =$ i. $\left(\sqrt[4]{3}\right)^{-8} =$

4) **Simplify** each expression completely.

a. $5^{1/4} \times 5^{7/4} =$ b. $(2^{1/3})^{3/4} =$

c. $\frac{7^{1/5}}{7^{3/5}} =$ d. $\left(2^{1/4} \times 2^{1/3}\right)^6 =$

e. $\frac{12^{11/8}}{12^{-5/8}} =$ f. $\frac{5x^{3/4}yz^{-1/3}}{10x^{1/4}z^{2/3}} =$

Pre-AP Algebra 2 **Name: ___________________________**
Homework #9-1

Homework #9-1: Rational Exponents

Part 1

1) **Find** the exact, simplified value of each expression **without a calculator**. *If you are stuck, try converting between radical and rational exponential notation first, and then simplify.* Sometimes, simplifying the exponent (or changing a decimal to a fraction) is very helpful.

a. $125^{1/3} =$ b. $64^{-1/2} =$ c. $64^{1/6} =$

d. $81^{1/2} =$ e. $32^{-1/5} =$ f. $81^{-1/4} =$

g. $4^{3/2} =$ h. $(-64)^{2/3} =$ i. $(-8)^{-5/3} =$

j. $9^{-3/2} =$ k. $\left(\frac{9}{4}\right)^{3/2} =$ l. $16^{-1.5} =$

m. $\left(\sqrt[3]{-27}\right)^2 =$ n. $\sqrt[3]{125^2} =$ o. $\left(\sqrt[3]{4}\right)^6 =$

p. $\left(\sqrt{5}\right)^{-2} =$ q. $\left(\sqrt[4]{2}\right)^{-4} =$ r. $\left(\sqrt[5]{3}\right)^5 =$

2) **Simplify** each expression completely.

a. $3^{5/3} \times 3^{1/3} =$ b. $\left(5^{2/3}\right)^{1/2} =$

c. $\frac{1}{36^{-1/2}} =$ d. $\left(\frac{5^2}{8^2}\right)^{-1/2} =$

e. $\frac{125^{1/9}}{5^{1/4}} =$ f. $\left(10^{3/4} \times 4^{3/4}\right)^{-4} =$

Part 2: STAAR Practice

1) A parabola has the function $f(x) = 2(x + 3)^2 – 5$. It is translated to a new location, given by the function $g(x) = 2(x – 3)^2 – 2$. Describe the translation.

a. 6 left and 3 up
b. 6 left and 3 down
c. 6 right and 3 down
d. 6 right and 3 up

2) What is the highest point on the function $y = -(x – 5)^2 + 3$?

a. $(1,-13)$ b. $(0,-22)$ c. $(5,3)$ d. $(-5,3)$

3) A certain radioactive element decays over time according to the equation $y = A\left(\frac{1}{2}\right)^{t/300}$ where A is the number of grams present initially and t is the time in years. If 1000 grams were present initially, how many grams will remain after 900 years?

a. 500 grams b. 250 grams c. 125 grams d. 62.5 grams

4) Given the equation $\sqrt{\frac{x}{y}} = 4$, which of the following represents y in terms of x?

a. $y = \frac{x}{2}$ b. $y = \frac{2}{x}$ c. $y = \frac{x}{16}$ d. $y = \frac{16}{x}$

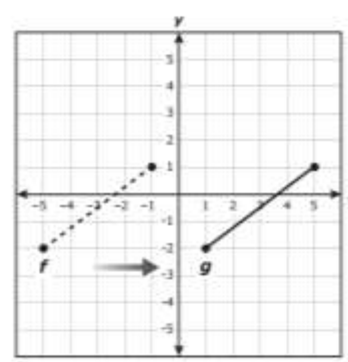
5) Simplify: $\frac{5}{x+7} - \frac{10}{x^2+2x-35}$

a. $\frac{5x-6}{x^2+2x-35}$ b. $\frac{5(x-5)}{x^2+2x-35}$ c. $\frac{5(x-3)}{x^2+2x-35}$ d. $\frac{5(x-7)}{x^2+2x-35}$

6) Which figure best describes the graph of $2x^2 + 5y^2 - 2x - 10y - 15 = 0$

a. circle b. ellipse c. parabola d. hyperbola

7) The graph of the function g was obtained from the graph of the function f using a transformation as shown above. Based on the graph, which equation can be used to describe $g(x)$ in terms of $f(x)$?



a. $g(x) = f(x) + 6$
b. $g(x) = f(x + 6)$
c. $g(x) = f(x) - 6$
d. $g(x) = f(x - 6)$

Lesson Name: *Rational Exponents* **Date:** ______________ **Student:** ______________________

Concepts	Examples
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More Examples
What about combinations? 1. $3^{\frac{4}{3}} \cdot 3^{\frac{5}{3}}$ 2. $(7^3)^{\frac{2}{3}}$ 3. $8^{-\frac{5}{3}} \cdot 8^{\frac{6}{3}}$ 4. $\frac{32^{\frac{7}{5}}}{32^{\frac{9}{5}}}$ 5. $\left(64^{\frac{3}{2}}\right)^{-\frac{1}{3}}$