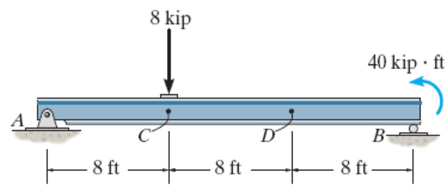
HW 10 SOLUTIONS

•7–1. Determine the internal normal force and shear force, and the bending moment in the beam at points C and D. Assume the support at B is a roller. Point C is located just to the right of the 8-kip load.



Support Reactions **: FBD (a).**

$\circlearrowleft + \Sigma M_A = 0;\quad B_y(24) + 40 - 8(8) = 0 \quad B_y = 1.00 \text{ kip}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 1.00 - 8 = 0 \quad A_y = 7.00 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x = 0$

Internal Forces **: Applying the equations of equilibrium to segment AC [FBD (b)], we have**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_C = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 7.00 - 8 - V_C = 0 \quad V_C = -1.00 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad M_C - 7.00(8) = 0 \quad M_C = 56.0 \text{ kip}\cdot\text{ft}$ **Ans**

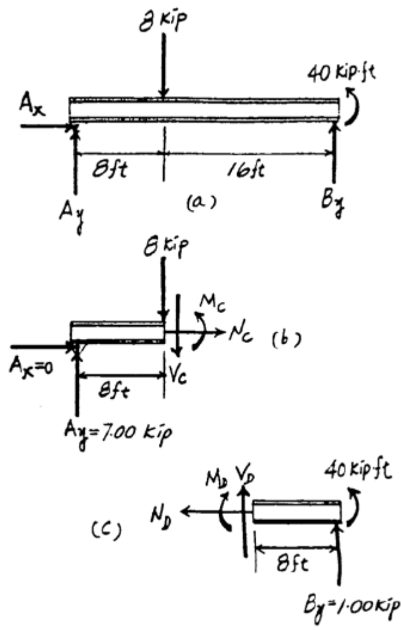
Applying the equations of equilibrium to segment BD [FBD (c)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans**

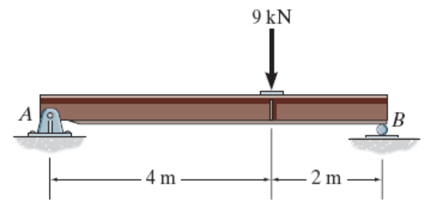
$+\uparrow \Sigma F_y = 0;\quad V_D + 1.00 = 0 \quad V_D = -1.00 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0;\quad 1.00(8) + 40 - M_D = 0$

$M_D = 48.0 \text{ kip}\cdot\text{ft}$ **Ans**



•**7–41.** Draw the shear and moment diagrams for the simply supported beam.



Since the loading discontinues at the 9-kN concentrated force, the shear and moment equations must be written for the regions $0 \le x < 4$ m and $4 \text{ m} < x \le 6$ m of the beam. The free-body diagrams of the beam's segment sectioned through the arbitrary points in these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < 4$ m, Fig. *b*

$+\uparrow \Sigma F_y = 0; \quad 3 - V = 0 \quad\quad V = 3 \text{kN} \quad\quad (1)$

$\circlearrowleft + \Sigma M = 0; \quad M - 3x = 0 \quad\quad M = \{3x\} \text{kN} \cdot \text{m} \quad\quad (2)$

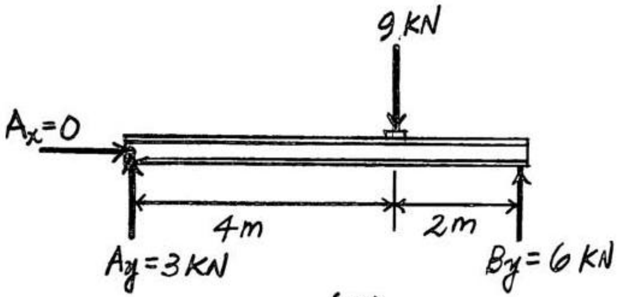
Region $4 \text{ m} < x \le 6$ m, Fig. *c*

$+\uparrow \Sigma F_y = 0; \quad V + 6 = 0 \quad\quad V = -6 \text{ kN} \quad\quad (3)$

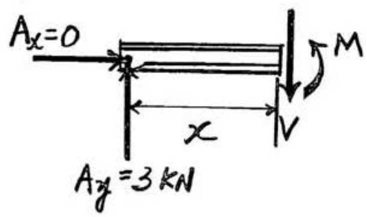
$\circlearrowleft + \Sigma M = 0; \quad 6(6 - x) - M = 0 \quad\quad M = \{36 - 6x\} \text{ kN} \cdot \text{m} \quad\quad (4)$

The shear and moment diagrams in Figs. *d* and *e* are plotted using Eqs. (1) and (3), and Eqs. (3) and (4), respectively. The values of the moment at $x = 4$ m are evaluated using either Eqs. (2) or (4),

$$M\big|_{x=4\text{ m}} = 3(4) = 12 \text{ kN} \cdot \text{m or } M\big|_{x=4\text{m}} = 36 - 6(4) = 12 \text{ kN} \cdot \text{m}$$

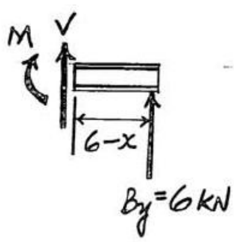


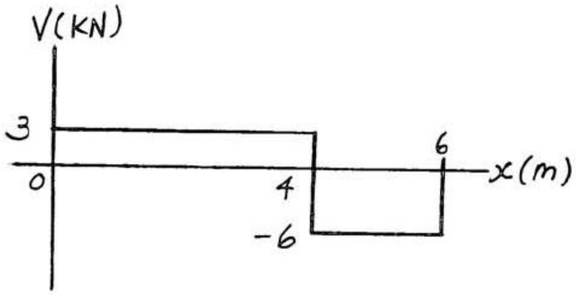
(a)



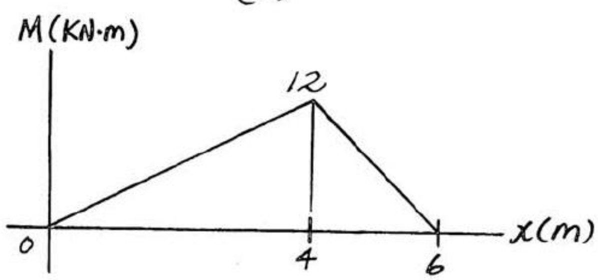
$0 \le x < 4m$ (b)

(C) $4m < x \le 6m$



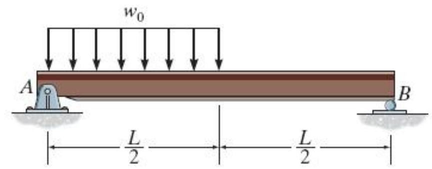


(d)



(e)

7–46. Draw the shear and moment diagrams for the simply supported beam.



Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < L/2$ and $L/2 < x \le L$ of the beam. The free - body diagram of the beam's segments sectioned through arbitrary points in these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < \frac{L}{2}$, Fig. *b*

$+\uparrow \Sigma F_y = 0;\quad \frac{3}{8}w_0L - w_0x - V = 0 \qquad V = w_0\left(\frac{3}{8}L - x\right) \qquad (1)$

$\circlearrowleft + \Sigma M = 0;\quad M + w_0x\left(\frac{x}{2}\right) - \frac{3}{8}w_0L(x) = 0 \qquad M = \frac{w_0}{8}(3Lx - 4x^2) \qquad (2)$

Region $L/2 < x \le L$, Fig. *c*

$+\uparrow \Sigma F_y = 0;\quad V + \frac{w_0L}{8} = 0 \qquad V = -\frac{w_0L}{8} \qquad (3)$

$\circlearrowleft + \Sigma M = 0;\quad \frac{w_0L}{8}(L - x) - M = 0 \qquad M = \frac{w_0L}{8}(L - x) \qquad (4)$

The shear diagram is plotted using Eqs. (1) and (3). The location at where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

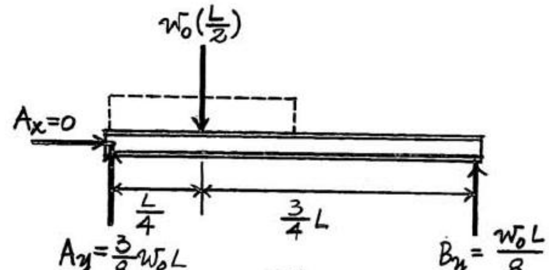
$$0 = w_0\left(\frac{3}{8}L - x\right) \qquad x = \frac{3}{8}L$$

The moment diagram is plotted using Eqs. (2) and (4). The value of the moment at $x = \frac{3}{8}L$ $(V = 0)$ can be evaluated using Eq. (2).

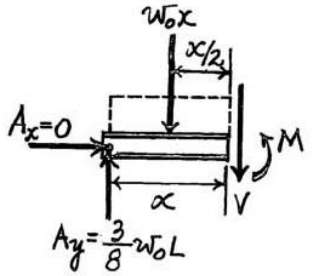
$$M\Big|_{x=\frac{3}{8}L} = \frac{w_0}{8}\left(3L\left(\frac{3}{8}L\right) - 4\left(\frac{3}{8}L\right)^2\right) = \frac{9}{128}w_0L^2$$

The value of the moment at $x = L/2$ is evaluated using either Eqs. (3) or (4).

$$M\Big|_{x=\frac{L}{2}} = \frac{w_0L}{8}\left(L - \frac{L}{2}\right) = \frac{w_0L^2}{16}$$

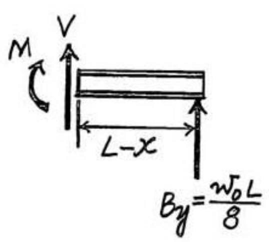


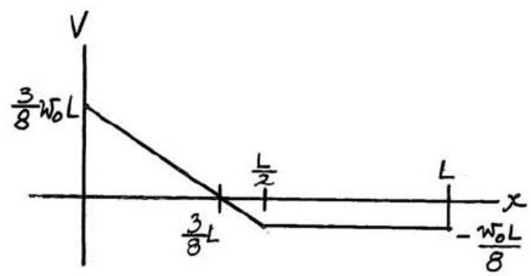
(a)



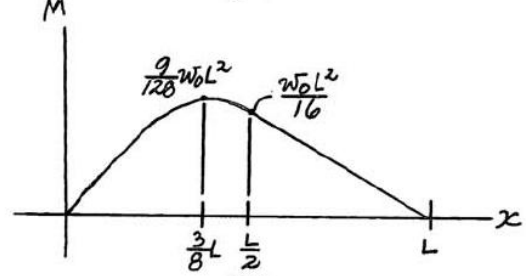
$0 \le x < \frac{L}{2}$ (b)

(c) $\frac{L}{2} < x \le L$



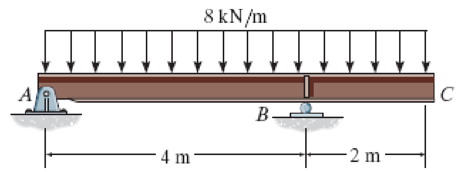


(d)



(e)

*7–48. Draw the shear and moment diagrams for the overhang beam.



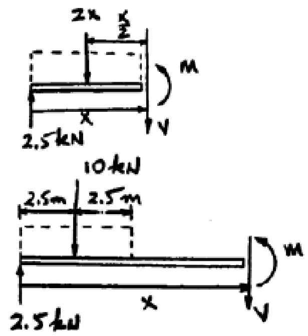
$0 \le x < 5$ m:

$+\uparrow \Sigma F_y = 0; \quad 2.5 - 2x - V = 0$

$$V = 2.5 - 2x$$

$\circlearrowleft + \Sigma M = 0; \quad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$$M = 2.5x - x^2$$



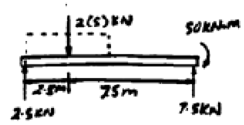
5 m $< x \le 10$ m:

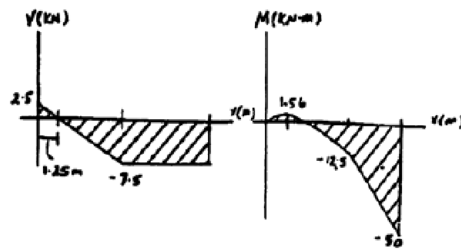
$+\uparrow \Sigma F_y = 0; \quad 2.5 - 10 - V = 0$

$$V = -7.5$$

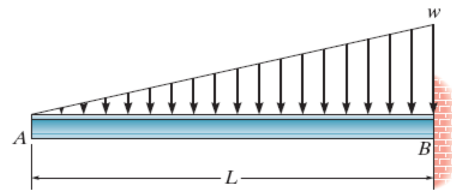
$\circlearrowleft + \Sigma M = 0; \quad M + 10(x - 2.5) - 2.5x = 0$

$$M = -7.5x + 25$$





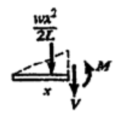
7–54. If $L = 18$ ft, the beam will fail when the maximum shear force is $V_{max} = 800$ lb, or the maximum moment is $M_{max} = 1200$ lb·ft. Determine the largest intensity w of the distributed loading it will support.



For $0 \le x \le L$

$+\uparrow \Sigma F_y = 0;$ $\quad V = -\dfrac{wx^2}{2L}$

$\circlearrowleft + \Sigma M = 0;$ $\quad M = -\dfrac{wx^3}{6L}$



$$V_{max} = \frac{-wL}{2}$$

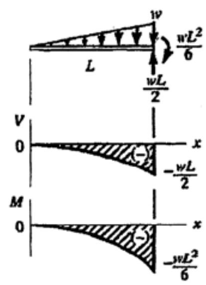
$$-800 = \frac{-w(18)}{2}$$

$w = 88.9$ lb/ft

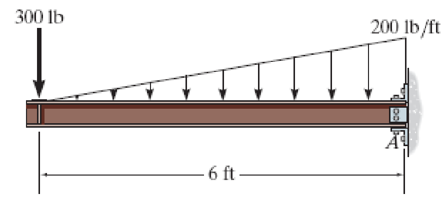
$$M_{max} = -\frac{wL^2}{6};$$

$$-1200 = \frac{-w(18)^2}{6}$$

$w = 22.2$ lb/ft **Ans**



***7–56.** Draw the shear and moment diagrams for the cantilevered beam.



The free - body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

$$w = 200\left(\frac{x}{6}\right) = 33.33x$$

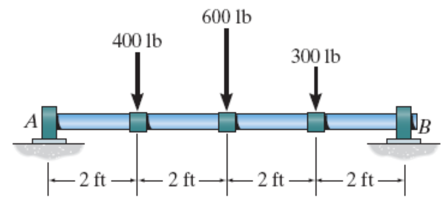
Referring to Fig. *b*,

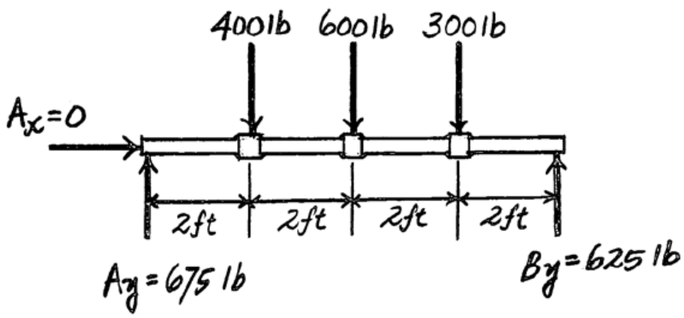
$+\uparrow \Sigma F_y = 0;$ $\quad -300 - \frac{1}{2}(33.33x)(x) - V = 0$ $\quad V = \{-300 - 16.67x^2\}\ \text{lb}$ (1)

$\circlearrowleft + \Sigma M = 0;$ $\quad M + \frac{1}{2}(33.33x)(x)\left(\frac{x}{3}\right) + 300x = 0$ $\quad M = \{-300x - 5.556x^3\}\ \text{lb}\cdot\text{ft}$ (2)

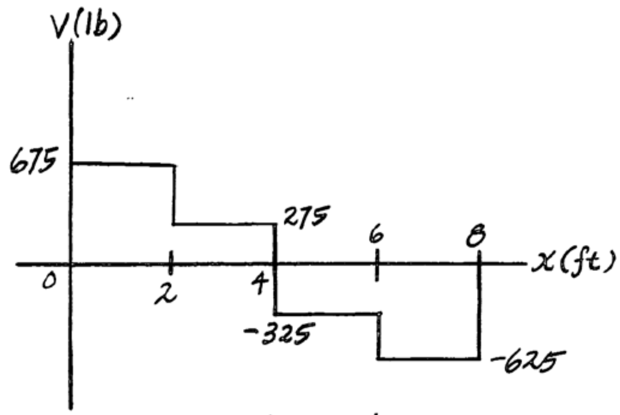
The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively.

•**7–65.** The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.

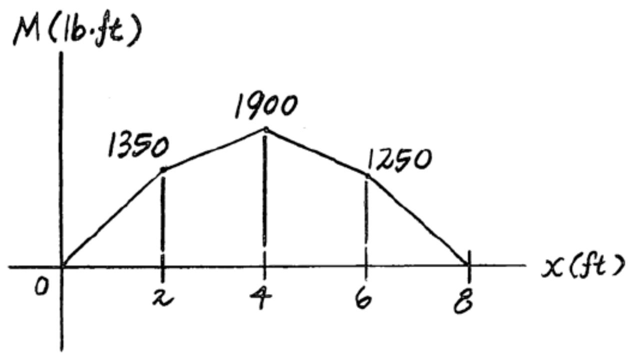




(a)

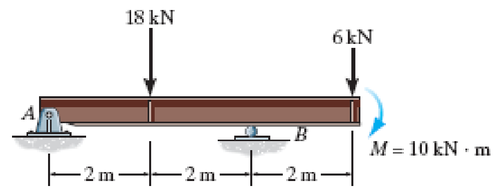


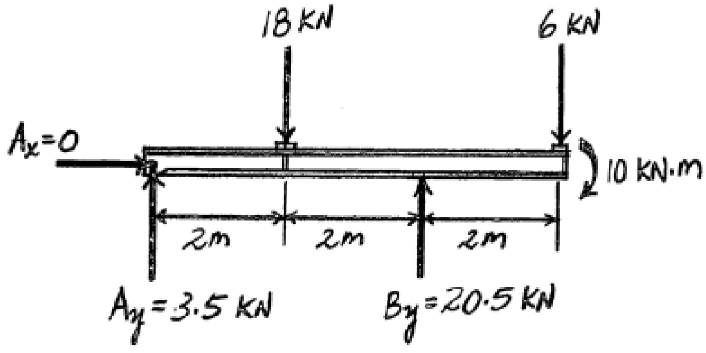
Shear diagram
(b)



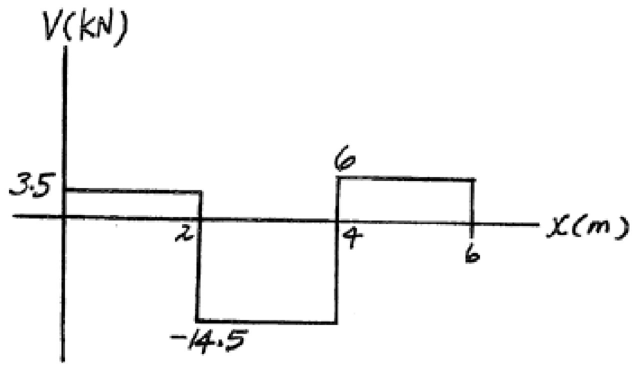
Moment diagram
(c)

7–67. Draw the shear and moment diagrams for the overhang beam.



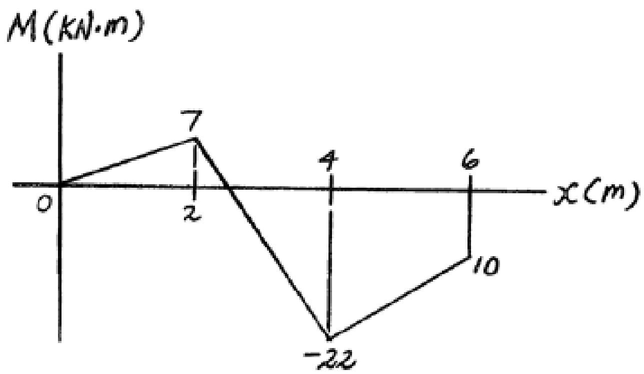


(a)



shear diagram

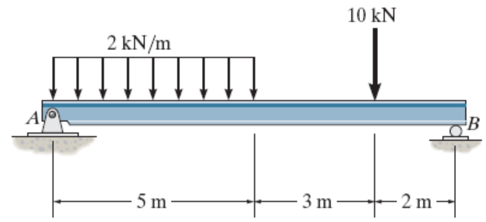
(b)



moment diagram

(c)

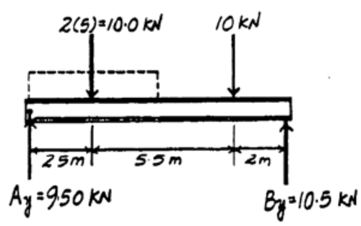
*7–76. Draw the shear and moment diagrams for the beam.

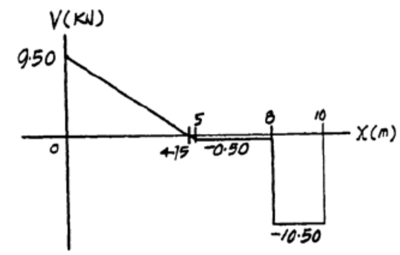


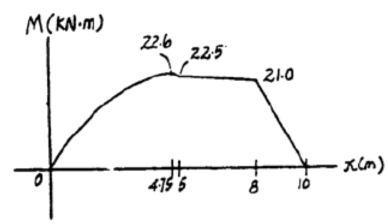
Support Reactions :

$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(10) - 10.0(2.5) - 10(8) = 0 \quad B_y = 10.5\ \text{kN}$$

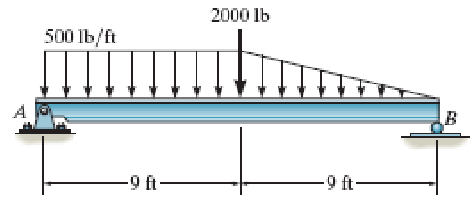
$$+\uparrow \Sigma F_y = 0; \quad A_y + 10.5 - 10.0 - 10 = 0 \quad A_y = 9.50\ \text{kN}$$

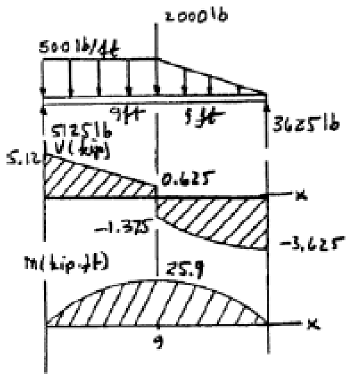




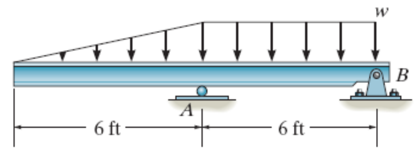


•**7–81.** Draw the shear and moment diagrams for the beam.





•**7–85.** The beam will fail when the maximum moment is $M_{max} = 30$ kip · ft or the maximum shear is $V_{max} = 8$ kip. Determine the largest intensity w of the distributed load the beam will support.



$V_{max} = 4w; \quad 8 = 4w$

$w = 2$ kip/ft

$M_{max} = -6w; \quad -30 = -6w$

$w = 5$ kip/ft

Thus, $\quad w = 2$ kip/ft $\qquad$ **Ans**