Laplace Transform solved problems

Pavel Pyrih

May 24, 2012

Acknowledgement. The following problems were solved using my own procedure in a program Maple V, release 5, using commands from

Bent E. Petersen: Laplace Transform in Maple
http://people.oregonstate.edu/~peterseb/mth256/docs/256winter2001_laplace.pdf

All possible errors are my faults.

1 Solving equations using the Laplace transform

Theorem.(Lerch) If two functions have the same integral transform then they are equal almost everywhere.

This is the right key to the following problems.

Notation.(Dirac & Heaviside) The Dirac unit impuls function will be denoted by $\delta(t)$. The Heaviside step function will be denoted by $u(t)$.

1.1 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial}{\partial t}\,\mathrm{y}(t) = 3 - 2\,t$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) = 3\,\frac{1}{s} - 2\,\frac{1}{s^2}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s^2 + 3\,s - 2}{s^3}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-t^2 + 3\,t + \mathrm{y}(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$-t^2 + 3\,t$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = -t^2 + 3\,t + \_C1$$

Info.

$$\textit{polynomial}$$

Comment.

$$\textit{elementary}$$

1.2 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial}{\partial t}\,\mathrm{y}(t) = e^{(-3\,t)}$$

with initial conditions

$$\mathrm{y}(0) = 4$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$\mathit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) = \frac{1}{s+3}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + 3\,\mathrm{y}(0) + 1}{s\,(s+3)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{3} + \mathrm{y}(0) - \frac{1}{3}\,e^{(-3\,t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{13}{3} - \frac{1}{3}\,e^{(-3\,t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = -\frac{1}{3}\,e^{(-3\,t)} + \_C1$$

Info.

$$\mathit{exponential\_function}$$

Comment.

$$\mathit{elementary}$$

1.3 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + \mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

$$\textit{convolution}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) + \mathrm{Y}(s) = \mathrm{laplace}(\mathrm{f}(t),\, t,\, s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0) + \mathrm{laplace}(\mathrm{f}(t),\, t,\, s)}{s+1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\,e^{(-t)} + \int_0^t \mathrm{f}(\_U1)\,e^{(-t+\_U1)}\,d\_U1$$

With the initial conditions incorporated we obtain a solution in the form

$$a\,e^{(-t)} + \int_0^t \mathrm{f}(\_U1)\,e^{(-t+\_U1)}\,d\_U1$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = e^{(-t)} \int \mathrm{f}(t)\,e^t\,dt + e^{(-t)}\,\_C1$$

Info.

$$\textit{exp\_convolution}$$

Comment.

$$\textit{advanced}$$

1.4 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial}{\partial t} \mathrm{y}(t)) + \mathrm{y}(t) = e^t$$

with initial conditions

$$\mathrm{y}(0) = 1$$

$$\mathrm{Dy}(0) = 0$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) + \mathrm{Y}(s) = \frac{1}{s-1}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s - \mathrm{y}(0) + 1}{s^2 - 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{2}\,e^t + \mathrm{y}(0)\,e^{(-t)} - \frac{1}{2}\,e^{(-t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{1}{2}\,e^t + \frac{1}{2}\,e^{(-t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \frac{1}{2}\,e^t + e^{(-t)}\,\_C1$$

Info.

exponential_function

Comment.

elementary

1.5 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial}{\partial t}\,\mathrm{y}(t)) - 5\,\mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 2$$

$$\mathrm{Dy}(0) = b$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) - 5\,\mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)}{s-5}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\,e^{(5\,t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$2\,e^{(5\,t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\;e^{(5\,t)}$$

Info.

$$\textit{exponential\_function}$$

Comment.

$$\textit{elementary}$$

1.6 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial}{\partial t}\,\mathrm{y}(t)) - 5\,\mathrm{y}(t) = e^{(5\,t)}$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = b$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) - 5\,\mathrm{Y}(s) = \frac{1}{s-5}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s - 5\,\mathrm{y}(0) + 1}{s^2 - 10\,s + 25}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$t\,e^{(5\,t)} + \mathrm{y}(0)\,e^{(5\,t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$t\,e^{(5\,t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = t\,e^{(5\,t)} + \_C1\,e^{(5\,t)}$$

Info.

exponential_function

Comment.

elementary

1.7 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial}{\partial t}\,\mathrm{y}(t)) - 5\,\mathrm{y}(t) = e^{(5\,t)}$$

with initial conditions

$$\mathrm{y}(0) = 2$$

$$\mathrm{Dy}(0) = b$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,\mathrm{Y}(s) - \mathrm{y}(0) - 5\,\mathrm{Y}(s) = \frac{1}{s-5}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s - 5\,\mathrm{y}(0) + 1}{s^2 - 10\,s + 25}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$t\,e^{(5\,t)} + \mathrm{y}(0)\,e^{(5\,t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$t\,e^{(5\,t)} + 2\,e^{(5\,t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = t\,e^{(5\,t)} + \_C1\,e^{(5\,t)}$$

Info.

exponential_function

Comment.

elementary

1.8 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

convolution

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) = \mathrm{laplace}(\mathrm{f}(t),\, t,\, s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + \mathrm{laplace}(\mathrm{f}(t),\, t,\, s)}{s^2}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0) + \mathrm{D}(y)(0)\,t + \int_0^t \mathrm{f}(\_U1)\,(t - \_U1)\,d\_U1$$

With the initial conditions incorporated we obtain a solution in the form

$$a + b\,t + \int_0^t \mathrm{f}(\_U1)\,(t - \_U1)\,d\_U1$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \int\int \mathrm{f}(t)\,dt + \_C1\;dt + \_C2$$

Info.

convolution

Comment.

advanced

1.9 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t) = 1 - t$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) = \frac{1}{s} - \frac{1}{s^2}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + \mathrm{D}(y)(0)\,s^2 + s - 1}{s^4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-\frac{1}{6}\,t^3 + \frac{1}{2}\,t^2 + \mathrm{D}(y)(0)\,t + \mathrm{y}(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$-\frac{1}{6}\,t^3 + \frac{1}{2}\,t^2$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \frac{1}{2}\,t^2 - \frac{1}{6}\,t^3 + \_C1\,t + \_C2$$

Info.

$$\textit{polynomial}$$

Comment.

$$\textit{elementary}$$

1.10 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t) = 2\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + \mathrm{y}(t)$$

with initial conditions

$$\mathrm{y}(0) = 3$$

$$\mathrm{Dy}(0) = 6$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) = 2\,s\,\mathrm{Y}(s) - 2\,\mathrm{y}(0) + \mathrm{Y}(s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) - 2\,\mathrm{y}(0)}{s^2 - 2\,s - 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{2}\,e^t\,\sqrt{2}\,\mathrm{D}(y)(0)\sinh(\sqrt{2}\,t) - \frac{1}{2}\,e^t\,\mathrm{y}(0)\,\sqrt{2}\sinh(\sqrt{2}\,t) + e^t\,\mathrm{y}(0)\cosh(\sqrt{2}\,t)$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{3}{2}\,e^t\,\sqrt{2}\sinh(\sqrt{2}\,t) + 3\,e^t\cosh(\sqrt{2}\,t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\;e^{((\sqrt{2}+1)\,t)} + \_C2\;e^{(-(\sqrt{2}-1)\,t)}$$

Info.

$$3\,e^{(2\,t)}$$

Comment.

elementary

1.11 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t) = 3 + 2\,t$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) = 3\,\frac{1}{s} + 2\,\frac{1}{s^2}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + \mathrm{D}(y)(0)\,s^2 + 3\,s + 2}{s^4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{3}\,t^3 + \frac{3}{2}\,t^2 + \mathrm{D}(y)(0)\,t + \mathrm{y}(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{1}{3}\,t^3 + \frac{3}{2}\,t^2 + b\,t + a$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \frac{3}{2}\,t^2 + \frac{1}{3}\,t^3 + \_C1\,t + \_C2$$

Info.

$$\textit{polynomial}$$

Comment.

$$\textit{elementary}$$

1.12 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t) = 3 - 2\,t$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) = 3\,\frac{1}{s} - 2\,\frac{1}{s^2}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + \mathrm{D}(y)(0)\,s^2 + 3\,s - 2}{s^4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-\frac{1}{3}\,t^3 + \frac{3}{2}\,t^2 + \mathrm{D}(y)(0)\,t + \mathrm{y}(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$-\frac{1}{3}\,t^3 + \frac{3}{2}\,t^2 + b\,t + a$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \frac{3}{2}\,t^2 - \frac{1}{3}\,t^3 + \_C1\,t + \_C2$$

Info.

polynomial

Comment.

elementary

1.13 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 16\,\mathrm{y}(t) = 5\,\delta(t-1)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$care!$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 16\,\mathrm{Y}(s) = 5\,e^{(-s)}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + 5\,e^{(-s)}}{s^2 + 16}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\cos(4\,t) + \frac{1}{4}\,\mathrm{D}(y)(0)\sin(4\,t) + \frac{5}{4}\,u(t-1)\sin(4\,t-4)$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{5}{4}\,u(t-1)\sin(4\,t-4)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \frac{5}{4}\cos(4)\,u(t-1)\sin(4\,t) - \frac{5}{4}\sin(4)\,u(t-1)\cos(4\,t) + \_C1\,\sin(4\,t) + \_C2\,\cos(4\,t)$$

Info.

$$u\_and\_trig\_functions$$

Comment.

$$advanced$$

1.14 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 16\,\mathrm{y}(t) = 16\,u(t-3) - 16$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$care!$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 16\,\mathrm{Y}(s) = 16\,\frac{e^{(-3\,s)}}{s} - 16\,\frac{1}{s}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s^2 + \mathrm{D}(y)(0)\,s + 16\,e^{(-3\,s)} - 16}{s\,(s^2+16)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\cos(4\,t) + \frac{1}{4}\,\mathrm{D}(y)(0)\sin(4\,t) + u(t-3) - u(t-3)\cos(4\,t-12) - 1 \\ + \cos(4\,t)$$

With the initial conditions incorporated we obtain a solution in the form

$$-1 + u(t-3) - u(t-3)\cos(4\,t-12) + \cos(4\,t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = (u(t-3)\sin(4\,t) - u(t-3)\sin(12) - \sin(4\,t))\sin(4\,t) \\ + (\cos(4\,t)\,u(t-3) - u(t-3)\cos(12) - \cos(4\,t))\cos(4\,t) \\ + \_C1\sin(4\,t) + \_C2\cos(4\,t)$$

Info.

u_and_trig_functions

Comment.

advanced

1.15 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 2\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + 2\,\mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 1$$

$$\mathrm{Dy}(0) = -1$$

Hint.

$$\mathit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 2\,s\,\mathrm{Y}(s) - 2\,\mathrm{y}(0) + 2\,\mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + 2\,\mathrm{y}(0)}{s^2 + 2\,s + 2}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$e^{(-t)}\,\mathrm{D}(y)(0)\sin(t) + e^{(-t)}\,\mathrm{y}(0)\sin(t) + e^{(-t)}\,\mathrm{y}(0)\cos(t)$$

With the initial conditions incorporated we obtain a solution in the form

$$e^{(-t)}\cos(t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\,e^{(-t)}\sin(t) + \_C2\,e^{(-t)}\cos(t)$$

Info.

$$e^{(-t)}\cos(t)$$

Comment.

standard

1.16 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 2\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + 2\,\mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$\mathit{convolution}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 2\,s\,\mathrm{Y}(s) - 2\,\mathrm{y}(0) + 2\,\mathrm{Y}(s) = \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + 2\,\mathrm{y}(0) + \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)}{s^2 + 2\,s + 2}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$e^{(-t)}\,\mathrm{y}(0)\cos(t) + e^{(-t)}\,\mathrm{y}(0)\sin(t) + e^{(-t)}\,\mathrm{D}(y)(0)\sin(t) + \int_0^t -\,\mathrm{f}(\_U1)\,e^{(-t+\_U1)}\sin(-t+\_U1)\,d\_U1$$

With the initial conditions incorporated we obtain a solution in the form

$$\int_0^t -\,\mathrm{f}(\_U1)\,e^{(-t+\_U1)}\sin(-t+\_U1)\,d\_U1$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = -\int \sin(t)\,\mathrm{f}(t)\,e^t\,dt\,e^{(-t)}\cos(t) + \int \cos(t)\,\mathrm{f}(t)\,e^t\,dt\,e^{(-t)}\sin(t) + \_C1\,e^{(-t)}\cos(t) + \_C2\,e^{(-t)}\sin(t)$$

Info.

$$\mathit{sin\_convolution}$$

Comment.

$$\mathit{standard}$$

1.17 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\, \mathrm{y}(t)) + 4\, \mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 2$$

$$\mathrm{Dy}(0) = 2$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 4\,\mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\, s + \mathrm{D}(y)(0)}{s^2 + 4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{2}\, \mathrm{D}(y)(0) \sin(2\, t) + \mathrm{y}(0) \cos(2\, t)$$

With the initial conditions incorporated we obtain a solution in the form

$$\sin(2\, t) + 2 \cos(2\, t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1 \cos(2\, t) + \_C2 \sin(2\, t)$$

Info.

$$\textit{trig\_functions}$$

Comment.

$$\textit{elementary}$$

1.18 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 4\,\mathrm{y}(t) = 6\,\mathrm{y}(t)$$

with initial conditions

$$\mathrm{y}(0) = 6$$

$$\mathrm{Dy}(0) = 0$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 4\,\mathrm{Y}(s) = 6\,\mathrm{Y}(s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0)}{s^2 - 2}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{2}\,\sqrt{2}\,\mathrm{D}(y)(0)\sinh(\sqrt{2}\,t) + \mathrm{y}(0)\cosh(\sqrt{2}\,t)$$

With the initial conditions incorporated we obtain a solution in the form

$$6\cosh(\sqrt{2}\,t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\,\sinh(\sqrt{2}\,t) + \_C2\,\cosh(\sqrt{2}\,t)$$

Info.

sinh_cosh

Comment.

standard

1.19 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 4\,\mathrm{y}(t) = \cos(t)$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 4\,\mathrm{Y}(s) = \frac{s}{s^2+1}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + \mathrm{y}(0)\,s + \mathrm{D}(y)(0)\,s^2 + \mathrm{D}(y)(0) + s}{s^4 + 5\,s^2 + 4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-\frac{1}{3}\cos(2\,t) + \mathrm{y}(0)\cos(2\,t) + \frac{1}{2}\,\mathrm{D}(y)(0)\sin(2\,t) + \frac{1}{3}\cos(t)$$

With the initial conditions incorporated we obtain a solution in the form

$$-\frac{1}{3}\cos(2\,t) + a\cos(2\,t) + \frac{1}{2}\,b\sin(2\,t) + \frac{1}{3}\cos(t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = (\frac{1}{12}\cos(3\,t) + \frac{1}{4}\cos(t))\cos(2\,t) + (\frac{1}{4}\sin(t) + \frac{1}{12}\sin(3\,t))\sin(2\,t) + \_C1\cos(2\,t) + \_C2\sin(2\,t)$$

Info.

trig_functions

Comment.

standard

1.20 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 9\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + 20\,\mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

convolution

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 9\,s\,\mathrm{Y}(s) - 9\,\mathrm{y}(0) + 20\,\mathrm{Y}(s) = \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + 9\,\mathrm{y}(0) + \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)}{s^2 + 9\,s + 20}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-4\,\mathrm{y}(0)\,e^{(-5\,t)} + 5\,\mathrm{y}(0)\,e^{(-4\,t)} - \mathrm{D}(y)(0)\,e^{(-5\,t)} + \mathrm{D}(y)(0)\,e^{(-4\,t)} \\ - \int_0^t \mathrm{f}(\_U1)\,e^{(-5\,t+5\,\_U1)}\,d\_U1 + \int_0^t \mathrm{f}(\_U2)\,e^{(-4\,t+4\,\_U2)}\,d\_U2$$

With the initial conditions incorporated we obtain a solution in the form

$$-\int_0^t \mathrm{f}(\_U1)\,e^{(-5\,t+5\,\_U1)}\,d\_U1 + \int_0^t \mathrm{f}(\_U2)\,e^{(-4\,t+4\,\_U2)}\,d\_U2$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = -(-\int \mathrm{f}(t)\,e^{(4\,t)}\,dt\,e^{(5\,t)} + \int \mathrm{f}(t)\,e^{(5\,t)}\,dt\,e^{(4\,t)})\,e^{(-9\,t)} + \_C1\,e^{(-4\,t)} + \_C2\,e^{(-5\,t)}$$

Info.

exp_convolution

Comment.

standard

1.21 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 9\,\mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 3$$

$$\mathrm{Dy}(0) = -5$$

Hint.

$$\mathit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 9\,\mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0)}{s^2 + 9}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{3}\,\mathrm{D}(y)(0)\sin(3\,t) + \mathrm{y}(0)\cos(3\,t)$$

With the initial conditions incorporated we obtain a solution in the form

$$-\frac{5}{3}\sin(3\,t) + 3\cos(3\,t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\cos(3\,t) + \_C2\sin(3\,t)$$

Info.

$$\mathit{trig\_functions}$$

Comment.

$$\mathit{standard}$$

1.22 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 1$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0)}{s^2 + 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\cos(t) + \mathrm{D}(y)(0)\sin(t)$$

With the initial conditions incorporated we obtain a solution in the form

$$\sin(t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\,\cos(t) + \_C2\,\sin(t)$$

Info.

$$\textit{trig\_functions}$$

Comment.

$$\textit{standard}$$

1.23 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = 2\,(\frac{\partial}{\partial t}\,\mathrm{y}(t))$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 1$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = 2\,s\,\mathrm{Y}(s) - 2\,\mathrm{y}(0)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) - 2\,\mathrm{y}(0)}{s^2 + 1 - 2\,s}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$t\,e^t\,\mathrm{D}(y)(0) - t\,e^t\,\mathrm{y}(0) + \mathrm{y}(0)\,e^t$$

With the initial conditions incorporated we obtain a solution in the form

$$t\,e^t$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\,e^t + \_C2\,t\,e^t$$

Info.

$$t\,e^t$$

Comment.

standard

1.24 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = \delta(t)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$\textit{care!}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = 1$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + 1}{s^2 + 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\cos(t) + \mathrm{D}(y)(0)\sin(t) + \sin(t)$$

With the initial conditions incorporated we obtain a solution in the form

$$\sin(t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = u(t)\sin(t) + \_C1\cos(t) + \_C2\sin(t)$$

Info.

$$\textit{u\_and\_trig\_functions}$$

Comment.

$$\textit{standard}$$

1.25 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{D}\mathrm{y}(0) = 0$$

Hint.

convolution

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = \mathrm{laplace}(\mathrm{f}(t),\, t,\, s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + \mathrm{laplace}(\mathrm{f}(t),\, t,\, s)}{s^2 + 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\cos(t) + \mathrm{D}(y)(0)\sin(t) + \int_0^t -\,\mathrm{f}(\_U1)\sin(-t + \_U1)\,d\_U1$$

With the initial conditions incorporated we obtain a solution in the form

$$\int_0^t \mathrm{f}(\_U1)\sin(t - \_U1)\,d\_U1$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \int -\sin(t)\,\mathrm{f}(t)\,dt\cos(t) + \int \cos(t)\,\mathrm{f}(t)\,dt\sin(t) + \_C1\cos(t) + \_C2\sin(t)$$

Info.

sin_convolution

Comment.

standard

1.26 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = 2\,u(t-1)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

care!

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = 2\,\frac{e^{(-s)}}{s}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s^2 + \mathrm{D}(y)(0)\,s + 2\,e^{(-s)}}{s\,(s^2+1)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\cos(t) + \mathrm{D}(y)(0)\sin(t) + 2\,u(t-1) - 2\,u(t-1)\cos(t-1)$$

With the initial conditions incorporated we obtain a solution in the form

$$2\,u(t-1) - 2\,u(t-1)\cos(t-1)$$

Without the Laplace transform we can obtain this general solution

$$\begin{aligned}\mathrm{y}(t) = {} & (2\cos(t)\,u(t-1) - 2\,u(t-1)\cos(1))\cos(t) \\ & + (2\sin(t)\,u(t-1) - 2\,u(t-1)\sin(1))\sin(t) + \_C1\cos(t) \\ & + \_C2\sin(t)\end{aligned}$$

Info.

u_and_trig_functions

Comment.

standard

1.27 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = \sin(t)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = b$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = \frac{1}{s^2+1}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + \mathrm{y}(0)\,s + \mathrm{D}(y)(0)\,s^2 + \mathrm{D}(y)(0) + 1}{s^4 + 2\,s^2 + 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-\frac{1}{2}\,t\cos(t) + \frac{1}{2}\sin(t) + \mathrm{y}(0)\cos(t) + \mathrm{D}(y)(0)\sin(t)$$

With the initial conditions incorporated we obtain a solution in the form

$$-\frac{1}{2}\,t\cos(t) + \frac{1}{2}\sin(t) + b\sin(t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = (\frac{1}{2}\cos(t)\sin(t) - \frac{1}{2}\,t)\cos(t) + \frac{1}{2}\sin(t)^3 + \_C1\cos(t) + \_C2\sin(t)$$

Info.

t_and_trig_functions

Comment.

standard

1.28 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + \mathrm{y}(t) = t\,e^{(-t)}$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + \mathrm{Y}(s) = \frac{1}{(s+1)^2}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + 2\,\mathrm{y}(0)\,s^2 + \mathrm{y}(0)\,s + \mathrm{D}(y)(0)\,s^2 + 2\,\mathrm{D}(y)(0)\,s + \mathrm{D}(y)(0) + 1}{s^4 + 2\,s^3 + 2\,s^2 + 2\,s + 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-\frac{1}{2}\cos(t) + \mathrm{y}(0)\cos(t) + \mathrm{D}(y)(0)\sin(t) + \frac{1}{2}\,e^{(-t)} + \frac{1}{2}\,t\,e^{(-t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$-\frac{1}{2}\cos(t) + a\cos(t) + b\sin(t) + \frac{1}{2}\,e^{(-t)} + \frac{1}{2}\,t\,e^{(-t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = (-(-\frac{1}{2}\,t - \frac{1}{2})\,e^{(-t)}\cos(t) + \frac{1}{2}\sin(t)\,t\,e^{(-t)})\cos(t)$$
$$+ (-\frac{1}{2}\cos(t)\,t\,e^{(-t)} - (-\frac{1}{2}\,t - \frac{1}{2})\,e^{(-t)}\sin(t))\sin(t) + \_C1\cos(t) + \_C2\sin(t)$$

Info.

$$\textit{t\_exp\_trig\_functions}$$

Comment.

$$\textit{standard}$$

1.29 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) - 2\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + 2\,\mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

convolution

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - 2\,s\,\mathrm{Y}(s) + 2\,\mathrm{y}(0) + 2\,\mathrm{Y}(s) = \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) - 2\,\mathrm{y}(0) + \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)}{s^2 - 2\,s + 2}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0)\,e^t\cos(t) - \mathrm{y}(0)\,e^t\sin(t) + \mathrm{D}(y)(0)\,e^t\sin(t) + \int_0^t -\mathrm{f}(\_U1)\,e^{(t-\_U1)}\sin(-t+\_U1)\,d\_U1$$

With the initial conditions incorporated we obtain a solution in the form

$$\int_0^t -\mathrm{f}(\_U1)\,e^{(t-\_U1)}\sin(-t+\_U1)\,d\_U1$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = -(-\int\cos(t)\,\mathrm{f}(t)\,e^{(-t)}\,dt\,\sin(t) + \int\sin(t)\,\mathrm{f}(t)\,e^{(-t)}\,dt\,\cos(t))\,e^t + \_C1\,e^t\sin(t) + \_C2\,e^t\cos(t)$$

Info.

sin_exp_convolution

Comment.

standard

1.30 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) - 3\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + 2\,\mathrm{y}(t) = 4$$

with initial conditions

$$\mathrm{y}(0) = 2$$

$$\mathrm{Dy}(0) = 3$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - 3\,s\,\mathrm{Y}(s) + 3\,\mathrm{y}(0) + 2\,\mathrm{Y}(s) = 4\,\frac{1}{s}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s^2 + \mathrm{D}(y)(0)\,s - 3\,\mathrm{y}(0)\,s + 4}{s\,(s^2 - 3\,s + 2)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$2 - 4\,e^t + 2\,\mathrm{y}(0)\,e^t - e^t\,\mathrm{D}(y)(0) + 2\,e^{(2\,t)} - e^{(2\,t)}\,\mathrm{y}(0) + e^{(2\,t)}\,\mathrm{D}(y)(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$2 - 3\,e^t + 3\,e^{(2\,t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = 2 + \_C1\,e^t + \_C2\,e^{(2\,t)}$$

Info.

exp_functions

Comment.

standard

1.31 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) - 3\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) + 4\,\mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 1$$

$$\mathrm{Dy}(0) = 5$$

Hint.

$$\textit{no\_hint}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - 3\,s\,\mathrm{Y}(s) + 3\,\mathrm{y}(0) + 4\,\mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) - 3\,\mathrm{y}(0)}{s^2 - 3\,s + 4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$e^{(3/2\,t)}\,\mathrm{y}(0)\cos(\frac{1}{2}\,\sqrt{7}\,t) - \frac{3}{7}\,e^{(3/2\,t)}\,\mathrm{y}(0)\,\sqrt{7}\sin(\frac{1}{2}\,\sqrt{7}\,t) + \frac{2}{7}\,e^{(3/2\,t)}\,\sqrt{7}\,\mathrm{D}(y)(0)\sin(\frac{1}{2}\,\sqrt{7}\,t)$$

With the initial conditions incorporated we obtain a solution in the form

$$e^{(3/2\,t)}\cos(\frac{1}{2}\,\sqrt{7}\,t) + e^{(3/2\,t)}\,\sqrt{7}\sin(\frac{1}{2}\,\sqrt{7}\,t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\,e^{(3/2\,t)}\sin(\frac{1}{2}\,\sqrt{7}\,t) + \_C2\,e^{(3/2\,t)}\cos(\frac{1}{2}\,\sqrt{7}\,t)$$

Info.

$$\textit{exp\_trig\_functions}$$

Comment.

$$\textit{standard}$$

1.32 Problem.

Using the Laplace transform find the solution for the following equation

$$\left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) - 4\,\mathrm{y}(t) = 0$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - 4\,\mathrm{Y}(s) = 0$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0)}{s^2 - 4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{4}\,e^{(2\,t)}\,\mathrm{D}(y)(0) + \frac{1}{2}\,e^{(2\,t)}\,\mathrm{y}(0) + \frac{1}{2}\,e^{(-2\,t)}\,\mathrm{y}(0) - \frac{1}{4}\,e^{(-2\,t)}\,\mathrm{D}(y)(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$0$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \_C1\,\cosh(2\,t) + \_C2\,\sinh(2\,t)$$

Info.

exp_functions

Comment.

standard

1.33 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) - (\frac{\partial}{\partial t}\,\mathrm{y}(t)) - 2\,\mathrm{y}(t) = 4\,t^2$$

with initial conditions

$$\mathrm{y}(0) = 1$$

$$\mathrm{Dy}(0) = 4$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - s\,\mathrm{Y}(s) + \mathrm{y}(0) - 2\,\mathrm{Y}(s) = 8\,\frac{1}{s^3}$$

From this equation we solve $Y(s)$

$$\frac{s^4\,\mathrm{y}(0) + \mathrm{D}(y)(0)\,s^3 - s^3\,\mathrm{y}(0) + 8}{s^3\,(s^2 - s - 2)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-3 + 2\,t - 2\,t^2 + \frac{8}{3}\,e^{(-t)} + \frac{2}{3}\,\mathrm{y}(0)\,e^{(-t)} - \frac{1}{3}\,e^{(-t)}\,\mathrm{D}(y)(0) + \frac{1}{3}\,e^{(2\,t)}\,\mathrm{y}(0) + \frac{1}{3}\,e^{(2\,t)} + \frac{1}{3}\,e^{(2\,t)}\,\mathrm{D}(y)(0)$$

With the initial conditions incorporated we obtain a solution in the form

$$-3 + 2\,t - 2\,t^2 + 2\,e^{(-t)} + 2\,e^{(2\,t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = -3 + 2\,t - 2\,t^2 + \_C1\,e^{(2\,t)} + \_C2\,e^{(-t)}$$

Info.

polynomial_exp_functions

Comment.

standard

1.34 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\, \mathrm{y}(t)) - \mathrm{y}(t) = e^t$$

with initial conditions

$$\mathrm{y}(0) = 1$$

$$\mathrm{Dy}(0) = 0$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - \mathrm{Y}(s) = \frac{1}{s-1}$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\, s^2 - \mathrm{y}(0)\, s + \mathrm{D}(y)(0)\, s - \mathrm{D}(y)(0) + 1}{s^3 - s^2 - s + 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{2}\, \mathrm{y}(0)\, e^{(-t)} - \frac{1}{2}\, e^{(-t)}\, \mathrm{D}(y)(0) + \frac{1}{4}\, e^{(-t)} + \frac{1}{2}\, \mathrm{y}(0)\, e^t + \frac{1}{2}\, e^t\, \mathrm{D}(y)(0) - \frac{1}{4}\, e^t + \frac{1}{2}\, t\, e^t$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{3}{4}\, e^{(-t)} + \frac{1}{4}\, e^t + \frac{1}{2}\, t\, e^t$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = (-\frac{1}{2}\sinh(t)\cosh(t) + \frac{1}{2}\, t - \frac{1}{2}\cosh(t)^2)\cosh(t) \\ + (\frac{1}{2}\cosh(t)^2 + \frac{1}{2}\sinh(t)\cosh(t) + \frac{1}{2}\, t)\sinh(t) + \_C1\cosh(t) + \_C2\sinh(t)$$

Info.

polynomial_exp_functions

Comment.

standard

1.35 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) - \mathrm{y}(t) = \mathrm{f}(t)$$

with initial conditions

$$\mathrm{y}(0) = a$$

$$\mathrm{Dy}(0) = b$$

Hint.

convolution

Solution.

We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) - \mathrm{Y}(s) = \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)$$

From this equation we solve $Y(s)$

$$\frac{\mathrm{y}(0)\,s + \mathrm{D}(y)(0) + \mathrm{laplace}(\mathrm{f}(t),\,t,\,s)}{s^2 - 1}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{2}\,\mathrm{y}(0)\,e^t + \frac{1}{2}\,\mathrm{y}(0)\,e^{(-t)} + \frac{1}{2}\,e^t\,\mathrm{D}(y)(0) - \frac{1}{2}\,e^{(-t)}\,\mathrm{D}(y)(0) + \frac{1}{2}\int_0^t \mathrm{f}(\_U1)\,e^{(t-\_U1)}\,d\_U1 - \frac{1}{2}\int_0^t \mathrm{f}(\_U2)\,e^{(-t+\_U2)}\,d\_U2$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{1}{2}\,a\,e^t + \frac{1}{2}\,a\,e^{(-t)} + \frac{1}{2}\,e^t\,b - \frac{1}{2}\,e^{(-t)}\,b + \frac{1}{2}\int_0^t \mathrm{f}(\_U1)\,e^{(t-\_U1)}\,d\_U1 - \frac{1}{2}\int_0^t \mathrm{f}(\_U2)\,e^{(-t+\_U2)}\,d\_U2$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \int -\sinh(t)\,\mathrm{f}(t)\,dt\,\cosh(t) + \int \cosh(t)\,\mathrm{f}(t)\,dt\,\sinh(t) + \_C1\,\cosh(t) + \_C2\,\sinh(t)$$

Info.

exp_convolution

Comment.

standard

1.36 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^3}{\partial t^3}\,\mathrm{y}(t)) + (\frac{\partial}{\partial t}\,\mathrm{y}(t)) = e^t$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

no_hint

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0)) - (\mathrm{D}^{(2)})(y)(0) + s\,\mathrm{Y}(s) - \mathrm{y}(0) = \frac{1}{s-1}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) - \mathrm{y}(0)\,s^2 + \mathrm{D}(y)(0)\,s^2 - \mathrm{D}(y)(0)\,s + (\mathrm{D}^{(2)})(y)(0)\,s - (\mathrm{D}^{(2)})(y)(0) + \mathrm{y}(0)\,s - \mathrm{y}(0) + 1}{s\,(s^3 - s^2 + s - 1)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$(\mathrm{D}^{(2)})(y)(0) + \mathrm{y}(0) - 1 + \frac{1}{2}\,e^t - \frac{1}{2}\sin(t) + \mathrm{D}(y)(0)\sin(t) + \frac{1}{2}\cos(t) - (\mathrm{D}^{(2)})(y)(0)\cos(t)$$

With the initial conditions incorporated we obtain a solution in the form

$$(\mathrm{D}^{(2)})(y)(0) - 1 + \frac{1}{2}\,e^t - \frac{1}{2}\sin(t) + \frac{1}{2}\cos(t) - (\mathrm{D}^{(2)})(y)(0)\cos(t)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = \frac{1}{2}\,e^t + \_C1 + \_C2\cos(t) + \_C3\sin(t)$$

Info.

trig_exp

Comment.

standard

1.37 Problem.

Using the Laplace transform find the solution for the following equation

$$(\frac{\partial^3}{\partial t^3}\,\mathrm{y}(t)) + (\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) = 6\,e^t + 6\,t + 6$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

no_hint

Solution.

We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0)) - (\mathrm{D}^{(2)})(y)(0) + s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) = 6\,\frac{1}{s-1} + 6\,\frac{1}{s^2} + 6\,\frac{1}{s}$$

From this equation we solve $Y(s)$

$$\frac{s^5\,\mathrm{y}(0) + s^4\,\mathrm{D}(y)(0) + (\mathrm{D}^{(2)})(y)(0)\,s^3 - (\mathrm{D}^{(2)})(y)(0)\,s^2 - s^3\,\mathrm{y}(0) - \mathrm{D}(y)(0)\,s^2 + 12\,s^2 - 6}{s^4\,(s^2-1)}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$-(\mathrm{D}^{(2)})(y)(0) + \mathrm{y}(0) - 6\,t + \mathrm{D}(y)(0)\,t + t\,(\mathrm{D}^{(2)})(y)(0) + t^3 + 3\,e^t + e^{(-t)}\,(\mathrm{D}^{(2)})(y)(0) - 3\,e^{(-t)}$$

With the initial conditions incorporated we obtain a solution in the form

$$-(\mathrm{D}^{(2)})(y)(0) - 6\,t + t\,(\mathrm{D}^{(2)})(y)(0) + t^3 + 3\,e^t + e^{(-t)}\,(\mathrm{D}^{(2)})(y)(0) - 3\,e^{(-t)}$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = e^t\,(t^3\,e^{(-t)} + 3) + \_C1 + \_C2\,t + \_C3\,e^{(-t)}$$

Info.

polynomial_exp_functions

Comment.

standard

1.38 Problem.

Using the Laplace transform find the solution for the following equation

$$\frac{\partial^4}{\partial t^4}\,\mathrm{y}(t) = 6\,\delta(t-1)$$

with initial conditions

$$\mathrm{y}(0) = 0$$

$$\mathrm{Dy}(0) = 0$$

Hint.

$$\mathit{care!}$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$s\,(s\,(s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0)) - (\mathrm{D}^{(2)})(y)(0)) - (\mathrm{D}^{(3)})(y)(0) = 6\,e^{(-s)}$$

From this equation we solve $Y(s)$

$$\frac{s^3\,\mathrm{y}(0) + \mathrm{D}(y)(0)\,s^2 + (\mathrm{D}^{(2)})(y)(0)\,s + (\mathrm{D}^{(3)})(y)(0) + 6\,e^{(-s)}}{s^4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\mathrm{y}(0) + \mathrm{D}(y)(0)\,t + \frac{1}{2}\,(\mathrm{D}^{(2)})(y)(0)\,t^2 + \frac{1}{6}\,(\mathrm{D}^{(3)})(y)(0)\,t^3 + u(t-1)\,t^3 - 3\,u(t-1)\,t^2 + 3\,u(t-1)\,t - u(t-1)$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{1}{2}\,(\mathrm{D}^{(2)})(y)(0)\,t^2 + \frac{1}{6}\,(\mathrm{D}^{(3)})(y)(0)\,t^3 + u(t-1)\,t^3 - 3\,u(t-1)\,t^2 + 3\,u(t-1)\,t - u(t-1)$$

Without the Laplace transform we can obtain this general solution

$$\mathrm{y}(t) = u(t-1)\,t^3 - u(t-1) + 3\,u(t-1)\,t - 3\,u(t-1)\,t^2 + \frac{1}{6}\,\_C1\,t^3 + \frac{1}{2}\,\_C2\,t^2 + \_C3\,t + \_C4$$

Info.

$$\mathit{u\_polynomial\_function}$$

Comment.

$$\mathit{standard}$$

1.39 Problem.

Using the Laplace transform find the solution for the following equation

$$y(t) = t + \int_0^t - y(\tau) \sin(-t + \tau) \, d\tau$$

with initial conditions

$$y(0) = a$$

$$Dy(0) = b$$

Hint.

$$care!$$

Solution.
We denote $Y(s) = L(y)(t)$ the Laplace transform $Y(s)$ of $y(t)$. We perform the Laplace transform for both sides of the given equation. For particular functions we use tables of the Laplace transforms and obtain

$$Y(s) = \frac{1}{s^2} + \frac{Y(s)}{s^2 + 1}$$

From this equation we solve $Y(s)$

$$\frac{s^2 + 1}{s^4}$$

and invert it using the inverse Laplace transform and the same tables again and obtain

$$\frac{1}{6} t^3 + t$$

With the initial conditions incorporated we obtain a solution in the form

$$\frac{1}{6} t^3 + t$$

Without the Laplace transform we can obtain this general solution

$$not\_found$$

Info.

$$polynomial\_functions$$

Comment.

$$standard$$