Intermediate Macroeconomics

Lecture 1 - Introduction to Economic Growth

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Sciences Po

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About the course I.

- 2-hour lecture every week, Wednesdays from 12:30-14:30
- 3 big topics covered:
  1. economic growth
  2. economic fluctuations
  3. governments
- last lecture: 2-hour exam
- I plan to have a weekly office hour, in office E.404, time to be announced
- email: zsofia.barany@sciences-po.org
- lecture notes will become available weekly on my website

About the course II.

- besides the lectures, there will be 6 tutorials
- September 15, **September 29**, October 13, **November 10**, November 17, December 1
- the tutorials complement the lectures:
  - 1st covers the mathematics used in the course
  - other 5 goes over the problem sets
- you will receive the solution to the problem sets
- your tutor: Alexis Le Chapelain, questions related to problem sets go to him

What is expected from you?

- attend all the lectures, this is compulsory
- do all the assigned problem sets, there will be 5 in total
- hand in 2 problem sets on the Monday before the bold dates, I will remind you during the lecture before
- these two will be marked
- final grade: exam mark and mark from two problem sets
- tutorials work like classes: your tutor will go over the solution of the problem sets
- tutorials are not compulsory, but **highly recommended**, as the exam will be similar to the problem sets

Let's start!

Why is economic growth important?

- economic growth is defined as the growth in **real GDP per capita**
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- economic growth is defined as the growth in **real GDP per capita**
- real GDP per capita approximates the standard of living
- this measure neglects lots of important factors
  such as health, education, environment
- ultimate interest: happiness or well-being
  this is *VERY* difficult to measure
- some attempts:
  - Richard Layard: Happiness - Lessons from a New Science
  - OECD Better Life Initiative

Economic growth

- is about changes in the real GDP per capita on a very long horizon
- comparisons over long periods of time are difficult
  - even if *nominal income* data is available, we need *price indices* to transform it into *real* data
  - adjustment for **quality changes** and introduction of **new goods** is difficult
- however: average real incomes in the US and in Western Europe are
  - 10 to 30 times larger than 100 years ago
  - 50 to 300 times larger than 200 years ago

Patterns of growth over time

Worldwide growth rates are far from constant

- growth rates rising throughout most of modern history, in industrialised economies: $g_{20th} > g_{19th} > g_{18th}$
- average income at the beginning of the industrial revolution was not far above subsistence levels $\rightarrow$ average growth before must have been very low
- exception: productivity growth slowdown from the 1970s in the US and other industrialised countries - 1 percentage point below its earlier level

Patterns of growth across countries

- average real incomes in the US, Germany, Japan exceed those in Bangladesh, Kenya by 10 to 20 times
- these differences are not immutable - small, sustained differences in growth rates could lead to a country catching up or falling behind
- cross-country differences in income per capita have widened on average:
  at the time of the industrial revolution all countries were close to subsistence level, now there are huge differences
- over the past few decades no clear tendency towards continued divergence or convergence

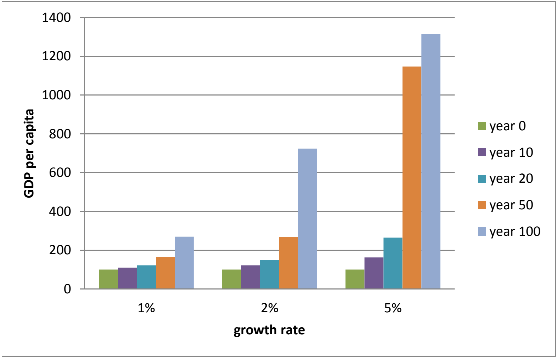
GDP per capita in 1820 and 1998 (in 1990$)

	1820	1998	growth rate
Western Europe	1202	18137	1.5%
America, Canada, Oceania	1202	25767	1.7%
Eastern Europe	683	5550	1.2%
Former USSR	688	3907	1.0%
Latin America	691	5837	1.2%
Japan	669	20662	1.9%
China	600	2993	0.9%
East Asia (incl. China and Japan)	556	1405	0.5%
West Asia	607	5623	1.3%
Africa	420	1444	0.7%
World Average	667	5729	1.2%

Data: Maddison

Small, sustained differences matter

Implications for human welfare are ENORMOUS



"Once one starts to think about [economic growth] it is hard to think about anything else."

Robert Lucas

Economic growth is important.

Preview of the Solow model

- aggregate production function: output produced from capital and labour
- if you have more capital and/or more labour, you can produce more output $\rightarrow$
- this is an **extensive** model of growth
- assumption: labour supply grows at a fixed rate
- so growth mainly depends on capital accumulation

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Main conclusion:

the accumulation of physical capital cannot account for

- the huge growth over time in output per person
- the big geographic differences in output per person

The Solow model

An explanation for economic growth based on the **factors of production**:

- K(t) - capital stock at time t: physical capital, i.e. machines, buildings
- L(t) - labour input at time t: quantity of work-hours per year, or number of workers

Production function:

$$Y = F(K, L)$$

where Y is the amount of output produced.

Important features:

- output *only* depends on the quantity of labour and capital
- no other inputs, such as land or natural resources, matter

Characteristics of the production function I.

- $F(K, L)$ is monotone increasing in K and L

  positive marginal product of both inputs

  - $F_K = \frac{\partial F}{\partial K} > 0$: if K increases, $F(K, L)$ increases
  - $F_L = \frac{\partial F}{\partial L} > 0$: if L increases, $F(K, L)$ increases

- $F(K, L)$ is concave - decreasing returns to K and L

  decreasing marginal product of both inputs

  - $F_{KK} = \frac{\partial^2 F}{\partial K^2} = \frac{F_K}{\partial K} < 0$ and $F_{LL} = \frac{\partial^2 F}{\partial L^2} = \frac{F_L}{\partial L} < 0$
  - as K increases more-and-more, $F(K, L)$ increases by less-and-less
  - as L increases more-and-more, $F(K, L)$ increases by less-and-less

Characteristics of the production function II.

- homogeneous of degree one - if K and L double, then output, Y doubles:

$$F(\lambda K, \lambda L) = \lambda F(K, L) \quad \text{for} \quad \forall \lambda > 0$$

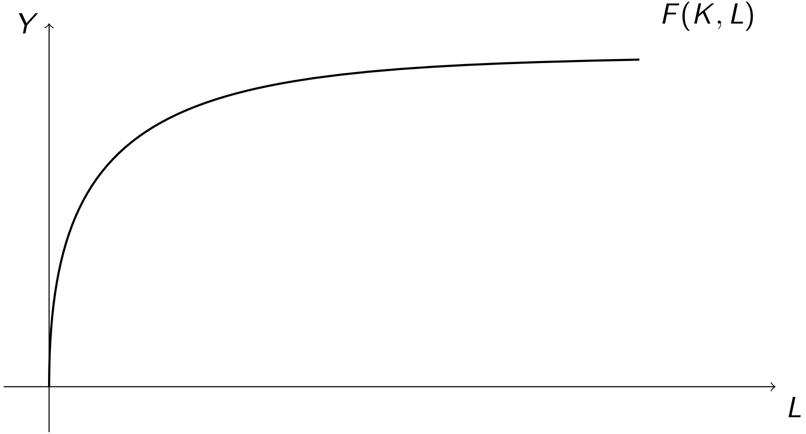
consequence of homogeneity:

- if we pick $\lambda = 1/L$, we get:

$$\frac{Y}{L} = F(\frac{K}{L}, \frac{L}{L}) \quad \Leftrightarrow \quad y = f(k) = F(\frac{K}{L}, 1)$$

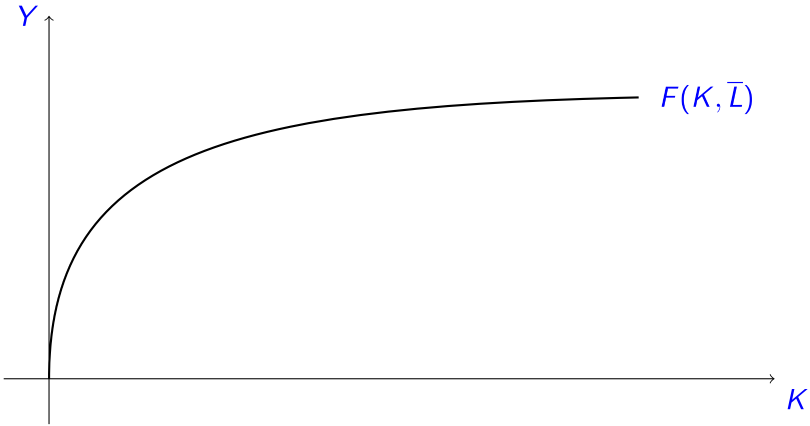
$\rightarrow$ allows us to work with **per capita** quantities

Shape of the production function



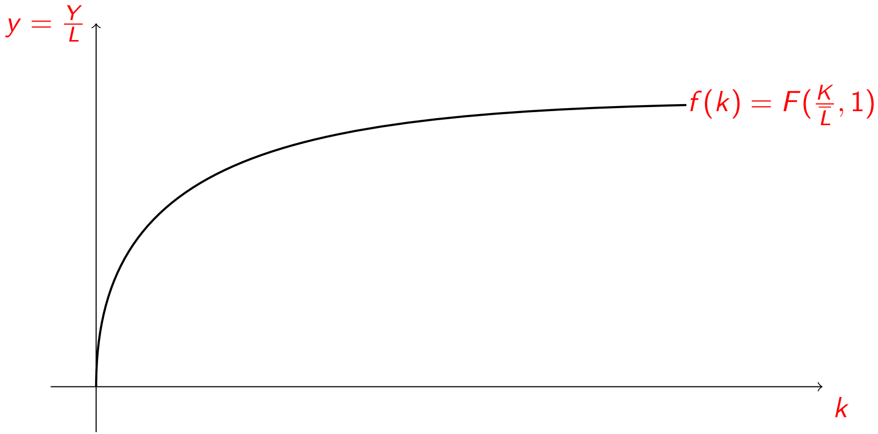
For a fixed capital level, $\overline{K}$, Y looks something like this.

Shape of the production function



For a fixed labour supply, $\overline{L}$, Y looks something like this.

Shape of the production function



The per capita production function looks something like this.

The growth rate of labour

We assume that the population grows at a constant rate:

$$\dot{L}(t) = nL(t)$$

Where n is an exogenous parameter, and

$$\dot{L}(t) = \frac{\partial L(t)}{\partial t}$$

The growth rate of L is then

$$\frac{\dot{L}(t)}{L(t)} = n$$

This implies that for a given initial population, $L(0)$:

$$L(t) = L(0)e^{nt}$$

The growth rate of capital I.

Remember that output is divided between consumption, investment, and government spending:

$$Y = C + I + G$$

- no government: $G = 0$
- people save a constant fraction, s, of their income $\rightarrow$ remaining income is consumed: $C = (1 - s)Y$

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$$Y = C + I = (1 - s)Y + I$$

by simplifying we get:

$$I = sY$$

All savings are invested.

The growth rate of capital II.

Capital

- increases - due to investment
- decreases - due to depreciation

The change in the stock of capital is:

$$\dot{K}(t) = I(t) - \delta K(t) = sY(t) - \delta K(t)$$

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The growth of output $Y(t) = F(K(t), L(t))$, depends on the growth of

- $L(t)$ - which is exogenous
- $K(t)$ - this is endogenous

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- $K(t)$ - this is endogenous

Goal: determine the behaviour of the economy. Need to analyse the behaviour of capital or per capita capital.

The evolution of per capita capital

The change in the per capita capital is defined as:

$$\dot{k}(t) = \frac{\partial \frac{K(t)}{L(t)}}{\partial t}$$

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Substituting $\dot{L}(t) = nL(t)$ and $\dot{K}(t) = sY(t) - \delta K(t)$:

$$\dot{k}(t) = \frac{sY(t)}{L(t)} - \frac{\delta K(t)}{L(t)} - \frac{K(t)}{L(t)^2}nL(t)$$

The evolution of per capita capital

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Substituting $\dot{L}(t) = nL(t)$ and $\dot{K}(t) = sY(t) - \delta K(t)$:

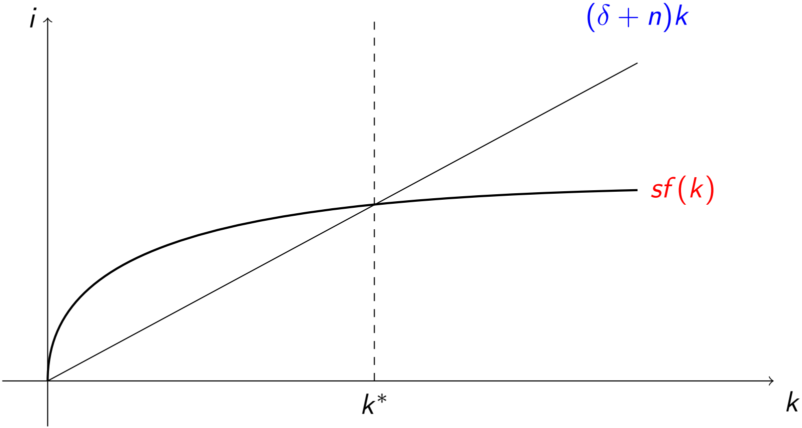
$$\dot{k}(t) = \frac{sY(t)}{L(t)} - \frac{\delta K(t)}{L(t)} - \frac{K(t)}{L(t)^2} nL(t)$$

Using that $\frac{Y(t)}{L(t)} = y(t) = f(k(t))$ we get:

$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t)$$

This is the **key equation** of the Solow model.

$$\dot{k}(t) = \underbrace{sf(k(t))}_{\text{actual investment}} - \underbrace{(\delta + n)k(t)}_{\text{break-even investment}}$$



$$\dot{k}(t) = \underbrace{sf(k(t))}_{\text{actual investment}} - \underbrace{(\delta + n)k(t)}_{\text{break-even investment}}$$

break-even investment: just to keep k at its existing level

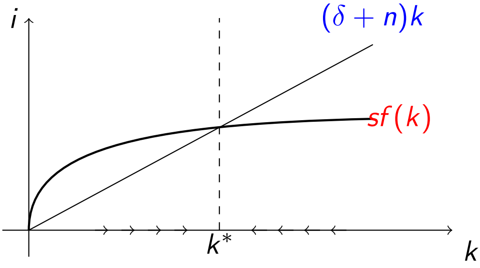
Two reasons that some investment is needed to prevent k from falling:

1. existing capital is depreciating (δk), this capital needs to be *replaced*
2. the quantity of labour is increasing $\rightarrow$ it is not enough to keep K constant, since then k is falling at rate n

Three cases:

- $sf(k) > (n + \delta)k \rightarrow k$ is rising
- $sf(k) < (n + \delta)k \rightarrow k$ is falling
- $sf(k) = (n + \delta)k \rightarrow k$ is constant

The balanced growth path



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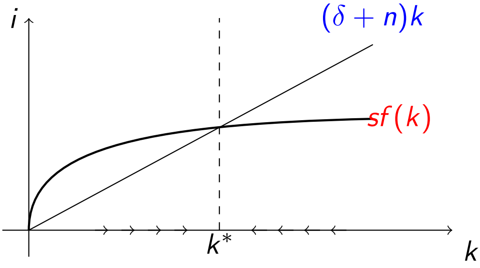

This implies that from any starting point the economy converges to k^*.

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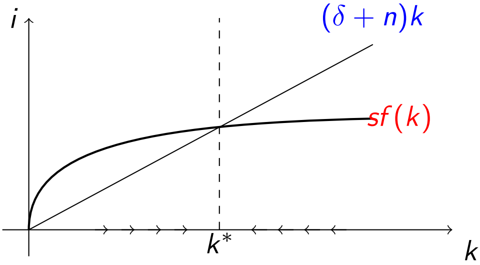

This implies that from any starting point the economy converges to k^*.

$k(t) = k^*$

$y(t) = f(k^*)$

$K(t) = L(t)k^* = k^* L_0 e^{nt}$

$Y(t) = L(t)f(k^*) = f(k^*)L_0 e^{nt}$