Lesson Overview

LESSON 18

Fractions as Division

Lesson Objectives

Content Objectives

- Use visual fraction models to represent a fraction as division.
- Solve word problems involving division of whole numbers in which the quotient is a fraction or mixed number.
- Understand a fraction as a way to represent division where the numerator is divided by the denominator.

Language Objectives

- Make a visual fraction model to represent a fraction as a division of two whole numbers and explain the relationship of the model to the fraction.
- Draw a visual model and write an equation to represent word problems involving a quotient of whole numbers where the quotient is a fraction.

Prerequisite Skills

- Understand division as equal groups or sharing.
- Understand that multiplication and division are inverse operations.
- Divide whole numbers.
- Multiply a fraction by a whole number.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 305m to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line and can also represent the division of two numbers.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **quotient** the result of division.
- **remainder** the amount left over when one number does not divide another number a whole number of times.

Learning Progression

In previous grades students have understood the meaning of division as equal groups or equal shares. They have interpreted fractions as equal parts of a whole or equal parts of a group.

In this lesson students extend their understanding of both division and fractions to see that when a whole number is divided by another whole number the quotient may be a fraction or a mixed number, e.g., $2 \div 5 = \frac{2}{5}$ and $23 \div 7 = \frac{23}{7}$. From exploring problems like this they recognize that a fraction can be interpreted as representing a division expression, one where the numerator is divided by the denominator. They further connect this idea to the inverse relationship of multiplication and division, e.g., $2 \div 5 = \frac{2}{5}$ and $5 \times \frac{2}{5} = 2$.

In later lessons students will learn to divide whole numbers by unit fractions and divide unit fractions by whole numbers. In Grade 6 students will apply and extend their understanding of division with fractions to divide a whole number by a fraction and to divide a fraction by a fraction.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 **Explore** *45–60 min*	**Fractions as Division** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 377–378
SESSION 2 **Develop** *45–60 min*	**Fractions as Division** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 383–384 **Fluency** Fractions as Division
SESSION 3 **Refine** *45–60 min*	**Fractions as Division** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Activity Sheet:* Number Lines
Activities	*Per student:* base-ten blocks (1 tens rods, 2 ones units), scissors *Activity Sheets:* Fraction Bars, Digit Cards, 1-Inch Grid Paper
Math Toolkit	fraction circles, fraction tiles, fraction bars, tenths grids, number lines, index cards
Digital Math Tools	Fraction Models, Number Line

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 4
- Lesson 23 Understand Fraction Multiplication

RETEACH

Tools for Instruction

Grade 4
- Lesson 23 Understand Fraction Multiplication

Grade 5
- Lesson 18 Interpreting Fractions as Division

REINFORCE

Math Center Activities

Grade 5
- Lesson 18 Fractions as Quotients
- Lesson 18 Relate Situations to Fractional Quotients

EXTEND

Enrichment Activity

Grade 5
- Lesson 18 Pizza Party

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 5
- Fractions as Division

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

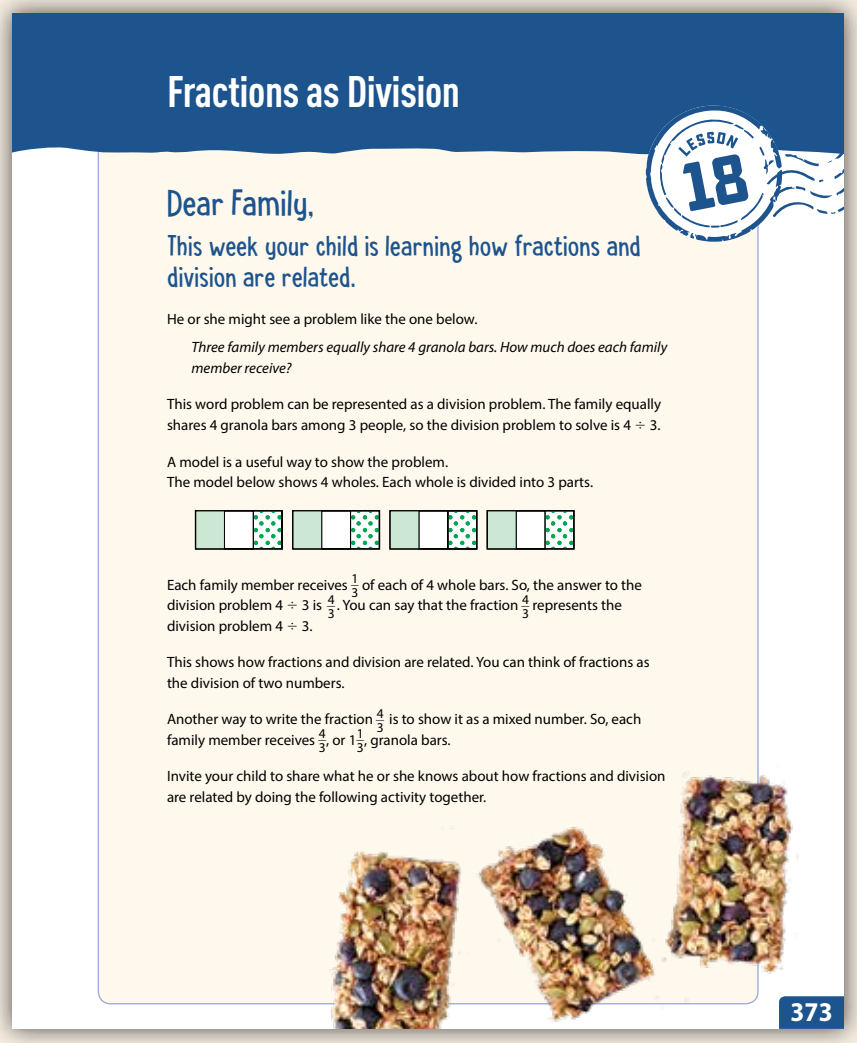
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Fractions as Division

LESSON 18

Dear Family,

This week your child is learning how fractions and division are related.

He or she might see a problem like the one below.

Three family members equally share 4 granola bars. How much does each family member receive?

This word problem can be represented as a division problem. The family equally shares 4 granola bars among 3 people, so the division problem to solve is $4 \div 3$.

A model is a useful way to show the problem.
The model below shows 4 wholes. Each whole is divided into 3 parts.

Each family member receives $\frac{1}{3}$ of each of 4 whole bars. So, the answer to the division problem $4 \div 3$ is $\frac{4}{3}$. You can say that the fraction $\frac{4}{3}$ represents the division problem $4 \div 3$.

This shows how fractions and division are related. You can think of fractions as the division of two numbers.

Another way to write the fraction $\frac{4}{3}$ is to show it as a mixed number. So, each family member receives $\frac{4}{3}$, or $1\frac{1}{3}$, granola bars.

Invite your child to share what he or she knows about how fractions and division are related by doing the following activity together.

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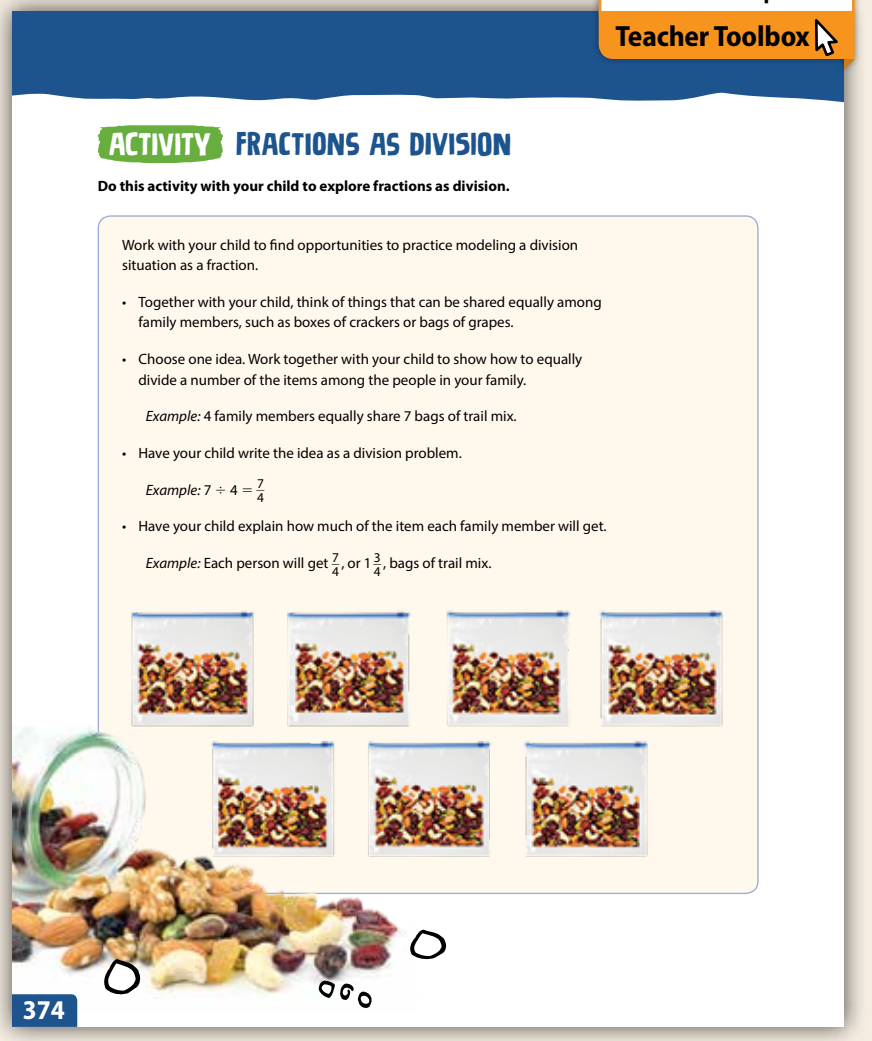

ACTIVITY FRACTIONS AS DIVISION

Do this activity with your child to explore fractions as division.

Work with your child to find opportunities to practice modeling a division situation as a fraction.

- Together with your child, think of things that can be shared equally among family members, such as boxes of crackers or bags of grapes.
- Choose one idea. Work together with your child to show how to equally divide a number of the items among the people in your family.

  Example: 4 family members equally share 7 bags of trail mix.
- Have your child write the idea as a division problem.

  Example: $7 \div 4 = \frac{7}{4}$
- Have your child explain how much of the item each family member will get.

  Example: Each person will get $\frac{7}{4}$, or $1\frac{3}{4}$, bags of trail mix.

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Goal

The goal of the Family Letter is to help students and their families understand how fractions and division are related.

Activity

Understanding how fractions and division are related will help students build skills that are necessary to perform more complex mathematical operations involving fractions. Look at the *Fractions as Division* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to look for objects at home that can be divided into equal parts to show relationships between fractions and division.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *What are some objects or items that can be shared equally among our family members?*
- *How can we represent that as a division? How can we represent it as a fraction?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 1 and 3 **Use anytime during these sessions.**

- Many problems in this lesson discuss dividing labor or items into equal parts. Ask students to make a personal connection to how they divide chores or activities around their home. Ask if the chores are divided by the days of the week when they are done, the number of people doing the chores, etc.
- Sharing objects or items equally is an example of division that students encounter in their daily life. An equal number of students assigned to each classroom is another example of dividing with whole numbers.
- Division situations do not always involve dividing a given quantity by a number less than the given quantity. Dividing one whole (e.g., a snack) between two or more people can be represented by a fraction. As students think of how to represent this division situation as a fraction, encourage them to make personal connections to how they use fractions and division in their daily lives.

Session 2 **Use with *Try It*.**

- Decorating school hallways is a way to show school pride and encourage collaboration between different classrooms. The collaboration may include developing a theme and organizing how students will work together to complete the work. In the same way, cities and communities make murals and landscape public spaces to show pride. Ask students to share any community work they might have been a part of and how the work was divided.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Writing Read ***Try It*** aloud. Choose five volunteers to act out the problem. Use 4 index cards to represent the four ounces of paint. Demonstrate cutting each index card into 5 equal shares. As you cut each index card into 5 pieces, share one piece with each volunteer. As you hand each student a share say: *One fifth of an ounce for you*. When you have divided each index card, ask the volunteers: *How many fifths do you have?* [four fifths] Organize students into pairs to draw fraction bars or a grid model to represent the problem. Encourage them to visualize the role-playing activity as they model the problem.	**Speaking/Writing** Read ***Try It*** with students. Organize students into pairs to restate the problem in their own words. Provide the following sentence frame to encourage students to think about using fractions to solve the problem: *How many fifths are in* 4 wholes; 4 ounces of paint *?* Provide fraction tiles or fraction circles. Allow partners to choose their preferred tool to model and solve the problem. Have them work together to write a summary of their steps using the sequencing words *first*, *next*, and *then*.	**Reading/Writing** Have students read ***Try It*** and use a known strategy to solve the problem independently. Ask students to think about the steps they took to model and solve the problem. Have them write directions for a partner to model and solve the problem in the same way. Organize students into pairs. If possible, partner students who used different tools or methods to solve the problem. Ask students to read and follow their partner's directions. Ask: *Were you able to solve the problem in a new way? Why or why not?* Give students the opportunity to revise their directions as needed.

LESSON 18

SESSION 1 Explore

Purpose In this session, students draw on their understanding of division as equal sharing and fractions as equal parts of a whole. They share models to explore how to represent the division of a whole number by a greater whole number. They will look ahead to think about how division of whole numbers can lead to quotients that are fractions or mixed numbers.

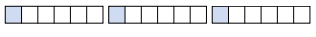
Start

Connect to Prior Knowledge

Why Supports students in relating repeated addition of a fraction to multiplying the fraction by a whole number.

How Have students complete the addition equation and the multiplication equation to represent the number of sixths shown in the fraction model.

Complete the equations to tell how many sixths are shaded.



$\frac{1}{6} + \_\_\_\_ + \_\_\_\_ = \_\_\_\_$

$\_\_\_\_ \times \frac{1}{6} = \_\_\_\_$

Solutions

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

$3 \times \frac{1}{6} = \frac{3}{6}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that the number of students sharing the paint, 5, is greater than the number of ounces of paint, 4.

DISCUSS IT

Support Partner Discussion

Encourage students to use the *Discuss It* question and sentence starter on the Student Worktext page as part of their discussion.

Look for, and prompt as necessary for, understanding of:

- 4 ounces as the amount to be shared equally
- 5 students to get equal shares
- the amount of each share is the unknown

LESSON 18 SESSION 1 ● ○ ○

Explore Fractions as Division

You know that division is used for equal sharing and that fractions represent a number of equal parts of a whole. In this lesson, you will learn how division and fractions are related. Use what you know to try to solve the problem below.

Mrs. Tatum needs to share 4 fluid ounces of red paint equally among 5 art students. How many ounces of red paint will each student get?

Learning Target

- Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP 1, 2, 3, 4, 5, 6

TRY IT

Possible student work:

Sample A

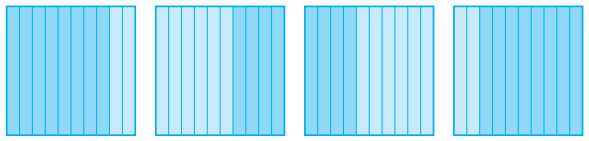
1 ounce | 1 ounce | 1 ounce | 1 ounce

A B C D E | A B C D E | A B C D E | A B C D E

Each student gets $\frac{1}{5}$ of each ounce. Since $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$, each student gets $\frac{4}{5}$ of an ounce.

Sample B

4 ounces = 40 tenths of an ounce



40 tenths ÷ 5 = 8 tenths

Each student gets 0.8 ounce of red paint.

Math Toolkit

- fraction circles or tiles
- fraction bars
- fraction models
- tenths grids
- number lines
- index cards

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: At first, I thought . . .

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Common Misconception Look for students who are not comfortable with the idea of dividing a whole number by a greater whole number. As students present solutions, have them specify why they represented the 4 ounces of paint the way they did.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models showing 5 groups of 4 fifths
- drawings to represent the problem
- number lines marked in fifths
- equations showing 4 as an equivalent fraction with numerator divisible by 5

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the ounces of paint divided? Each student's share of paint?*

Listen for Each ounce can be divided into 5 equal parts. Each student gets the same number of equal parts for their fair share. They each get four $\frac{1}{5}$s, or $\frac{4}{5}$, or 8 tenths.

CONNECT IT

1 LOOK BACK

Look for understanding that each whole ounce of paint can be divided into equal parts and each student's equal share is the sum of the equal parts that is his share in each ounce.

Hands-On Activity

Use fraction bars to divide.

If . . . *students are unsure about dividing 4 wholes into equal parts that are fractions,*

Then . . . *use this activity to model* Try It.

Materials For each student: scissors, Activity Sheet *Fraction Bars* (4 bars for fifths)

- Explain to students that they will model the *Try It* problem.
- Explain that each fraction bar represents 1 ounce of paint. Have students write the letters A through E in the sections of each fraction bar, each letter representing one of the 5 art students sharing the ounces of paint. Prompt students to identify each art student's share of each ounce as $\frac{1}{5}$ of an ounce.
- Have students cut the fraction bars into fifths and collect each art student's shares in a separate pile. Have them count fifths to find the size of one share. Discuss the results.
- Repeat activity, representing other situations such as 4 students sharing 3 bags of popcorn.

2 LOOK AHEAD

Point out that sometimes when you divide whole numbers, the quotient may be in the form of a fraction greater than 1 or a mixed number.

Students should be able to show or interpret the equal parts comprising one share for both ways of dividing the 8 ounces of paint into 5 equal shares.

Have students think about using multiplication to explain why the quotient of two whole numbers can be represented by a fraction.

Ask *Your work in problem 2 shows that* $8 \div 5 = \frac{8}{5}$. *What is the related multiplication equation that shows this same relationship?*

Listen for $5 \times \frac{8}{5} = 8$; $5 \times \frac{8}{5} = \frac{(5 \times 8)}{5} = \frac{40}{5} = 8$

CONNECT IT

1 LOOK BACK

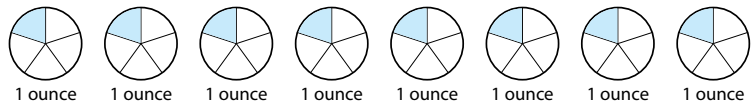
Explain how to find the amount of paint each student gets.

Possible answer: I drew a picture to show that each student gets $\frac{1}{5}$ of each ounce. I added $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ to get $\frac{4}{5}$ ounce for each student.

2 LOOK AHEAD

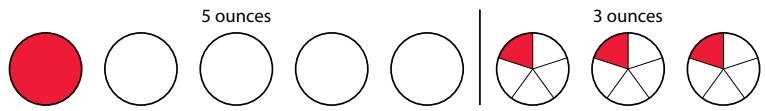
Suppose Mrs. Tatum wants to share 8 fluid ounces of paint equally among the 5 students. You can think about this quotient in two ways.

a. Think of each student getting $\frac{1}{5}$ of each ounce. Shade $\frac{1}{5}$ of each whole in the model below to show one student's share.



8 ounces ÷ 5 = 8 × $\frac{1}{5}$ = $\frac{8}{5}$ ounces

b. Think of 8 ounces as 5 ounces + 3 ounces. Explain how the shaded part of the model below shows one student's share.



Possible explanation: Each student gets 1 full ounce of paint and $\frac{1}{5}$ of each of the remaining 3 ounces.

c. Write the quotient 8 ÷ 5 as a fraction and as a mixed number. **$\frac{8}{5} = 1\frac{3}{5}$**

3 REFLECT

How would you write the fraction $\frac{2}{5}$ as a division expression? Write a word problem that can be represented by your expression and by the fraction $\frac{2}{5}$.

2 ÷ 5; Possible problem: How much paint will each student get if 2 ounces of paint are shared equally among 5 students?

Close: Exit Ticket

3 REFLECT

Look for understanding that a fraction can represent division of the numerator by the denominator. The problem situation students describe should have 2 as the quantity being divided and 5 as the number of equal shares it is divided into.

Common Misconception **If** students confuse which number in a fraction represents the dividend and which represents the divisor in the equivalent division expression, **then** write the three division expressions from the Student Worktext page for this session: 4 ÷ 5, 8 ÷ 5, 2 ÷ 5. Have students pair each expression with the fraction they found that represents it. For each expression/fraction pair have students circle the numerator in the fraction and identify where that number is in the expression.

Real-World Connection

Encourage students to think about everyday places or situations in which dividing a quantity into equal shares may result in an amount that includes a fractional part. Have volunteers share ideas. Examples: cooking 5 eggs for breakfast to feed 3 people, making 10 place mats out of 6 feet of fabric, needing 3 cups of yogurt to make 24 servings of fruit smoothie.

Solutions

Support Vocabulary Development

1 Have students say the terms: *fraction, division expression, quotient* and *remainder*. Guide a conversation with students in which they discuss what they know about division. Then ask them to discuss the definition for each term and write it in the *In My Own Words* column.

Call on volunteers to share what they wrote for each term. Correct any misconceptions and ask students to revise their answers if necessary.

As students share their graphic organizer, encourage them to use this sentence frame:

- *My explanation for _____ is _____.*

2 Read the problem. Have students discuss with a partner how you use multiplication to check an answer to a division problem. Then ask students to confirm their work with another set of partners.

Supplemental Math Vocabulary

- *dividend*
- *divisor*
- *quotient*

Name: _____ LESSON 18 SESSION 1

Prepare for Fractions as Division

1 Think about what you know about division. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Word	In My Own Words	Example
fraction	**a number that names equal parts of a whole**	$\frac{4}{5}$
division expression	**an expression that has the operation of division**	**4 ÷ 5**
quotient	**the result of division**	**16 ÷ 8 = 2**
remainder	**the amount left over when one number does not divide another number a whole number of times**	**20 ÷ 3 = 6 R 2**

2 Write the fraction $\frac{3}{4}$ as a division expression. **3 ÷ 4**

How could you use multiplication to check your answer?

Possible answer: If 3 ÷ 4 = $\frac{3}{4}$, then 4 × $\frac{3}{4}$ should be 3.

$4 \times \frac{3}{4} = \frac{4 \times 3}{4}$

$= \frac{12}{4}$

$= 3$

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3 Assign problem 3 to provide another look at fractions as division.

This problem is very similar to the problem about Mrs. Tatum sharing red paint among her art students. In both problems, students are given a word problem that requires them to divide one whole number by a greater whole number. The question asks how many grams of glitter each student will get.

Students may want to use fraction bars or grid paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

Each student gets $\frac{1}{8}$ of each gram. Since $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$, each student gets $\frac{3}{8}$ gram of glitter.
Medium

4 Have students solve the problem a different way, or use multiplication, to check their answer.

3 Solve the problem. Show your work.

Mrs. Tatum needs to share 3 grams of glitter equally among 8 art students. How many grams of glitter will each student get?

Possible student work using pictures:

Each student gets $\frac{1}{8}$ of each gram.

$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

Solution **Each student gets $\frac{3}{8}$ gram of glitter.**

4 Check your answer. Show your work.

Possible student work:

$8 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

$= \frac{24}{8}$

$= 3$

So, each student gets $\frac{3}{8}$ gram.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Apply It*** problem 9 to students. Ask students to underline the phrase *into each*. Have students discuss why they think that a division expression is needed to solve the problem. Provide a sentence frame to guide students' discussions: *I need a division expression because objects are* separated, shared, or distributed *into equal groups*. Confirm that students understand the terms *remainder form* and *mixed number*. Organize students into pairs and give them base-ten blocks to model the problem. Encourage students to use gestures, pictures, numbers, and words to communicate ideas.	**Listening/Speaking** Read ***Apply It*** problem 9 with students. Ask them to state the problem in their own words. In partners, have them decide on the first step they will use to solve the problem. When complete, have students work with another set of partners to compare their first steps. Then ask them to solve the problem individually. Ask students to explain their thinking as they explore writing the answer as a mixed number. Ask guiding questions, such as: *How do you know what the denominator will be? How many wholes are there? How many tenths are remaining? How is the mixed number related to the remainder form of the answer?*	**Reading/Writing** Have students read ***Apply It*** problem 9 individually. Ask students to write the steps that are needed to solve the problem, and to also draw a visual model. Have students trade their steps with another student and compare the steps and visual models they plan to use to solve the problem. Ask them to solve the problem individually. After students solve the problem and write their answers in remainder form and as a mixed number, have them pair up again to debrief. Ask them to take turns explaining how they found the mixed number.

LESSON 18

SESSION 2 Develop

Purpose In this session students solve a problem that requires finding the quotient 5 ÷ 3 and representing it as both a fraction and a mixed number. Students model the division either on paper or with manipulatives. The purpose of this problem is to develop strategies for finding quotients that are fractions or mixed numbers.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *Number Lines*

Why Support students' facility with writing fractions greater than 1 as mixed numbers.

How Have students label a number line to show fourths, locate the fractions $\frac{7}{4}$ and $\frac{14}{4}$ on the number line, and then write the equivalent mixed numbers.

Locate each fraction on a number line labeled in fourths. Then write the fraction as a mixed number.

$\frac{7}{4}$ = ... $\frac{14}{4}$ = ...

Solutions

Check students' number lines.

$\frac{7}{4} = 1\frac{3}{4}$

$\frac{14}{4} = 3\frac{2}{4}$, or $3\frac{1}{2}$

Develop Language

Why Review the terms *dividend*, *divisor*, and *quotient* to facilitate student discussions.

How Write the terms on the board and explain that they are used to identify the parts of a division equation. Write a division equation on the board and have students work to label each of the numbers with the term *dividend, divisor,* or *quotient*. Discuss and correct any misconceptions. Have students revise their work as needed, and copy the labeled equation onto a class chart for students to use as a reference.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, help them recognize that 3 students are decorating 5 hallways.

Ask *The problem asks you to find how much each student will decorate. What does* how much *mean in this situation? What will be the unit for your answer?*

Develop Fractions as Division

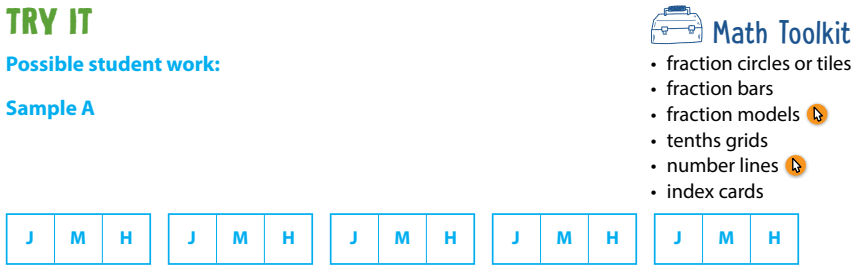
Read and try to solve the problem below.

Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?

TRY IT

Possible student work:

Sample A



Each student decorates $\frac{5}{3}$ of a hallway.

Sample B

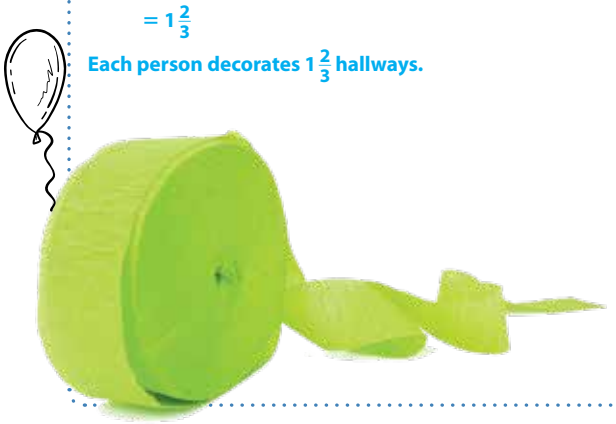
$5 \div 3 = \frac{5}{3}$

$= 1\frac{2}{3}$

Each person decorates $1\frac{2}{3}$ hallways.

Math Toolkit

- fraction circles or tiles
- fraction bars
- fraction models
- tenths grids
- number lines
- index cards



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I disagree with this part because . . .

DISCUSS IT

Support Partner Discussion

Encourage students to name the model they used as they discuss their solutions.

Support as needed with questions such as:

- *How would you describe your model?*
- *What was it about this problem that made you think of using that model?*

Common Misconception Look for students who accurately model the problem but have difficulty identifying what constitutes one equal share from all the equal parts represented. As students present solutions, ask them to identify Jared's share in the model.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models showing 3 groups of 5 thirds
- drawings to represent the problem
- number lines marked in thirds
- equations showing 5 ÷ 3 can be represented by $\frac{5}{3}$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the number of hallways? The part of the hallways Jared decorates? Monica decorates? Heather decorates?*

Listen for Students should recognize that accurate representations show 5 wholes divided into thirds, with each student's share identified. Responses may include each student's share is equivalent to $\frac{5}{3}$.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the 5 wholes
- the number of equal parts
- the number of parts decorated by each student

Ask *What is the total number of equal parts shown in the fraction model? On the number line? Is it the same or different? Why?*

Listen for 15 is the total number of equal parts shown in both the fraction model and on the number line. It is the same number because each model shows 5 wholes divided into thirds, so there are 5 × 3 thirds, or 15 thirds in all.

For the fraction model, prompt students to identify how color is used to help represent the problem.

- *Why are three different colors used to shade the thirds in each whole?*

For the number line model, prompt students to recognize that the same information is shown on both number lines.

- *How many thirds are part of Jared's share in the top number line? The bottom number line?*
- *How is the first number line similar to the fraction model? How is it different?*
- *Think of each whole on the number line as representing 1 hallway. How does each way of shading the number line show a different way the students can divide up the work?*

Explore different ways to understand fractions as quotients.

Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?

PICTURE IT

You can use a fraction model to picture how the students divide up the work.

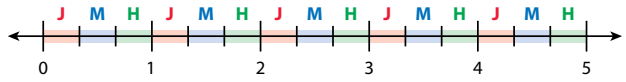
There are 5 hallways for 3 students to decorate, which is 5 ÷ 3.

If they share the work equally, each student can decorate $\frac{1}{3}$ of each hallway.

MODEL IT

You can use a number line to model each student's share of the work.

The number line is numbered from 0 to 5 because there are 5 hallways. It is divided into thirds because each student can decorate one third of each hallway.



The thirds can be rearranged to show each student's share of the work.

Deepen Understanding

Remainders

SMP 2 Reason abstractly and quantitatively.

When discussing the number line model, have students use a mixed number to identify what is shown as each student's share of the work. [$1\frac{2}{3}$ hallways]

Ask *If the 3 students were sharing 5 paintbrushes instead of 5 hallways, would it make sense for them to each get $1\frac{2}{3}$ paintbrushes? How many paintbrushes would each student get? How many paintbrushes would be left over?*

Listen for $\frac{2}{3}$ of a paintbrush does not make sense. Each student would get 1 paintbrush with 2 brushes left over.

Prompt students to consider how in the past when they have divided whole numbers that did not divide evenly, they wrote the part left over as a remainder. Ask them how 5 ÷ 3 would be written using this method. [1 R 2]

Discuss division situations where they would want to show a quotient as a mixed number, and situations where they would want to show it as a whole number and a remainder.

CONNECT IT

- Remind students that one thing that is alike about all the representations is they show whole-number division that results in a quotient that is a fraction.
- Explain that on this page they will look at two different ways to think about the division and two different ways to show the quotient.

Monitor and Confirm

1–3 Check for understanding that:

- there are 15 thirds in all
- 15 thirds ÷ 3 = 5 thirds
- the quotient can be written as the fraction $\frac{5}{3}$
- as with whole numbers, you can write related multiplication and division equations

Support Whole Class Discussion

4–5 Have students think about modeling the way of dividing up the work described in problem 4. Guide them to connect writing the quotient in remainder form and as a mixed number.

Ask *How would you change the fraction model in* Picture It *to show this way of dividing up the work? What would a number line model of this way look like?*

Listen for In *Picture It*, each of the first three rectangles would be fully shaded in a single color, pink, blue, and green. On a number line, you could shade from 0 to 1 in pink, from 1 to 2 in blue, and from 2 to 3 in green. For the other two sections, shade $\frac{1}{3}$ of each section pink, $\frac{1}{3}$ blue, and $\frac{1}{3}$ green.

Ask *Does the mixed number or the remainder form better represent the solution?*

Listen for The mixed number gives an exact amount each person decorates. The remainder form indicates that each decorate 1 full hallway and then some of the remaining 2 hallways.

6 Look for the idea that the bar in a fraction can be interpreted as meaning *divided by*—the numerator is divided by the denominator—just as the division symbol in an expression does.

7 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand fractions as quotients.

1 How many thirds of a hallway are there to decorate in 5 hallways? 15 thirds

2 How many thirds of a hallway will each student decorate? 5 thirds

Write this as a fraction. $\frac{5}{3}$ of a hallway

3 Write a division equation that shows the quotient as a fraction. $5 \div 3 = \frac{5}{3}$

Write a multiplication equation to check this equation. $3 \times \frac{5}{3} = 5$

4 How many whole hallways can each student decorate? 1

How many hallways remain after those are done? 2

How much of the 2 remaining hallways will each student decorate? $\frac{2}{3}$

Write a mixed number to show how many hallways each student will decorate.

$1\frac{2}{3}$ hallways

5 Calculate using remainder notation: 5 ÷ 3 = 1 R 2

Compare this answer to the mixed number. How are they alike?

The whole number is the same as the quotient without the remainder. The numerator is the same as the remainder.

6 How does the bar in a fraction represent division?

The bar means that the number in the numerator is divided by the number in the denominator.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding fraction quotients? Explain.

Students may respond that they like using fraction models or number lines to visualize dividing an amount into equal shares, or that they like representing a problem as a division equation that shows the quotient as a fraction.

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Hands-On Activity

Connect fractions to equivalent division expressions.

If . . . *students are unsure about how to interpret a fraction as division,*

Then . . . *use this activity to rewrite fractions as equivalent division expressions.*

Materials For each student: base-ten blocks (1 tens rod, 2 ones units), Activity Sheet *Digit Cards* (3, 4, 5)

- Distribute materials to students. Have students use the digit cards and base-ten blocks to "build" the fraction used to solve the *Try It* problem, $\frac{5}{3}$, using the rod as the fraction bar and placing a digit card for *5* above the rod and a digit card for *3* below it.
- Ask students to alter the fraction they built to show the division expression used to represent the problem, *5 ÷ 3*, moving the digit cards and using the ones units along with the rod to make a division symbol (÷). Discuss where students placed the numerator and denominator to make the expression.
- Repeat activity, using the situation from the Explore *Try It:* 4 ounces of paint shared equally by 5 students. This time, have students first show the division expression that models the problem and then turn it into the fraction quotient.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; dividing wholes and number lines into equal parts accurately can be very difficult.

8 $\frac{3}{5}$ of a pack; Students may use a model to see that there are 15 fifths in 3 packs, so each friend gets 3 of the fifths. See possible model on the Student Worktext page. Students may also show a number line divided into fifths marked to show 5 shares of 3 fifths each.

9 $10 \div 4 = 2$ R 2; $10 \div 4 = 2\frac{2}{4}$; Each container has $2\frac{2}{4}$, or $2\frac{1}{2}$, ounces of apple chips. See possible visual model on the Student Worktext page. Students should see that the mixed number best answers the question because it gives an exact amount.

Close: Exit Ticket

10 **C**; The expression $12 \div 7$ shows the numerator, 12, divided by the denominator, 7.

Error Alert **If** students choose A, B, or D, **then** remind students that the bar in a fraction can mean the numerator is divided by the denominator. Have them read the fraction $\frac{12}{7}$, inserting the words *divided by* for the fraction bar.

APPLY IT

Use what you just learned to solve these problems.

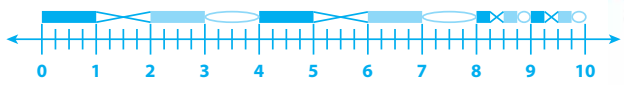
8 Five friends are equally sharing 3 packs of football cards. How many packs of cards will each friend get? Use a visual model to support your answer.

Possible student work:

Solution $\frac{3}{5}$ **of a pack**

9 Elena made 10 ounces of apple chips. She puts the same amount of apple chips into each of 4 containers. How many ounces of apple chips are in 1 container?
Write a division expression to represent the problem and solve.
Write the solution in remainder form and as a mixed number.
Use a visual model to support your answer.

Possible student work:



$10 \div 4 = 2$ R 2; $10 \div 4 = 2\frac{2}{4}$

Solution **2 R 2 or $2\frac{2}{4}$ or $2\frac{1}{2}$ ounces of apple chips**

Does the remainder form or the mixed number form best answer the question? Explain.

The mixed number tells exactly how many ounces, so it best answers the question of how many ounces of apple chips.

10 Which expression is equivalent to $\frac{12}{7}$?

Ⓐ $12 - 7$ Ⓑ $7 - 12$
Ⓒ $12 \div 7$ Ⓓ $7 \div 12$

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Solutions

1. Number Line A; This number line shows 4 wholes divided into sixths so it can be used to solve $4 \div 6$.
Basic

2. The model would change to show 5 equal parts in each rectangle; the answer would change to $\frac{1}{5} \times 4$, or $\frac{4}{5}$ of a package.
Medium

Name: _____ LESSON 18 SESSION 2

Practice Fractions as Division

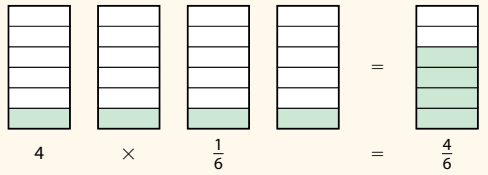
Study the Example showing whole-number division with a fraction quotient. Then solve problems 1–5.

EXAMPLE

There are 4 packages of printer paper to be divided equally among 6 classrooms. How much paper will each classroom get?

There are 4 packages for 6 classrooms to share, which is $4 \div 6$.

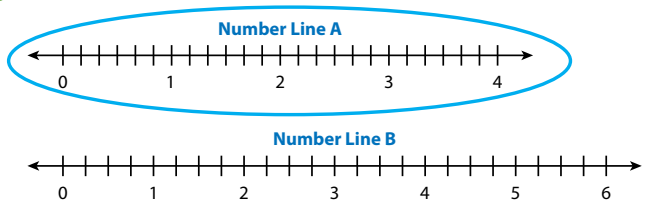
If you divide each package into sixths, each classroom would get one sixth of each package. So, $\frac{1}{6}$ of each package from 4 packages is the same as $\frac{4}{6}$ of a package.



$4 \div 6 = \frac{4}{6}$

Each classroom gets $\frac{4}{6}$ of a package.

1. Circle the number line you would use to solve the problem in the Example.



2. Look at the Example. Suppose only 5 classrooms share 4 packages. How would the model in the Example change? How would the answer change?

Each rectangle would be divided into 5 equal sections instead of 6; the answer would change to $\frac{4}{5}$ of a package.

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Fluency & Skills Practice

Teacher Toolbox

Assign *Fractions as Division*

In this activity students solve real-world division problems in which the quotient is a fraction or mixed number. Students may encounter similar situations in everyday life. For example, students may need to divide 5 cubic meters of peat moss evenly among 4 garden plots. Or, they may need to determine how many 2-cup servings of milk are in 25 cups of milk.

Fluency and Skills Practice

Fractions as Division

Name: _____

Solve each problem.

1. Roger has 4 gallons of orange juice. He puts the same amount of juice into each of 5 pitchers. How many gallons of orange juice are in 1 pitcher?

2. Marta has 8 cubic feet of potting soil and 3 flower pots. She wants to put the same amount of soil in each pot. How many cubic feet of soil will she put in each flower pot?

3. Greg made 27 ounces of potato salad to serve to 10 guests at a picnic. If each serving is the same size, how much potato salad will each guest receive?

4. Chandra spends 15 minutes doing 4 math problems. She spends the same amount of time on each problem. How many minutes does she spend on each problem?

5. Taylor has 5 yards of gold ribbon to decorate 8 costumes for the school play. She plans to use the same amount of ribbon for each costume. How many yards of ribbon will she use for each costume?

6. DeShawn is using 7 yards of wire fencing to make a play area for his puppy. He wants to cut the fencing into 6 pieces of equal length. How long will each piece of fencing be?

7. What is a division word problem that can be represented by $\frac{4}{3}$?

3 $\frac{7}{3}$, or $2\frac{1}{3}$ cans; Students may use fraction models or number lines divided into thirds, or the expression $7 \div 3$, to show how much food the dogs will get each day. Students should explain that the mixed number better expresses the amount of food per day because it gives an exact amount. The remainder just indicates that more than 2 cans will be used.
Medium

4 No; See possible explanation on the student page.
Challenge

5 Less than 1 cup; Students may use fraction models divided into sevenths, the expression $48 \div 7$, or some other method to find each person will get $\frac{48}{7}$, or $6\frac{6}{7}$ ounces. That amount is less than 8 ounces.
Medium

3 Trish is taking care of the Han family's dogs. The Hans leave 7 cans of dog food for the 3 days they will be away. How much food will the dogs get each day if Trish feeds them an equal amount each day? Show your work. Write the answer in remainder form and as a mixed number.

Students might use number lines, equations, or some other method to show the quotient of $7 \div 3$.

Solution **2 R 1 cans; $\frac{7}{3}$, or $2\frac{1}{3}$ cans**

Which best answers the question, the remainder form or the mixed number? Explain.

Possible answer: The mixed number, because it tells exactly how many cans to use each day. The remainder form shows to use more than 2, but not an exact amount.

4 Raul plans to run 30 miles this week. He wants to run the same number of miles each day of the week. He says he will run $\frac{7}{30}$ mile each day. Is he correct? Explain.

No; Possible explanation: Raul divided the number of days by the number of miles, $7 \div 30$. He needed to divide the number of miles by the number of days, $30 \div 7$. Raul will run $\frac{30}{7}$, or $4\frac{2}{7}$, miles a day.

5 Gus makes 48 fluid ounces of spiced cider. If he serves an equal amount to each of 7 people, will each person get more than 1 cup of cider or less than 1 cup? (1 cup = 8 fluid ounces) Show your work.

Students might use fraction models, equations, or some other method to show that $48 \div 7 = \frac{48}{7}$ or $6\frac{6}{7}$. $6\frac{6}{7}$ ounces < 8 ounces

Solution **less than 1 cup**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 1 to students. Ask them to point to the words *equal amount*. Ask students to discuss with a partner what operation they will need to solve the problem and why.

Provide the following sentence frame: *When you break up a bigger number into equal* __smaller__ *amounts, you need to* __divide__.

Levels 2–4

Reading/Writing Read ***Apply It*** problem 1. Have students read the problem with a partner and work together to decide on a strategy to use to solve for the amount of space Erica will give each vegetable.

Have students work together to solve the problem using the strategy they chose.

Levels 3–5

Reading/Writing Have students read ***Apply It*** problem 1 with a partner and work together to decide on a strategy to use to solve for the amount of space Erica will give each vegetable. Ask partners to solve the problem.

Have students work with a different group and discuss the different approaches each group selected to solve the problem.

Provide a sentence starter and encourage students to justify their thinking: *We know our answer is correct because* _____.

LESSON 18

SESSION 3 Refine

Purpose In this session students solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: Activity Sheet *Fraction Bars* (7 bars for thirds) 3 sheets of paper, scissors

Why Confirm understanding of fractions as division.

How Have students solve the problem using any strategy and write a division equation to show the solution.

Solve the problem and write a division equation.

Andy, Este, and May mow 7 lawns and share the work equally. How many lawns does each person mow?

Solution

$\frac{7}{3}$, or $2\frac{1}{3}$, lawns each;
$7 \div 3 = \frac{7}{3}$, or $2\frac{1}{3}$

LESSON 18 SESSION 3 ● ● ●

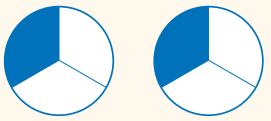
Refine Fractions as Division

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Luke, Carter, and Ava have 2 quarts of juice. They want to share it equally. How many quarts of juice will each of them get?

Look at how you could show your work using a model and equations.



$2 \div 3 = 2 \times \frac{1}{3}$
$= \frac{2}{3}$

Solution $\frac{2}{3}$ quart

2 quarts are shared equally by 3 friends, so I know that each friend will have less than 1 quart of juice. That means the quotient is a fraction.

PAIR/SHARE
Model the problem for 3 quarts of juice divided equally among Luke, Carter, Ava, and Ava's little brother.

APPLY IT

1 Erica has 7 square feet of space in her rectangular garden to plant carrots, beans, peppers, and lettuce. Suppose she gives each vegetable an equal amount of space. How much space will each vegetable get?
Show your work.

Possible student work using a model:

C	B	P	L	C	B	P	L	C	B	P	L	C	B	P	L
1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²	1 ft²

Solution $\frac{7}{4}$, or $1\frac{3}{4}$, square feet

Each vegetable will get at least 1 square foot of garden space. How will the rest of the space be divided up?

PAIR/SHARE
What are some ways you can check your solution?

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
$3 \div 7$ or $\frac{3}{7}$	have reversed the numerator and the denominator.	First, have students solve a simpler problem: *3 students mow 6 lawns and share the work equally. How many lawns does each person mow?* [$6 \div 3 = 2$] Then have students apply this same reasoning to the given problem.
$7 \times 3 = 21$, $7 - 3 = 4$, or $7 + 3 = 10$	not understand that they need to divide to solve the problem.	Have students cut out fraction bars to model 7 lawns and label a sheet of paper with the name of each person in the problem. Ask: *How many lawns does each person get?* Have students share the lawns equally by placing fraction bars or equal portions of fraction bars on each sheet of paper. Point out that this is an equal sharing problem and guide students to recall that division is used for equal sharing.

EXAMPLE

$\frac{2}{3}$ quart; the fraction model and equations show one way to solve the problem. Students could also solve the problem by drawing a number line from 0 to 2 that is divided into 3 equal sections of $\frac{2}{3}$.

Look for You divide 2 wholes into 3 equal shares. When the divisor is greater than the dividend, the quotient is a fraction.

APPLY IT

1. $\frac{7}{4}$, or $1\frac{3}{4}$ square feet; Students could solve the problem by drawing a model showing 1 square foot of the garden given to each of the 4 vegetables, and the remaining 3 square feet of garden divided into fourths, with a total of 3 fourths given to each vegetable.
   DOK 2

   Look for The solution is a mixed number since there is enough garden space for each vegetable to get at least 1 square foot of space but not enough for each to get 2 square feet.

2. Between 3 and 4 ounces; See possible work on the Student Worktext page. Students could also solve the problem by using rounding. If 36 is rounded down to 30, each dough would weigh $120 \div 30 = 4$ ounces. If 36 is rounded up to 40, each dough would weigh $120 \div 40 = 3$ ounces. So, $120 \div 36$ must be between 3 and 4 ounces.
   DOK 2

   Look for The solution requires two steps: the first is to find the mixed number quotient of $120 \div 36$ and the second is to identify the two whole numbers that the quotient falls between.

3. **D**; Students could solve the problem using the equation $21 \div 10 = \frac{21}{10}$.
   Explain why the other two answer choices are not correct:
   B is not correct because $\frac{21}{10} = 2\frac{1}{10}$, not $1\frac{1}{10}$.
   C is not correct because $\frac{21}{10} > 2$.
   DOK 3

2 Deon needs to make 36 pizza crusts. He has 120 ounces of dough and wants to use the same amount of dough for each crust. He weighs a portion of dough for 1 crust on a scale. The weight, in ounces, should fall between what two whole numbers? Show your work.

Possible student work:

120 ounces ÷ 36 pizza crusts = $\frac{120}{36}$ or $3\frac{2}{6}$ ounces per crust

$3\frac{2}{6}$ is between 3 and 4.

How many whole ounces of dough will each crust get? What will happen with the remaining ounces?

Solution **between 3 and 4 ounces**

PAIR/SHARE
Create a different division story to represent $\frac{120}{36}$.

3 Jonas is doing a science experiment with his class. The teacher has 21 fluid ounces of pond water to share equally among 10 pairs of students. How much pond water will Jonas and his science partner receive?

Ⓐ $\frac{10}{21}$ fluid ounce

Ⓑ $1\frac{1}{10}$ fluid ounces

Ⓒ 2 fluid ounces

Ⓓ $\frac{21}{10}$ fluid ounces

About how much water will each pair of students receive? Will it be more or less than 2 fluid ounces?

Olivia chose Ⓐ as the correct answer. How did she get that answer?

Possible explanation: She wrote a fraction to show 10 divided by 21 instead of a fraction to show 21 divided by 10.

PAIR/SHARE
Does Olivia's answer make sense?

4 **D**; 32 fluid ounces $\div$ 5 $= \frac{32}{5}$, or $6\frac{2}{5}$ fluid ounces. $6\frac{2}{5}$ is between 6 and 7.
DOK 2

5 **B**; Each lap takes almost 1 minute.
8 minutes $\div$ 10 $= \frac{8}{10}$ minute.
DOK 2

6 **A (Yes);**
D (No);
E (Yes);
H (No);
I (Yes)
DOK 2

Error Alert Students may not recognize that the board length can be expressed as a multiplication expression.

4 Teddy makes 32 fluid ounces of hot cocoa. He pours equal amounts of cocoa into 5 cups. The amount of hot cocoa in each cup will fall between which two amounts?

Ⓐ 3 and 4 fluid ounces
Ⓑ 4 and 5 fluid ounces
Ⓒ 5 and 6 fluid ounces
Ⓓ 6 and 7 fluid ounces

5 Pierce swims 10 laps in a pool in 8 minutes. He spends the same amount of time on each lap. How much time does each lap take him?

Ⓐ $\frac{2}{10}$ minute
Ⓑ $\frac{8}{10}$ minute
Ⓒ $\frac{10}{8}$ minutes
Ⓓ $1\frac{2}{8}$ minutes

6 Dani needs 8 equal sections from a board that is 13 feet long. Does the expression represent the largest possible length of 1 section of the board, in feet?

	Yes	No
$1\frac{5}{8}$	Ⓐ	Ⓑ
$\frac{8}{13}$	Ⓒ	Ⓓ
$\frac{13}{8}$	Ⓔ	Ⓕ
$8 \div 13$	Ⓖ	Ⓗ
$13 \times \frac{1}{8}$	Ⓘ	Ⓙ

Differentiated Instruction

RETEACH

Hands-On Activity
Model dividing two whole numbers with a fraction quotient.

Students struggling with the concept of dividing a lesser number by a greater number

Will benefit from additional work with concrete models of fraction quotients

Materials For each student: 8 squares of paper, each one a 3 $\times$ 3 array of squares cut from Activity Sheet *1-Inch Grid Paper,* scissors

- Pose the following problem: *7 cups of juice are shared equally among 9 students. How much juice does each student get?*
- Have students use 7 paper squares to model 7 cups. Set aside the remaining square.
- Have students shade $\frac{1}{9}$ in each square to show that each student gets $\frac{1}{9}$ from each cup.
- Have students cut out the shaded $\frac{1}{9}$s and reassemble them on the remaining square to show how much juice one student gets. [$\frac{7}{9}$ cup]
- Reassemble the squares and repeat the activity for numbers of cups less than 7, such as 3 cups and 5 cups, and sharing them equally among 9 students.

EXTEND

Challenge Activity
Model fractions as quotients.

Students who have achieved proficiency

Will benefit from deepening understanding of fractions as division

- In groups of 3, students pass their work to the next person after each round.
- Round 1: Each student writes a division word problem in which the quotient is a fraction or a mixed number.
- Round 2: Students draw a visual model to represent the problem they receive.
- Round 3: Students write an equation that represents the model they receive.
- The group checks each other's work.

7 **A;** Divide 25 yards of paper by the number of banners, 9, to find how much paper is used for each banner.

E; Divide 25 ounces by the number of equal servings, 9, to find how many ounces in each serving.

DOK 2

8 **Part A**

See possible number line on the Student Worktext page.

Part B

See possible explanation on the Student Worktext page. Students may also explain that to model $7 \div 2$, you can show 7 divided into 2 equal parts. Each part is equal to $3\frac{1}{2}$, which is equivalent to $\frac{7}{2}$.

DOK 3

7 Which situations can be represented by $\frac{25}{9}$?

Ⓐ Melanie equally shares 25 yards of paper to make 9 banners.

Ⓑ Quill gives away 9 baseball cards from a pack of 25 cards.

Ⓒ George invites 25 kids and 9 adults to his birthday party.

Ⓓ Becca makes 9 rows with 25 buttons each.

Ⓔ Joe makes 9 equal servings from a 25-ounce bag of peanuts.

8 Paco is trying to explain to his friend that $7 \div 2 = \frac{7}{2}$.

Part A Draw a model or number line showing $7 \div 2 = \frac{7}{2}$.

Possible student work using a number line:

$7 \div 2 = \frac{7}{2}$

$= 3\frac{1}{2}$

Part B Explain the equivalence of $7 \div 2$ and $\frac{7}{2}$ using words.

Possible explanation: Seven halves is equivalent to seven divided into two equal sections because both equal three and one half.

9 MATH JOURNAL

Write a division word problem that can be represented by the expression $12 \div 5$. Then explain how to solve your problem.

Possible problem: Five friends want to share 12 ounces of juice equally. How many ounces of juice should each friend get?

Possible explanation: You can solve the equation $12 \div 5 = n$: $n = \frac{12}{5}$. Each friend gets $\frac{12}{5}$, or $2\frac{2}{5}$, ounces of juice.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

REINFORCE

Problems 4–9

Interpret fractions as division.

All students will benefit from additional work with using fractions to represent division of whole numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide children with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should include a word problem with 12 as the number of wholes to be shared and 5 as the number of equal shares. Students should explain that the quotient $12 \div 5$ can be represented by the fraction $\frac{12}{5}$.

Error Alert **If** students reverse the numerator and denominator in the fraction quotient, **then** have them use reasoning to determine which two whole numbers the quotient of $12 \div 5$ falls between and assess which of the two possible fractions, $\frac{12}{5}$ or $\frac{5}{12}$, is between those two numbers.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener page.

Lesson Overview

LESSON 22

Multiply Fractions in Word Problems

Lesson Objectives

Content Objectives

- Represent real-world problems involving multiplication of fractions and mixed numbers using visual models and equations.
- Solve real-world problems involving multiplication of fractions and mixed numbers using visual models and equations.

Language Objectives

- Draw pictures to represent word problems involving multiplication of fractions and mixed numbers.
- Write equations to represent word problems involving multiplication of fractions and mixed numbers.
- Compare a visual model and an equation that represent the same problem situation.

Prerequisite Skills

- Multiply fractions.
- Add fractions.
- Write equivalent fractions.
- Write a mixed number as a fraction greater than 1.
- Write a fraction greater than 1 as a mixed number.
- Use visual models to represent problem situations.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

5 Use appropriate tools strategically.

*See page 305m to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **equation** a mathematical statement that uses an equal sign (=) to show that two expressions have the same value.
- **factor** a number that is multiplied.
- **mixed number** a number with a whole number part and a fractional part.
- **product** the result of multiplication.

Learning Progression

In previous Grade 5 lessons students acquired a conceptual understanding of multiplying fractions, using visual models to make sense of the meaning of fraction multiplication. Students also used area models to multiply fractions and found areas of rectangles with fractional side lengths. They wrote equations to represent fraction multiplication and recognized that using a model or writing an equation both result in the same product. Students also conceptualized multiplication as scaling when they understood a product as a relationship between a quantity and a resizing, or scaling, factor. They reasoned about the size of a product when one factor is greater than 1, equal to 1, or less than 1.

In this lesson students draw on their work in previous lessons to solve word problems that involve multiplying fractions and mixed numbers, including finding a fraction of a quantity, finding the area of a rectangle, and finding the dimensions of a resized object.

In the remaining lessons in this unit students will use what they know about fraction multiplication to divide with unit fractions and solve word problems about division with unit fractions. In later grades, students will apply their understanding of fraction multiplication to solve problems involving proportional relationships. They will also extend their understanding of fraction multiplication to multiply rational numbers.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Solve Multiplicative Comparison Problems

Multiplying Fractions in Word Problems
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 439–440

SESSION 2
Develop
45–60 min

Multiplying Fractions in Word Problems
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 445–446

Fluency
Multiplying Fractions in Word Problems

SESSION 3
Develop
45–60 min

Multiplying with Mixed Numbers in Word Problems
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 451–452

Fluency
Multiplying with Mixed Numbers in Word Problems

SESSION 4
Refine
45–60 min

Multiplying Fractions in Word Problems
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz
or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Activity Sheet:* Fraction Bars
Activities	*Per student:* drawing paper (1 sheet) *Per pair:* 1 set of fraction tiles, 1 set of fraction circles, tracing paper (2 sheets), ruler, colored pencils or crayons (2 different colors), tape
Math Toolkit	fraction tiles, fraction circles, fraction bars, grid paper, number lines, index cards
Digital Math Tools	Fraction Models, Number Line, Multiplication Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 4
- Lesson 7 Multiplication and Division in Word Problems

RETEACH

Tools for Instruction

Grade 4
- Lesson 7 Solve Comparison Problems

Grade 5
- Lesson 22 Multiplying Fractions to Solve Word Problems

REINFORCE

Math Center Activities

Grade 5
- Lesson 22 Write a Word Problem
- Lesson 22 Real-World Multiplication Situations

EXTEND

Enrichment Activity

Grade 5
- Lesson 22 Plant Growth

LESSON 22

Connect to Family, Community, and Language Development

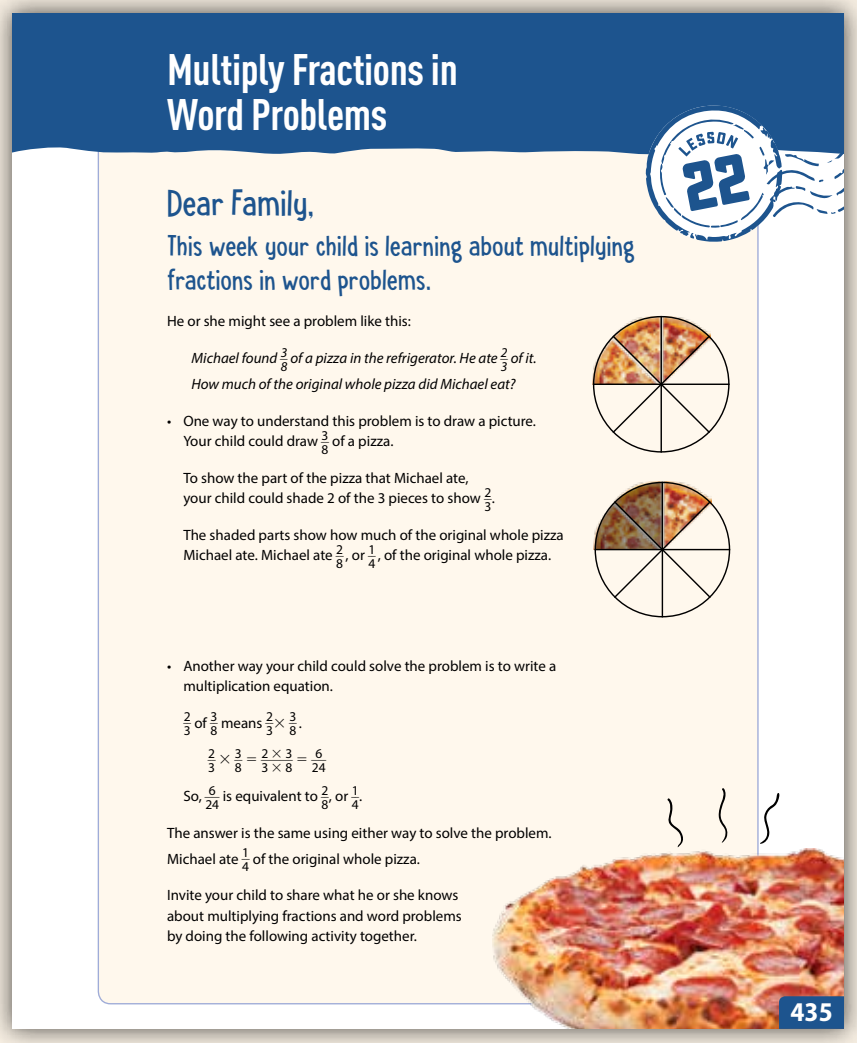
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Multiply Fractions in Word Problems

LESSON 22

Dear Family,

This week your child is learning about multiplying fractions in word problems.

He or she might see a problem like this:

Michael found $\frac{3}{8}$ of a pizza in the refrigerator. He ate $\frac{2}{3}$ of it. How much of the original whole pizza did Michael eat?

- One way to understand this problem is to draw a picture. Your child could draw $\frac{3}{8}$ of a pizza.

  To show the part of the pizza that Michael ate, your child could shade 2 of the 3 pieces to show $\frac{2}{3}$.

  The shaded parts show how much of the original whole pizza Michael ate. Michael ate $\frac{2}{8}$, or $\frac{1}{4}$, of the original whole pizza.

- Another way your child could solve the problem is to write a multiplication equation.

  $\frac{2}{3}$ of $\frac{3}{8}$ means $\frac{2}{3} \times \frac{3}{8}$.

  $\frac{2}{3} \times \frac{3}{8} = \frac{2 \times 3}{3 \times 8} = \frac{6}{24}$

  So, $\frac{6}{24}$ is equivalent to $\frac{2}{8}$, or $\frac{1}{4}$.

The answer is the same using either way to solve the problem.

Michael ate $\frac{1}{4}$ of the original whole pizza.

Invite your child to share what he or she knows about multiplying fractions and word problems by doing the following activity together.

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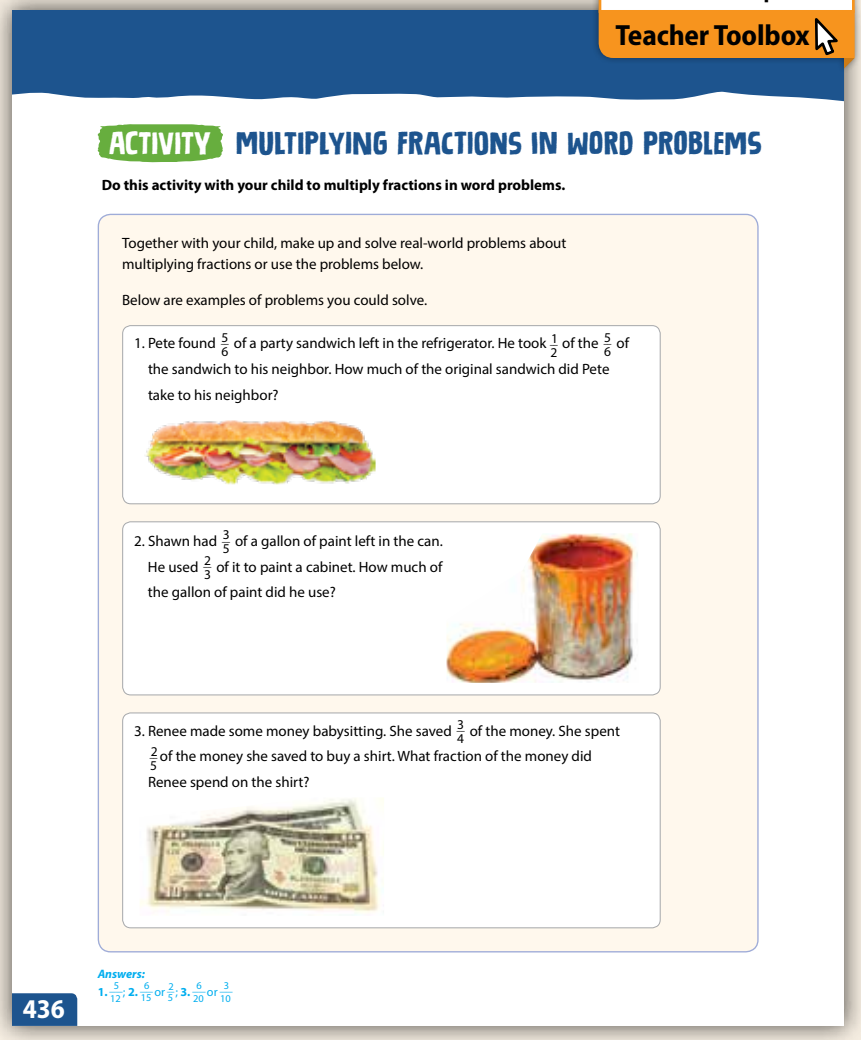

ACTIVITY MULTIPLYING FRACTIONS IN WORD PROBLEMS

Do this activity with your child to multiply fractions in word problems.

Together with your child, make up and solve real-world problems about multiplying fractions or use the problems below.

Below are examples of problems you could solve.

1. Pete found $\frac{5}{6}$ of a party sandwich left in the refrigerator. He took $\frac{1}{2}$ of the $\frac{5}{6}$ of the sandwich to his neighbor. How much of the original sandwich did Pete take to his neighbor?

2. Shawn had $\frac{3}{5}$ of a gallon of paint left in the can. He used $\frac{2}{3}$ of it to paint a cabinet. How much of the gallon of paint did he use?

3. Renee made some money babysitting. She saved $\frac{3}{4}$ of the money. She spent $\frac{2}{5}$ of the money she saved to buy a shirt. What fraction of the money did Renee spend on the shirt?

Answers:
1. $\frac{5}{12}$; **2.** $\frac{6}{15}$ or $\frac{2}{5}$; **3.** $\frac{6}{20}$ or $\frac{3}{10}$

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Goal

The goal of the Family Letter is to provide opportunities for students and their families to practice multiplying fractions in word problems.

Activity

Word problems provide real-world contexts for students to apply mathematical skills such as multiplying fractions. Look at the *Multiplying Fractions in Word Problems* activity and adjust if necessary to connect with your students.

Math Talk at Home

Students and family members solve the problems in the activity or use them as a model to make up other real-world problems.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *What are some items that you might need to divide into smaller parts and then divide those parts into smaller parts?*
- *How do you calculate what is left when you have a part of a whole and then you use a part of that part?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 1 and 2 **Use anytime during these sessions.**

- Many carpenters, architects, and engineers use fractions in their measurements. The tools they use, such as measuring tapes, have markings with fractions indicated. Understanding fractions is key to making accurate measurements and interpreting them appropriately.
- Share that in woodworking there are special precision marking and measuring tools that allow for measuring in fractions of a unit such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{32}$. Have students note that each fraction in the list is one-half of (or $\frac{1}{2}$ times) the previous number in the list.
- Have students work in small groups to research how different trades use fractions.

Session 3 **Use with *Try It*.**

- Have students customize the problem by researching what types of flowers or plants are commonly grown in their neighborhoods or their countries of origin. If time allows, discuss what type of climate and soil is needed to grow the flowers or plants that students selected.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Reading/Speaking Read the ***Try It*** problem aloud. Ask students to underline the quantities in the problem. Draw a sketch with Greyson's house and the park on it. Call on volunteers to provide the distances in the problem. Use the information to create a diagram. Organize students into pairs to model and solve the problem using tools from the *Math Toolkit*. Ask questions to assess understanding, such as: *How does your model show $\frac{4}{5}$ mile? How many equal parts does the model show? How does the model show the distance Greyson walked?* Encourage students to use gestures, pictures, numbers, and words to defend their answers.	**Reading/Speaking** Read the ***Try It*** problem with students. Organize them into pairs to discuss solution strategies and create a visual model to represent the problem. Ask: *How confident are you in your answer?* Have partners signal with two thumbs up for *very confident*, one thumb up and one thumb down for *fairly confident* and two thumbs down for *not confident*. Display the following word bank to support partners as they construct arguments to defend their answers: *because*, *shows*, *represents*, *explains*, *conclusion*, *equation*, *whole*, *part*, *numerator*, *denominator*. Provide support to students who expressed low confidence in their answers.	**Reading/Speaking** Have students read and solve the ***Try It*** problem independently. Organize them into pairs to defend their thinking. Tell students to convince their partners that their answer makes sense. Encourage them to use language for constructing arguments such as *because*, *shows*, *represents*, *proves*, *validates*, *explains*, *interpret*, *conclusion*, and *reasonable*. After both partners defend their answers, ask: *Was your partner's answer reasonable?* Have them signal thumbs up or down. Challenge students to identify a strength and an area for improvement in their partner's argument. Have them share ideas.

LESSON 22

SESSION 1 Explore

Purpose In this session, students draw on their knowledge of fraction multiplication. They share visual models and reasoning about a problem situation that involves finding a fraction of a fraction. They will look ahead to think about how to write equations, make estimates, and draw models to solve word problems involving multiplying fractions.

Start

Connect to Prior Knowledge

Why Supports students' facility in using visual models to show their thinking about fraction multiplication.

How Have students show $\frac{1}{2} \times \frac{3}{5}$ using any visual model they choose. Have them write the product.

Draw a visual model to show the expression. Write the product.

$\frac{1}{2} \times \frac{3}{5}$

Solution: $\frac{3}{10}$

Look for models that show the factors $\frac{1}{2}$ and $\frac{3}{5}$ and the product $\frac{3}{10}$.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that $\frac{3}{4}$ *of the way to the park* refers to $\frac{3}{4}$ of the distance from Grayson's house to the park.

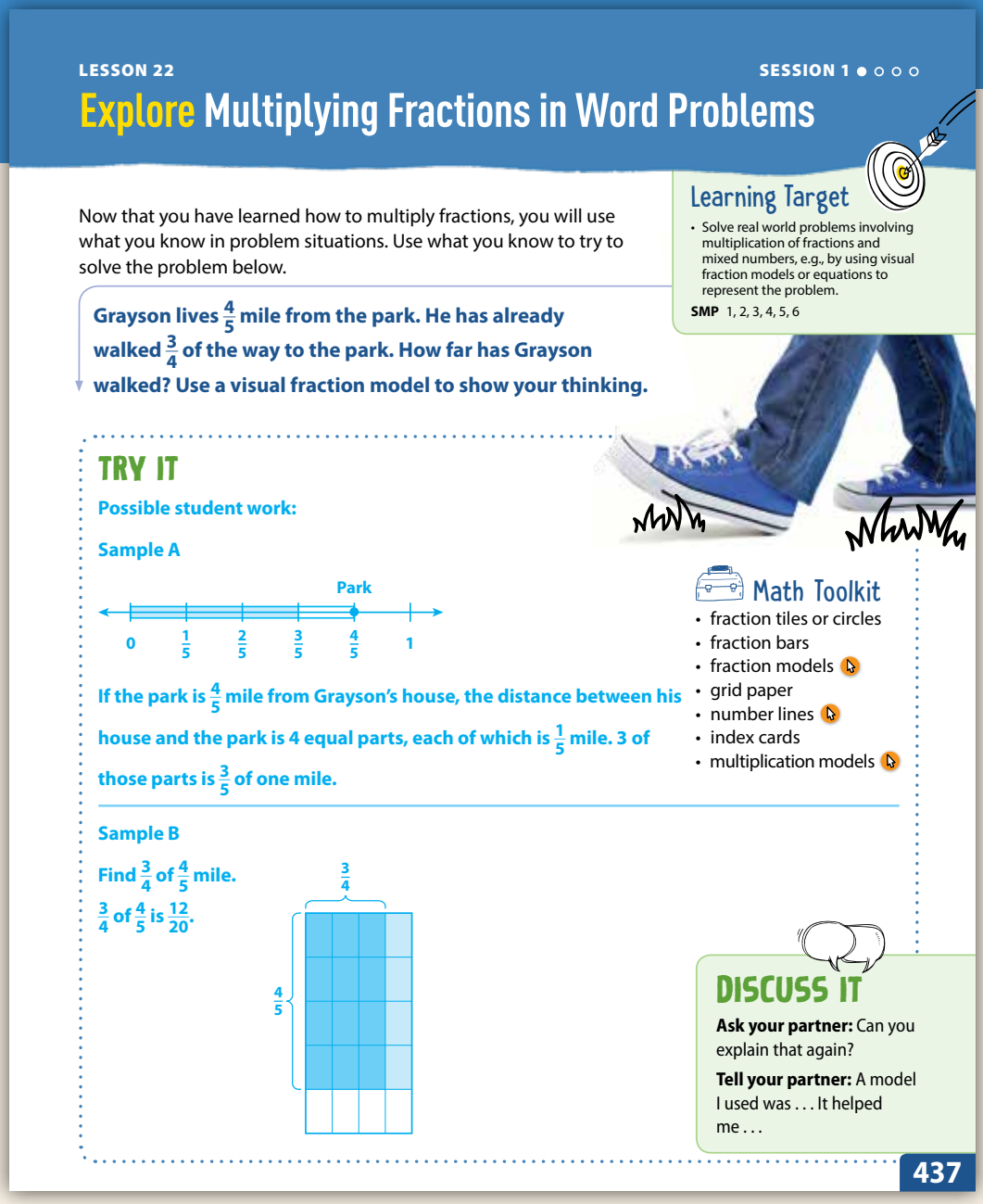
DISCUSS IT

Support Partner Discussion

Encourage students to use the *Discuss It* question and sentence starter on the Student Worktext page as part of their discussion.

Look for, and prompt as necessary for, understanding of:

- $\frac{4}{5}$ mile as the distance to the park
- $\frac{3}{4}$ as the fraction of the distance Grayson has walked
- $\frac{4}{5}$ mile as the whole of which $\frac{3}{4}$ is the part
- the fraction of a mile walked as the unknown



LESSON 22 SESSION 1 ● ○ ○ ○

Explore Multiplying Fractions in Word Problems

Learning Target

- Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP 1, 2, 3, 4, 5, 6

Now that you have learned how to multiply fractions, you will use what you know in problem situations. Use what you know to try to solve the problem below.

Grayson lives $\frac{4}{5}$ mile from the park. He has already walked $\frac{3}{4}$ of the way to the park. How far has Grayson walked? Use a visual fraction model to show your thinking.

TRY IT

Possible student work:

Sample A

If the park is $\frac{4}{5}$ mile from Grayson's house, the distance between his house and the park is 4 equal parts, each of which is $\frac{1}{5}$ mile. 3 of those parts is $\frac{3}{5}$ of one mile.

Sample B

Find $\frac{3}{4}$ of $\frac{4}{5}$ mile.

$\frac{3}{4}$ of $\frac{4}{5}$ is $\frac{12}{20}$.

Math Toolkit

- fraction tiles or circles
- fraction bars
- fraction models
- grid paper
- number lines
- index cards
- multiplication models

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: A model I used was . . . It helped me . . .

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Common Misconception Look for students who are not comfortable with this problem as a multiplying situation for fractions and may try to subtract to find the answer. As students present their solutions be sure to have them specify why they chose the operation they did to solve problem.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing fifths and fourths
- drawings to represent the problem
- number lines marked in fifths and then fourths
- equations to find the distance walked is $\frac{12}{20}$, or $\frac{3}{5}$, mile

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the distance from the house to the park? How far Grayson has walked as a fraction of that distance?*

Listen for The distance to the park is $\frac{4}{5}$ mile, or 4 equal parts each $\frac{1}{5}$ mile. The fraction Grayson walked is $\frac{3}{4}$ of $\frac{4}{5}$ mile, or 3 of the 4 equal parts.

CONNECT IT

1 LOOK BACK

Look for understanding of how to model $\frac{3}{4}$ of $\frac{4}{5}$ mile and interpret the model to find that Grayson has walked $\frac{3}{5}$ mile.

Hands-On Activity

Find a fraction of a fraction.

If . . . *students are unsure about drawing models to show fraction multiplication,*

Then . . . *use this activity to have them model* the Try It *and similar problems.*

Materials For each pair: 1 set of fraction tiles

- Have students use the 5 one-fifth tiles. Explain that, as a group, the 5 tiles represent a whole of 1 mile. Ask: *What does each tile represent?* [$\frac{1}{5}$ mile]
- Have students model the $\frac{4}{5}$-mile distance from Grayson's house to the park. [4 tiles] Point out that Grayson walked $\frac{3}{4}$ of the $\frac{4}{5}$ mile. Review that finding $\frac{3}{4}$ of $\frac{4}{5}$ is the same as finding $\frac{3}{4} \times \frac{4}{5}$.
- Ask students to use the tiles to find $\frac{3}{4}$ of $\frac{4}{5}$ mile and to explain how they did it. [$\frac{3}{5}$ mile; 3 of the 4 tiles represent $\frac{3}{4}$ of the distance to the park. Since each tile represents $\frac{1}{5}$ mile, 3 tiles represent $\frac{3}{5}$ mile.]
- Repeat the activity for additional fractions of $\frac{4}{5}$ mile, such as $\frac{2}{4}$ and $\frac{4}{4}$.

2 LOOK AHEAD

Point out that students know how to both write fraction multiplication expressions and model the multiplication. This along with reasoning and evaluating whether an answer makes sense prepares them to solve word problems involving fraction multiplication.

Ask *Why can you replace the word* of *in the phrase* $\frac{1}{2}$ of $\frac{3}{4}$ *with a multiplication symbol?*

Listen for You can think of finding $\frac{1}{2}$ of $\frac{3}{4}$ as finding $\frac{1}{2}$ of a group of size $\frac{3}{4}$, which is a multiplication situation.

CONNECT IT

1 LOOK BACK

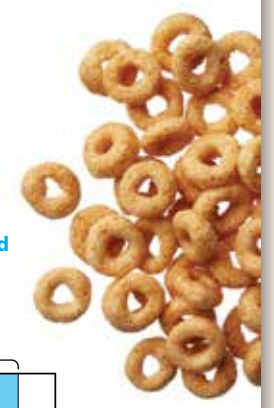
Explain how you can use a visual model to show how far Grayson has already walked.

Possible explanation: Using a number-line model, you can see that each $\frac{1}{4}$ of the distance is $\frac{1}{5}$ mile, so $\frac{3}{4}$ of the distance is $\frac{3}{5}$ mile.

2 LOOK AHEAD

You can use what you know about multiplying fractions to think through and solve word problems involving fractions. Consider this word problem:

Ehrin spills $\frac{1}{2}$ of a $\frac{3}{4}$-pound box of cereal. How many pounds did she spill?



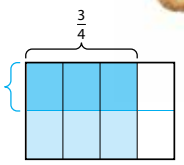
a. Finding $\frac{1}{2}$ of a quantity is the same as multiplying by $\frac{1}{2}$. What equation could you write for the cereal problem? Use p for the unknown amount in the problem.

$\frac{1}{2} \times \frac{3}{4} = p$

b. Estimate the product. Is the amount of cereal Ehrin spills on the floor more than $\frac{3}{4}$ pound or less than $\frac{3}{4}$ pound? Why?

Possible answer: The amount of cereal Ehrin spills is less than $\frac{3}{4}$ pound because you are finding $\frac{1}{2}$ of $\frac{3}{4}$.

c. Complete the area model to show the problem. How many pounds of cereal did Ehrin spill on the floor? **$\frac{3}{8}$** pound **Possible model is shown.**



3 REFLECT

How does writing an equation, making an estimate, and drawing a model help you think through the problem?

Possible answer: The equation shows the operation involved in solving the problem. The estimate helps me think about a reasonable solution. Drawing a model shows a visual of the problem.

Close: Exit Ticket

3 REFLECT

Look for understanding of how to apply the skills they have to solving word problems. Student responses should include references to equations showing the operations needed, estimates giving a frame of reference for the answer, and drawing models as a way to picture a problem situation.

Common Misconception **If** students are unclear or incomplete in their descriptions of how they can use equation writing, estimating, and visual models to represent and solve problems, **then** have students look at and describe how they used each skill to solve the cereal problem on the Student Worktext page.

Real-World Connection

Encourage students to think about everyday places or situations where people might need to multiply fractions. Have volunteers share their ideas. Examples: preparing $\frac{1}{3}$ of a recipe that includes fractional measurements, watching $\frac{2}{3}$ of a half-hour television show, cutting $\frac{3}{4}$ of a $\frac{5}{8}$-yard piece of fabric for a craft project.

Solutions

Support Vocabulary Development

1 Ask students to brainstorm models or pictures they have used in the past to represent a fraction of a fraction. Remind them that the root *frac-* means "to break."

A fraction of a fraction is a portion of a portion of equal parts. Have students record their examples and compare with other students.

2 Read the problem. Have students write the expression with a partner. Write the following on the board, aligning the words and numbers as shown:

fraction of a fraction

$\frac{1}{5}$ of a $\frac{3}{8}$

Ask: *Is any fraction of* $\frac{3}{8}$ *less than* $\frac{3}{8}$*?*

Then have partners write the multiplication expression and answer the question in the problem.

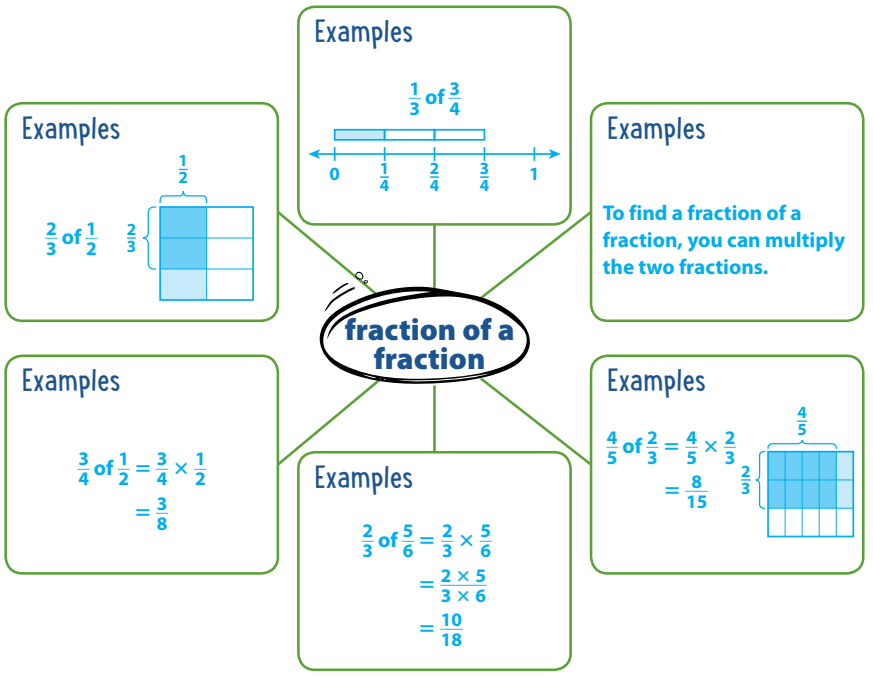
Supplemental Math Vocabulary

- *equal*
- *part*

Name: _____ LESSON 22 SESSION 1

Prepare for Multiplying Fractions in Word Problems

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Write a multiplication expression that can be used to find $\frac{1}{5}$ of $\frac{3}{8}$.

$\frac{1}{5} \times \frac{3}{8}$

Why is the product less than $\frac{3}{8}$?

Possible answer: The product is less than $\frac{3}{8}$ because it is $\frac{1}{5}$ of $\frac{3}{8}$ and $\frac{1}{5}$ is less than 1. A fractional part less than 1 is a smaller part. Multiplying a quantity by a fraction less than 1 reduces the size of the quantity.

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3 Assign problem 3 to provide another look at multiplying fractions in word problems.

This problem is very similar to the problem about Grayson walking to the park. In both problems, students will use a visual model to show finding a fraction of a fraction to solve a word problem. Students may also represent the situation with a product of fractions. The question asks how far Lola has walked.

Students may want to use fraction bars or number lines or draw models with pencil and paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

$\frac{2}{4}$, or $\frac{1}{2}$, mile; Check that visual fraction models show that $\frac{3}{4}$ of $\frac{2}{3}$ is $\frac{2}{4}$, or $\frac{1}{2}$. Students may also write the multiplication equation $\frac{3}{4} \times \frac{2}{3} = \frac{2}{4}$, or $\frac{1}{2}$.

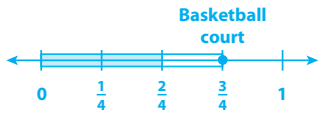
Medium

4 Have students solve the problem a different way to check their answer.

3 Solve the problem. Show your work.

Lola lives $\frac{3}{4}$ mile from the basketball court. She has already walked $\frac{2}{3}$ of the way to the basketball court. How far has Lola walked? Use a visual fraction model to show your thinking.

Possible student work using a picture:



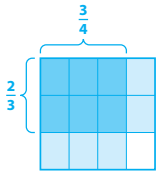
The basketball court is $\frac{3}{4}$ mile from Lola's house, so the distance between her house and the basketball court is 3 equal parts, each of which is $\frac{1}{4}$ mile. 2 of those parts is $\frac{2}{4}$ of one mile.

Solution **Lola has walked $\frac{2}{4}$ mile.**

4 Check your answer. Show your work.

Possible student work:

Find $\frac{2}{3}$ of $\frac{3}{4}$ mile. "$\frac{2}{3}$ of" means $\frac{2}{3}$ times, so I need to find $\frac{2}{3} \times \frac{3}{4}$ mile.



$\frac{2}{3}$ of $\frac{3}{4}$ mile is $\frac{6}{12}$ mile.

$\frac{6}{12} = \frac{2}{4}$

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 8 to students. Explain that the phrase *calls for* is used to indicate what is needed for a recipe. Write these sentence frames and have students complete and read them aloud.

- *Stan needs to make _____.*
- *The recipe needs _____ pound of eggplant.*
- *Stan only needs to make _____ of the recipe.*

Before students solve the problem, ask them if Stan will need more or less eggplant than the recipe calls for:

Stan will need __less__ *eggplant.*

Levels 2–4

Listening/Speaking Have students read ***Apply It*** problem 8 with a partner. Ask them to underline the phrase *calls for*. Explain that the phrase indicates what is needed for a recipe. Restate the problem if needed. Then ask: *Will Stan need more or less eggplant than the recipe calls for? How do you know?* Have students respond using the sentence frame:

Stan will need __less__ *eggplant because _____.* [Possible answer: he wants to make a fraction of the recipe]

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 8 with a partner. Ask them to underline the phrase *calls for*. Have volunteers explain what it means. Then have them restate the problem.

Remind students that something is reasonable when it makes sense. Have students discuss the type of answer they expect to get. Then have them solve the problem with a partner and talk about the answer. Ask: *Is your answer reasonable? Why?* Provide sentence starters:

I think the answer is reasonable because _____.

I don't think the answer is reasonable because _____.

LESSON 22

SESSION 2 Develop

Purpose In this session students solve a word problem that requires finding $\frac{2}{3}$ of $\frac{3}{4}$. Students model the fractions in the word problem either on paper or with manipulatives to develop strategies for solving word problems that involve multiplying fractions.

Start

Connect to Prior Knowledge

Why Support students' facility with writing equivalent fractions, in preparation for recognizing equivalent ways to write the product of two fractions.

How Have students identify a common factor of the numerator and denominator of a fraction and then use division to write an equivalent fraction.

Write an equivalent fraction by dividing the numerator and denominator by the same factor.

$\frac{3}{15} =$ ..... $\frac{8}{20} =$ ..... $\frac{6}{16} =$ .....

Solutions

$\frac{3}{15} = \frac{1}{5}$

$\frac{8}{20} = \frac{4}{10}$ or $\frac{2}{5}$

$\frac{6}{16} = \frac{3}{8}$

Develop Language

Why Clarify the meaning of the term *leftover*.

How Ask students: *Did Brandon's mother leave the whole pizza on the counter?* [No, she left $\frac{3}{4}$ of the pizza.] Explain: *Brandon then eats $\frac{2}{3}$ of the leftover pizza. The leftover part is the part that is still on the counter, the part that was left after Brandon's mother ate some.*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them describe what the problem is asking them to find.

Ask *How much of a whole pizza is left on the counter? Does Brandon eat $\frac{2}{3}$ of a whole pizza or $\frac{2}{3}$ of the pizza left on the counter?*

LESSON 22 SESSION 2 ● ● ○ ○

Develop Multiplying Fractions in Word Problems

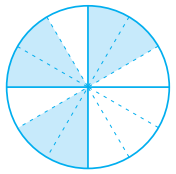
Read and try to solve the problem below.

Brandon's mother left $\frac{3}{4}$ of a pizza on the counter. If Brandon eats $\frac{2}{3}$ of the leftover pizza, how much of the whole pizza did Brandon eat?

TRY IT

Possible student work:

Sample A



He eats $\frac{6}{12}$ of the whole pizza.

Sample B

I need to find $\frac{2}{3}$ of $\frac{3}{4}$ pizza, so I find $\frac{2}{3} \times \frac{3}{4}$.

$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$

Brandon eats $\frac{6}{12}$, or $\frac{1}{2}$, of a pizza.

Math Toolkit

- fraction tiles or circles
- fraction bars
- fraction models
- grid paper
- number lines
- index cards
- multiplication models

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I am not sure how to find the answer because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did.

Support as needed with questions such as:

- *Did the first strategy you tried work?*
- *Would you use the same strategy next time for this type of problem?*

Common Misconception Look for students who find the fraction of the whole pizza that remains after Brandon finishes eating $\left(\frac{1}{4}\right)$, rather than the fraction of the whole pizza that Brandon eats. As students present solutions, be sure to have them identify which parts show what Brandon ate.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing fourths and thirds
- drawings to represent the problem
- number lines marked in fourths and then thirds
- equations to find the amount Brandon eats is $\frac{6}{12}$, or $\frac{1}{2}$, of a pizza

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they relate.

Ask *Where does your model show how much pizza Brandon's mother left on the counter? The fraction of that amount Brandon eats? How much of a whole pizza he eats?*

Listen for Students should recognize that accurate representations include showing fourths and thirds. Responses may include $\frac{3}{4}$ of a pizza as the amount Brandon's mother left on the counter; $\frac{2}{3}$ as the fraction of that amount Brandon eats; and $\frac{6}{12}$, or $\frac{1}{2}$, of a whole pizza as the amount he eats.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the fraction of a whole pizza there was to start
- the fraction of the leftover pizza that Brandon ate
- the fraction of the whole pizza Brandon ate

Ask *Do the picture and equation show the same number of equal parts for a whole pizza? Explain.*

Listen for Yes, both the picture and equation show 4 equal parts in a whole pizza. The picture shows 3 pieces of pizza plus 1 missing piece. The equation shows $\frac{3}{4}$, three out of four equal parts, to describe the pizza left on the counter.

For a picture of the pizza, prompt students to identify the fractions represented by the pictures.

- *What fraction is represented in the top pizza picture?*
- *What fractions are represented in the bottom pizza picture?*

For an equation, prompt students to relate the product shown by the equation to a model.

- *How would you show $\frac{2 \times 3}{3 \times 4}$ on a visual model like the ones in* Picture It*?* [Divide a model into fourths and then divide each fourth into thirds and then shade 6 of the equal parts.]

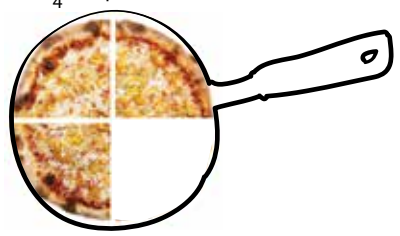
Explore different ways to understand strategies for solving word problems that involve finding a fraction of a fraction.

Brandon's mother left $\frac{3}{4}$ of a pizza on the counter. If Brandon eats $\frac{2}{3}$ of the leftover pizza, how much of the whole pizza did Brandon eat?

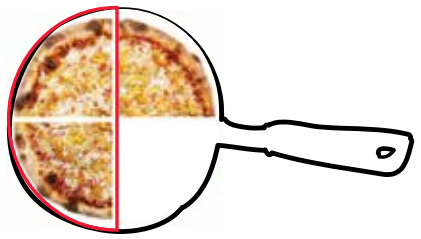
PICTURE IT

You can draw a picture to help you understand the problem.

Show $\frac{3}{4}$ of a pizza.



Since Brandon eats $\frac{2}{3}$ of what is left, outline 2 of the 3 pieces that are left. You can see from the outlined parts how much of the whole pizza Brandon ate.



MODEL IT

You can write an equation to help you understand the problem.

You need to find a fraction of a fraction: $\frac{2}{3}$ of $\frac{3}{4}$ of a pizza.

$\frac{2}{3}$ of $\frac{3}{4}$ means $\frac{2}{3} \times \frac{3}{4}$.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4}$$

Deepen Understanding

Equation Model

SMP 4 Model with mathematics.

When discussing the equation model, prompt students to consider how when people talk about modeling a problem they tend to think of methods such as acting it out or drawing a picture. Point out that writing an equation is also a way to model a problem.

Ask *What are some ways of modeling problems other than acting it out, drawing a picture, or writing an equation?*

Listen for Students should be able to describe a variety of other methods: for example, making a chart, list, or graph or using objects.

Ask *Think about the models shown on this page. How is writing an equation similar to drawing a picture?*

Listen for Both models represent the important quantities in a problem and show how they are related.

CONNECT IT

- Remind students that one thing that is alike about all these representations is the numbers.
- Explain that on this page they will compare the answers they got using the two strategies of drawing a picture and writing an equation.

Monitor and Confirm

1 – 3 Check for understanding that:

- $\frac{2}{3}$ of $\frac{3}{4}$ of the pizza is 2 of the 3 parts.
- Brandon ate $\frac{2}{4}$, or $\frac{1}{2}$, of the whole pizza.
- $\frac{2}{3}$ of $\frac{3}{4}$ means $\frac{2}{3} \times \frac{3}{4}$.

Support Whole Class Discussion

4 When discussing problem 4, review with students how they can write equivalent fractions by dividing the numerator and denominator by the same factor.

Ask *How do you use division to show that* $\frac{6}{12} = \frac{2}{4}$ *or that* $\frac{6}{12} = \frac{1}{2}$*?*

Listen for You can divide both the numerator and denominator of $\frac{6}{12}$ by 3 to show that $\frac{6}{12} = \frac{2}{4}$. You can divide both the numerator and denominator of $\frac{6}{12}$ by 6 to show that $\frac{6}{12} = \frac{1}{2}$.

5 Look for descriptions of at least two strategies, such as drawing a picture and writing an equation, to solve word problems involving finding a fraction of a fraction.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand strategies for solving word problems that involve finding a fraction of a fraction.

1 Look at **Picture It**. Why do you outline 2 of the 3 parts of the pizza?

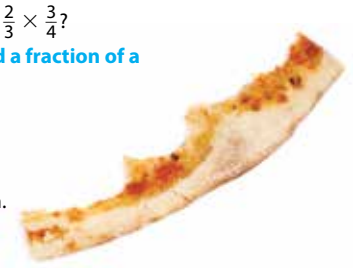
Possible answer: The problem says that Brandon eats $\frac{2}{3}$ of what was left.

2 How much of the whole pizza did Brandon eat? Explain your reasoning.

$\frac{2}{4}$, or $\frac{1}{2}$; Possible explanation: The whole pizza is divided into 4 equal parts. Brandon ate 2 of the 4 parts of the whole pizza.

3 Look at **Model It**. How do you know that you should multiply $\frac{2}{3} \times \frac{3}{4}$?

Possible answer: You need to find $\frac{2}{3}$ of $\frac{3}{4}$ of a pizza. To find a fraction of a number, you multiply the number by that fraction.



4 What is $\frac{2 \times 3}{3 \times 4}$? **$\frac{6}{12}$**

Is this product the same as your answer to problem 2? Explain.

Yes; Possible explanation: $\frac{6}{12}$ is equivalent to $\frac{2}{4}$, or $\frac{1}{2}$.

5 What strategies can you use to solve a word problem that involves finding a fraction of a fraction?

Possible answer: You can draw a picture to represent the problem and use reasoning to find a solution. You also can write an equation that represents the problem as a product of fractions and then multiply the numerators and multiply the denominators.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for solving word problems that involve finding a fraction of a fraction? Explain.

Possible answer: I like multiplying the numerators and the denominators of the fractions because it does not require trying to draw equal-size pieces. It is a quick way to find a fraction of another fraction.

Hands-On Activity

Act out the problem situation.

If . . . *students are unsure about using visual model strategies,*

Then . . . *use this activity for a hands-on approach.*

Materials For each pair: 1 set of fraction circles

- Have students use fraction pieces to represent $\frac{3}{4}$ of a pizza. [3 one-fourth pieces] Tell students they can think of this as *the amount that is left*.
- Remind students Brandon ate $\frac{2}{3}$ of the amount that is left. Guide them in seeing there are 3 equal-sized pieces left, so each piece is $\frac{1}{3}$ of *the amount that is left*. Have students remove $\frac{2}{3}$ of the amount that is left. [2 pieces]
- Have students place the 2 pieces they removed over a whole fraction circle. Ask: *How much of a whole pizza did Brandon eat?* $\left[\frac{1}{2}\right]$ Be sure students understand the answer is a fraction of a pizza, not a number of pieces of pizza.
- Use one-fifth pieces to repeat the activity for this problem situation: *A recipe for 2 loaves of bread uses $\frac{4}{5}$ pound of flour. Seth needs to use $\frac{1}{2}$ of that amount to make 1 loaf. How much flour does Seth need?* $\left[\frac{2}{5} \text{ pound}\right]$

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing equal parts in models can be difficult.

7 $\frac{24}{40}$, $\frac{6}{10}$, or $\frac{3}{5}$ mile; See possible work on the Student Worktext page. Students may also draw a visual model showing tenths and each tenth divided into fourths. They may also write the equation $\frac{3}{4} \times \frac{8}{10} = \frac{24}{40}$, or $\frac{3}{5}$.

8 $\frac{18}{48}$, or $\frac{3}{8}$, pound; See possible work on the Student Worktext page. Students may also draw a visual model showing sixteenths, and each sixteenth divided into thirds.

Close: Exit Ticket

9 $\frac{20}{30}$, $\frac{4}{6}$, or $\frac{2}{3}$ hour; See possible work on the Student Worktext page. Students may also draw a picture of an analog clock. They may also write the equation $\frac{4}{5} \times \frac{5}{6} = \frac{20}{30}$, or $\frac{2}{3}$.

Students' solutions should indicate understanding of:

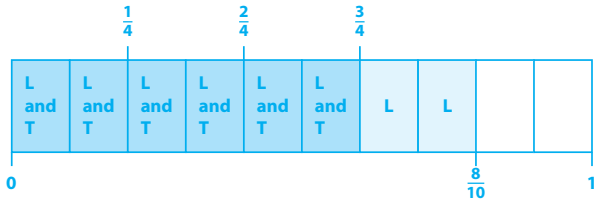
- Accurate use of visual fraction models and/or equations to represent and solve a word problem that involves finding a fraction of a fraction.

Error Alert **If** students' solution is a fraction greater than 1, **then** remind them what they know about multiplying a given factor by a factor less than 1—that the product will be less than the given factor. Ask them what they know about the solution to this problem. [It is less than $\frac{5}{6}$ hour.]

APPLY IT

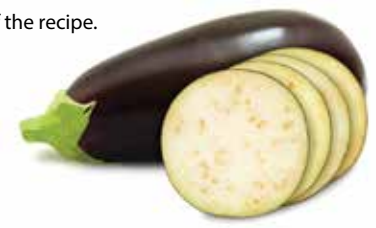
Use what you just learned to solve these problems.

7 Lewis walked $\frac{8}{10}$ of a mile. Todd walked $\frac{3}{4}$ of the way with Lewis. How many miles did Todd walk with Lewis? Show your work. **Possible student work:**



Solution $\frac{24}{40}$, $\frac{6}{10}$, or $\frac{3}{5}$ mile

8 Stan has a recipe for vegetable lasagna that calls for $\frac{9}{16}$ pound of eggplant. He wants to make a batch of lasagna that is $\frac{2}{3}$ of the amount of the recipe. How much eggplant will Stan need? Show your work.



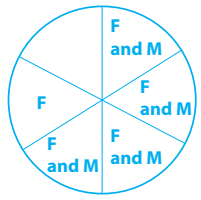
Possible student work:

$\frac{2}{3} \times \frac{9}{16} = \frac{2 \times 9}{3 \times 16} = \frac{18}{48}$

$\frac{18}{48} = \frac{18 \div 6}{48 \div 6} = \frac{3}{8}$

Solution $\frac{18}{48}$ or $\frac{3}{8}$ pound

9 Jamie worked $\frac{5}{6}$ hour filing papers for her mother. She listened to music for $\frac{4}{5}$ of the time she spent filing. How much time did Jamie spend listening to music? Show your work. **Possible student work:**



Solution $\frac{20}{30}$, $\frac{4}{6}$, or $\frac{2}{3}$ hour

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Solutions

1 $\frac{3}{5} \times \frac{5}{8} = \frac{3 \times 5}{5 \times 8} = \frac{15}{40}$
Basic

2 Yes; See possible explanation on the student page. Students should recognize $\frac{3}{8}$ and $\frac{15}{40}$ name the same amount.
Medium

Name: _____ LESSON 22 SESSION 2

Practice Multiplying Fractions in Word Problems

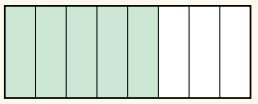
Study the Example showing one way to solve a word problem with fractions. Then solve problems 1–5.

EXAMPLE

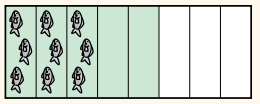
Vicky's favorite beach towel is green and white and has a fish design. The green part covers $\frac{5}{8}$ of the towel. A fish design is drawn on $\frac{3}{5}$ of that part. What part of the towel has a fish design?

You can draw a picture.

Show a towel with $\frac{5}{8}$ shaded green.



Draw fish on $\frac{3}{5}$ of the green part.



Because 3 of the 8 parts of the towel have fish drawn on them, $\frac{3}{8}$ of the towel has a fish design.

1 You can also write an equation to solve the Example. Write the numbers to complete the equation showing what part of the towel has the fish design.

$\frac{3}{5}$ of $\frac{5}{8}$ means $\frac{3}{5} \times \frac{5}{8}$.

$\frac{3}{5} \times \frac{\boxed{5}}{\boxed{8}} = \frac{\boxed{3}}{\boxed{5}} \times \frac{5}{8} = \frac{\boxed{15}}{\boxed{40}}$

2 Is your answer to problem 1 the same as the answer of $\frac{3}{8}$ shown in the Example? Explain.

Yes; Possible explanation: $\frac{15}{40}$ is equivalent to $\frac{3}{8}$ because $\frac{3}{8} \times \frac{5}{5} = \frac{15}{40}$.

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Fluency & Skills Practice Teacher Toolbox

Assign *Multiplying Fractions in Word Problems*

In this activity students solve word problems that involve multiplying two fractions in which both factors are less than 1. Students may encounter this situation in their daily lives, as when determining distances traveled, pounds of meat bought, yards of fabric used, and so on. Students may use various strategies to solve these problems, including drawing pictures, using number lines, or producing area models.

Fluency and Skills Practice

Multiplying Fractions in Word Problems Name: _____

Solve each problem.

1 Blanca has $\frac{3}{4}$ pound of tuna salad. She uses $\frac{1}{3}$ of the tuna salad to make sandwiches. How much of the tuna salad did Blanca use?

2 Frank has a board that is $\frac{5}{9}$ yard long. He cuts off a piece that is $\frac{2}{5}$ the length of the board. How long is the piece of the board Frank cut off?

3 Sharon drinks $\frac{2}{3}$ of $\frac{1}{2}$ pint of lemonade. How much lemonade did Sharon drink?

4 James lives $\frac{4}{5}$ mile from the library. He has already walked $\frac{3}{4}$ of the way. How far has James walked?

5 Ali worked on his math homework for $\frac{2}{3}$ hour. He spent $\frac{3}{4}$ of the time solving multiplication problems. How much time did Ali spend solving multiplication problems?

6 Madison has $\frac{5}{6}$ yard of fabric. She uses $\frac{2}{3}$ of the fabric to make a pillow cover. How much fabric did Madison use for the pillow cover?

7 How could you draw a picture to solve problem 2?

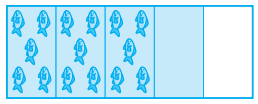
3 $\frac{3}{5}$ of the towel has the fish design; See possible picture on the student page.
Medium

4 $\frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{12}{20}$, or $\frac{3}{5}$
Medium

5 Student problems should refer to taking $\frac{1}{6}$ of the quantity $\frac{3}{8}$ or scaling a measurement of $\frac{3}{8}$ by $\frac{1}{6}$, $\frac{3}{48}$, or $\frac{1}{16}$, unit; See possible work on the student page. Students may also draw a visual fraction model divided into eighths and with the eighths divided into sixths.
Challenge

3 Suppose that the green part of Vicky's towel covers $\frac{4}{5}$ of the towel and the fish design is drawn on $\frac{3}{4}$ of that part. Draw a picture to find the part of the towel that has the fish design. Then write the answer.

Possible student work:



Solution **$\frac{3}{5}$ of the towel has the fish design.**

4 Write an equation to show the answer to problem 3.

Solution **$\frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{12}{20}$, or $\frac{3}{5}$**

5 Write a word problem that can be solved by finding the product $\frac{1}{6} \times \frac{3}{8}$. Then solve your problem.

Problem **Student problems should refer to taking $\frac{1}{6}$ of the quantity $\frac{3}{8}$ or scaling a measurement of $\frac{3}{8}$ by $\frac{1}{6}$.**

Show your work.

Possible student work:

$\frac{1}{6} \times \frac{3}{8} = \frac{1 \times 3}{6 \times 8} = \frac{3}{48}$

$\frac{3}{48} = \frac{3 \div 3}{48 \div 3} = \frac{1}{16}$

Solution **$\frac{3}{48}$ or $\frac{1}{16}$ unit**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Connect It*.

Levels 1–3

Speaking/Writing Read ***Connect It*** problem 5 to students. Ask them to work in small groups. Help groups make a multiple-column chart to compare and contrast the strategies that were used to solve the problem. Have groups use the information in the chart to talk about the strategy they liked best. Provide a sentence starter:

- *The strategy that I like best is . . .*

Ask simple questions about the strategies students select.

Levels 2–4

Reading/Writing Have students read ***Connect It*** problem 5 with a partner. Have students work together to make a multiple-column chart to compare and contrast the strategies that were used to solve the problem.

Have students use the information in the chart to talk about the strategy they liked best. Provide sentence starters:

- *The strategy that I like best is . . .*
- *I like this strategy because . . .*

Levels 3–5

Reading/Writing Have students read ***Connect It*** problem 5. Ask students to talk about the strategies with a partner and write their reflections.

Then have partners exchange their writing and ask and answer questions. Provide examples or sentence frames as needed:

Why do you prefer _____?

Why do you think _____ is helpful?

LESSON 22

SESSION 3 Develop

Purpose In this session students solve a word problem that requires finding $\frac{1}{2}$ of $2\frac{3}{4}$. Students model the fraction and mixed number in the word problem either on paper or with manipulatives to develop strategies for solving word problems that involve multiplying fractions and mixed numbers.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *Fraction Bars* (3 wholes, 3 bars for fifths)

Why Support students' facility with writing mixed numbers as fractions.

How Have students use fraction bars to model $2\frac{4}{5}$. Then have them use the fraction bars to show how many fifths in 2 wholes and to find how many fifths in all.

Use fraction bars to show $2\frac{4}{5}$. Then show how to write the mixed number as a fraction.

$2\frac{4}{5} = \frac{\square}{5} + \frac{4}{5} = \frac{\square}{5}$

Solution

$2\frac{4}{5} = \frac{10}{5} + \frac{4}{5} = \frac{14}{5}$

Look for models that reflect understanding that $2\frac{4}{5} = 2 + \frac{4}{5}$.

Develop Language

Why Review the meaning of the term *model* in mathematical contexts.

How Tell students that *model* is a multiple-meaning word. Also explain that *model* can be a noun (thing) and a verb (action). Say: *In math, we can use numbers, expressions, pictures, or graphs to represent different situations. A model is a representation, a way of helping us see or understand something.*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them describe how a yard is different from a square yard.

Ask *What is the length of Janie's garden? the width?*

Develop Multiplying with Mixed Numbers in Word Problems

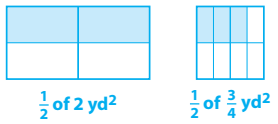
Read and try to solve the problem below.

Janie has a rectangular garden that is $2\frac{3}{4}$ yards in length and 1 yard in width. She grows roses in $\frac{1}{2}$ of her garden. How many square yards in Janie's garden has roses?

TRY IT

Possible student work:

Sample A



$\frac{1}{2}$ of 2 yd² $\frac{1}{2}$ of $\frac{3}{4}$ yd²

$\frac{1}{2}$ of 2 = 1; $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

$1\frac{3}{8}$ square yards of Janie's garden have roses.

Sample B

Find half of 2 and half of $\frac{3}{4}$.

$\frac{1}{2}$ of 2 is 1.

$\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

$1 + \frac{3}{8} = 1\frac{3}{8}$

$1\frac{3}{8}$ square yards of Janie's garden have roses.

Math Toolkit

- fraction tiles or circles
- fraction bars
- fraction models
- grid paper
- number lines
- index cards
- multiplication models

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: At first, I thought . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *square yards* as they discuss their solutions.

Support as needed with questions such as:

- *How would you describe your model?*
- *How did you know when you had found the solution?*

Common Misconception Look for students who add $2\frac{3}{4}$ yards and 1 yard and then find $\frac{1}{2}$ of $3\frac{3}{4}$ instead of $\frac{1}{2}$ of $2\frac{3}{4}$. As students present their solutions, be sure to have them specify how they determined the area of Janie's whole garden.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing wholes, fourths, halves, and eighths
- drawings, including area models, to represent the problem
- equations that show $2\frac{3}{4}$ rewritten as a fraction
- equations that use the distributive property to find $\frac{1}{2} \times \left(2 + \frac{3}{4}\right)$

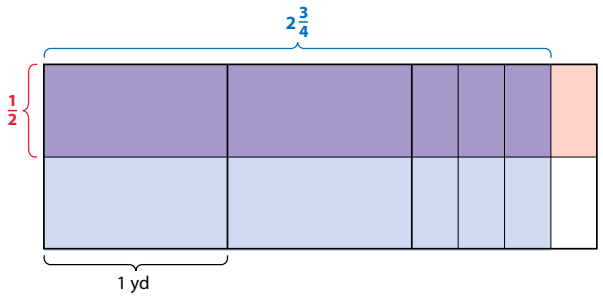
Explore different ways to understand multiplying fractions and mixed numbers.

Janie has a rectangular garden that is $2\frac{3}{4}$ yards in length and 1 yard in width. She grows roses in $\frac{1}{2}$ of her garden. How many square yards in Janie's garden has roses?

PICTURE IT

You can use an area model to help you understand the problem.

The purple shaded region of the area model shows **half** of $2\frac{3}{4}$.



MODEL IT

You can write equations to model the problem.

You can write $2\frac{3}{4}$ as a fraction.

$$2\frac{3}{4} = 2 + \frac{3}{4}$$
$$= \frac{8}{4} + \frac{3}{4}$$
$$= \frac{11}{4}$$

You need to find a fraction of a fraction: $\frac{1}{2}$ of $\frac{11}{4}$ square yards.

$\frac{1}{2}$ of $\frac{11}{4}$ means $\frac{1}{2} \times \frac{11}{4}$.

$$\frac{1}{2} \times \frac{11}{4} = \frac{1 \times 11}{2 \times 4}$$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the area of the whole garden? The fraction of the garden that has roses?*

Listen for Students should recognize that accurate responses include $2\frac{3}{4}$ square yards as the area of the garden and $\frac{1}{2}$ of that area for growing roses. Models may show 3 wholes with $2\frac{3}{4}$ of the model representing the garden and half of that area representing the part that has roses.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the area of the rectangular garden
- the part of the garden that has roses

Ask *Why does the equation model include the fraction $\frac{11}{4}$ but this fraction does not appear in the area model?*

Listen for The picture shows the area of the garden is $2\frac{3}{4}$ square yards. The equation shows the area in square yards as $\frac{11}{4}$. They are the same amount written two different ways.

For the area model, prompt students to identify how the model shows $2\frac{3}{4}$ square yards.

- *How is it helpful that the model shows 3 square yards in all?*
- *How is each square yard used in representing $2\frac{3}{4}$ square yards?*

For the equations, prompt students to describe the steps shown by the equations.

- *Before $\frac{1}{2}$ of $2\frac{3}{4}$ is written as a multiplication expression, what is done first?*
- *Why is that done first?*

Deepen Understanding

Equation Model

SMP 2 Reason abstractly and quantitatively.

When discussing the equation model, prompt students to consider how the commutative property applies.

Review that the problem asks you to find $\frac{1}{2}$ of $\frac{11}{4}$ yd^2 and that you can replace *of* with the multiplication symbol to write an expression that models the problem. Have a volunteer come to the board to write the multiplication expression, $\frac{1}{2} \times \frac{11}{4}$, and the expression with the factors reversed, $\frac{11}{4} \times \frac{1}{2}$.

Ask *Why can you use either expression to solve the problem? Is one expression a "better" model that the other for this problem? Explain.*

Listen for Since the order of factors can be changed without changing the product, either expression can be used to solve the problem. Students may say that although either expression can be used, the expression that is a more direct translation of the phrase $\frac{1}{2}$ *of* $\frac{11}{4}$ is the expression $\frac{1}{2} \times \frac{11}{4}$ and is the more accurate model of the problem.

CONNECT IT

- Remind students that one thing that is alike about all the representations is that they show how to find $\frac{1}{2}$ of the mixed number $2\frac{3}{4}$.
- Explain that on this page they will analyze strategies for multiplying a mixed number by a fraction.

Monitor and Confirm

1 Check for understanding that:

- In the area model, the area of the garden that has roses is represented by the shaded parts for one half of the first square yard, one half of the second square yard, and $\frac{3}{8}$ of the third square yard.
- $1\frac{3}{8}$ square yards of the garden has roses.

Support Whole Class Discussion

2 – 3 Tell students that these problems will prepare them to provide the explanation required in problem 4.

Make sure students understand that problem 2 focuses on one strategy for multiplying a mixed number by a fraction and problem 3 focuses on a different strategy. Both strategies result in the answer that is shown by the area model.

Ask *How are the strategies alike? How are they different?*

Listen for Both strategies break apart $2\frac{3}{4}$ as $2 + \frac{3}{4}$. The strategy in problem 2 (used in *Model It*) does this in order to write the mixed number as a fraction greater than 1. The strategy in problem 3 breaks apart $2\frac{3}{4}$ in order to use the distributive property. Both strategies involve multiplying fractions: the strategy in *Model It* has only one multiplication, while the other strategy has two multiplications.

4 Look for the idea that to multiply a fraction and a mixed number you first rewrite the mixed number either as a fraction or as the sum of a whole number and a fraction. If you do the latter, you multiply each part by the fraction in the original expression and add the products.

5 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to understand how to multiply fractions and mixed numbers.

1 Use the last equation in **Model It** to find the area of Janie's garden that has roses.

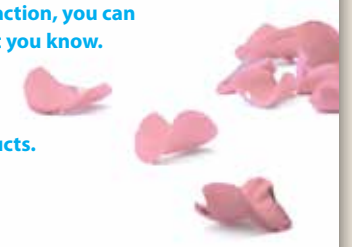
Janie's garden has $\frac{11}{8}$ or $1\frac{3}{8}$ square yards of roses.

Explain how you can use the area model in **Picture It** to find the area of Janie's garden that has roses.

Possible answer: The area model shows the area of Janie's garden with roses in parts: $\frac{1}{2} + \frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$, or $1\frac{3}{8}$ square yards.

2 Look at the first equation in **Model It**. Why is the mixed number rewritten as a fraction?

Possible answer: Since you know how to find a fraction of a fraction, you can rewrite the mixed number as a fraction and use a method that you know.



3 How can you multiply $\frac{1}{2} \times 2\frac{3}{4}$ without changing $2\frac{3}{4}$ to a fraction?

Possible answer: Find $\frac{1}{2}$ of 2 and $\frac{1}{2}$ of $\frac{3}{4}$ and then add the products.

What is $\frac{1}{2} \times 2$? 1 What is $\frac{1}{2} \times \frac{3}{4}$? $\frac{3}{8}$

Add the two products. 1 + $\frac{3}{8}$ = $1\frac{3}{8}$

Is this result the same as your answer to problem 1? Yes

4 How can you multiply a mixed number by a fraction?

Possible answer: You can write the mixed number as a fraction and then multiply or you can break apart the mixed number, multiply each part by the fraction, and then add the products.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for multiplying fractions and mixed numbers? Explain.

Possible answer: I like to use a visual model because I can see a fraction of a mixed number. I can think about how to move and combine the parts of my model to write the final answer as either a mixed number or a fraction.

Visual Model

Use an area model to rewrite a mixed number as a fraction.

If . . . *students are unsure about what to write as the numerator when rewriting a mixed number as a fraction before multiplying,*

Then . . . *use this activity to connect an area model and the strategy of rewriting a mixed number as a fraction to multiply.*

Materials For each student: drawing paper (1 sheet)

- Explain that you can use the area model shown in *Picture It* to show the strategy of rewriting a mixed number as a fraction before multiplying.
- Tell students to draw the 3 squares that represent the 3 square yards in the area model. Have them shade the model to show $2\frac{3}{4}$, dividing the last square into fourths by drawing vertical lines to do this.
- Then tell students to draw vertical lines to show the number of fourths in each whole, count the number of fourths in $2\frac{3}{4}$, and write it as a fraction. $\left[\frac{11}{4}\right]$
- Guide students in drawing a horizontal line to divide the model in half to represent the expression $\frac{1}{2} \times \frac{11}{4}$. Have students identify what is shown as the answer. $\left[\frac{11}{8}, \text{ or } 1\frac{3}{8}, \text{ square yards}\right]$

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing equal parts in models can be difficult.

6 $\frac{21}{10}$, or $2\frac{1}{10}$, yards; See possible work on the Student Worktext page. Students may also use either the expression $\frac{3}{5} \times \frac{7}{2}$ or $\frac{3}{5} \times \left(3 + \frac{1}{2}\right)$ to write equations to solve the problem.

7 $\frac{45}{12}$ feet, $3\frac{9}{12}$ feet, or $3\frac{3}{4}$ feet; See possible student work on the Student Worktext page. Students may also use the expression $\frac{5}{6} \times \left(4 + \frac{1}{2}\right)$ to write equations to solve the problem. They may also draw a visual fraction model divided into thirds, with the thirds divided into sixths.

Close: Exit Ticket

8 **A**; Students may use either the expression $\frac{3}{4} \times \frac{14}{9}$ or $\frac{3}{4} \times \left(1 + \frac{5}{9}\right)$ to write equations to solve the problem. They may also draw a visual fraction model divided into ninths, with the ninths divided into fourths.

Error Alert **If** students choose B, C, or D, **then** review the two strategies for multiplying a mixed number by a fraction using equations, recording steps for each. Have students choose one of the strategies and follow the steps, checking off each step as they complete it.

APPLY IT

Use what you just learned to solve these problems.

6 Izzy has $3\frac{1}{2}$ yards of rope. She uses $\frac{3}{5}$ of the rope to attach a tire swing to a tree in her yard. How many yards of rope does Izzy use for the tire swing? Show your work.

Possible student work:

$\frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{9}{5}$

$\frac{9}{5} = \frac{18}{10}$

$\frac{18}{10} + \frac{3}{10} = \frac{21}{10}$

$\frac{3}{10}$

Solution $\frac{21}{10}$ or $2\frac{1}{10}$ yards

7 Colin has a chain that is $\frac{5}{6}$ foot long. He adds links to his chain so that it is $4\frac{1}{2}$ times as long as the original chain. How many feet long is his chain now? Show your work.

Possible student work:

$4\frac{1}{2} = 4 + \frac{1}{2} = \frac{8}{2} + \frac{1}{2} = \frac{9}{2}$

$\frac{5}{6} \times \frac{9}{2} = \frac{5 \times 9}{6 \times 2} = \frac{45}{12}$

$\frac{45}{12} = \frac{3 \times 15}{3 \times 4} = \frac{15}{4} = 3\frac{3}{4}$

Solution $\frac{45}{12}$, $3\frac{9}{12}$ or $3\frac{3}{4}$ feet

8 George has $1\frac{5}{9}$ yards of fabric. He plans to use $\frac{3}{4}$ of the fabric to make a pillow. How many yards of fabric will George use for the pillow?

Ⓐ $1\frac{6}{36}$ Ⓑ $1\frac{8}{13}$

Ⓒ $1\frac{17}{36}$ Ⓓ $1\frac{5}{12}$

450

Solutions

1 $3\frac{1}{4} \times \frac{2}{3} = \left(3 \times \frac{2}{3}\right) + \left(\frac{1}{4} \times \frac{2}{3}\right)$

$= \frac{6}{3} + \frac{2}{12}$

$= 2\frac{2}{12}$

$2\frac{2}{12}$ square yards.

Basic

2 $\frac{26}{12}$, or $2\frac{2}{12}$, square yards; See possible work on the student page.

Medium

Name: _____ LESSON 22 SESSION 3

Practice Multiplying with Mixed Numbers

Study the Example showing one way to solve a word problem with a mixed number. Then solve problems 1–5.

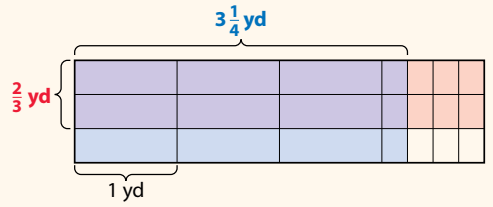
EXAMPLE

Mr. Urrego is painting his deck for the summer. He has painted a rectangular area that is $3\frac{1}{4}$ yards long and $\frac{2}{3}$ yard wide. How many square yards of deck are painted?

You can use an area model.

The larger sections of the area model are $\frac{1}{3} \times 1 = \frac{1}{3}$ square yard.

The smaller sections of the area model are $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ square yard.



The model shows the number of square yards painted is:

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{12} = \frac{6}{3} + \frac{2}{12} = 2 + \frac{2}{12} = 2\frac{2}{12}$

1 Write the missing numbers to complete the multiplication equation showing how much of the deck is painted.

Multiply the length and width of the painted area:

$3\frac{1}{4} \times \frac{\boxed{2}}{\boxed{3}} = \left(\boxed{3} \times \frac{2}{3}\right) + \left(\frac{\boxed{1}}{\boxed{4}} \times \frac{2}{3}\right) = \frac{\boxed{6}}{3} + \frac{2}{\boxed{12}} = \boxed{2}\frac{2}{12}$

$2\frac{2}{12}$ square yards

2 To multiply by a mixed number, you can also write the mixed number as a fraction and then multiply. Use this method to find the product $3\frac{1}{4} \times \frac{2}{3}$ in order to find how many square yards of the deck are painted. Show your work.

Possible student work: $3\frac{1}{4} = \frac{13}{4}; \frac{13}{4} \times \frac{2}{3} = \frac{13 \times 2}{4 \times 3} = \frac{26}{12}$

Solution $\frac{26}{12}$ **or** $2\frac{2}{12}$ **square yards**

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Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying with Mixed Numbers in Word Problems*

In this activity students solve word problems that involve multiplying a mixed number by either a fraction or another mixed number. Students may use this skill in their daily lives, such as when multiplying mixed numbers involved in recipes, distances, areas, and weights. Students may use different strategies, such as drawing area models or making number lines, to solve these problems.

Fluency and Skills Practice

Multiplying with Mixed Numbers in Word Problems

Name: _____

Solve each problem.

1 Neil has $2\frac{1}{4}$ pounds of apples. He uses $\frac{2}{3}$ of the apples to make pies. How many pounds of apples does Neil use to make pies?

2 Kathy is riding her bike $1\frac{3}{8}$ miles to her friend's house. She has already traveled $\frac{7}{8}$ of the distance. How far has Kathy already traveled?

3 Keisha spent $3\frac{1}{3}$ hours at the science museum. She spent $\frac{2}{5}$ of that time in the planetarium. How much time did Keisha spend in the planetarium?

4 Javier is planting a rectangular garden that will be $7\frac{1}{2}$ yards long and 1 yard wide. He will plant $\frac{1}{5}$ of the garden with tomatoes. How many square yards of the garden will be planted with tomatoes?

5 Ed has two dogs. The smaller dog weighs $8\frac{1}{4}$ pounds. The larger dog weighs $1\frac{1}{3}$ times as much as the smaller dog. How much does the larger dog weigh?

6 Shane designed a rectangular mural that is $2\frac{3}{4}$ yards long and $1\frac{1}{3}$ yards high. What is the area in square yards of the mural?

7 How could you use an area model to solve problem 4?

3 $\frac{15}{8}$, or $1\frac{7}{8}$, miles; Students may also use the expression $\frac{3}{4} \times \left(2 + \frac{1}{2}\right)$ to write equations to solve the problem.

Medium

4 See possible visual model on the student page. Students may draw other visual models divided into fourths, with the fourths divided into halves.

Medium

5 Less than 1 square yard; Students may also use either the expression $\frac{5}{4} \times \frac{3}{4}$ or $\frac{3}{4} \times \left(1 + \frac{1}{4}\right)$ to write equations to solve the problem. They may also draw visual models divided into sixteenths.

Challenge

3 On Saturday, Kira ran $\frac{3}{4}$ mile. On Sunday, she ran $2\frac{1}{2}$ times as far as on Saturday. Use a multiplication equation to find how far Kira ran on Sunday. Show your work.

Possible student work:

$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

$\frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$

Solution $\frac{15}{8}$ or $1\frac{7}{8}$ miles

4 Use a visual model to show another way to find the distance Kira ran on Sunday.

Accept visual models that show $\frac{3}{4} \times 2\frac{1}{2}$. Possible model:

5 The multipurpose room at the Cortez School is being set up for the annual book sale. Graphic novels will be displayed in a rectangular area $1\frac{1}{4}$ yards long and $\frac{3}{4}$ yard wide. Will the graphic novels be displayed in an area greater than or less than 1 square yard? Show your work.

Students may use area models, equations, or some other method to find that $1\frac{1}{4} \times \frac{3}{4} = \frac{15}{16}$ square yard.

Solution less than 1 square yard

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Speaking/Writing Read ***Apply It*** problem 8 to students. Model writing your own problem using another ***Apply It*** problem as a guide. Change the names, ideas, or actions. For example, you may choose to revise problem 3 to:

__Ms. Martinez__ has __$3\frac{3}{4}$__ bags of popcorn. Her students ate __$\frac{1}{2}$__ of her popcorn. What fraction of the popcorn bags did Ms. Martinez's students eat?

Have students to brainstorm a list of names and actions that can be used in their word problems.

Ask students to use the bank of names and phrases and work with a partner to write a word problem for the expression provided.

Levels 2–4

Reading/Writing Have students read ***Apply It*** problem 8 with a partner.

Ask students to make a T-chart and label one column "Names" and the other "Topics."

Ask students to complete the T-chart and swap with another set of partners. Have them use the charts to write word problems for the expression provided.

Levels 3–5

Reading/Writing Have students read ***Apply It*** problem 8 with a partner.

Have partners write their word problem. Encourage them to share their work with other partners and have each pair review and comment on each other's work.

LESSON 22

SESSION 4 Refine

Purpose In this session students solve word problems involving multiplication of fractions and mixed numbers using visual fraction models and equations and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of multiplying fractions to solve word problems.

How Have students solve the word problem using any strategy they want. Remind them to show their work.

Don practices piano for $\frac{3}{4}$ hour every day. He spends $\frac{1}{5}$ of that time practicing scales. How long does he practice scales?

Solution
$\frac{3}{20}$ hour

Refine Multiplying Fractions in Word Problems

Complete the Example below. Then solve problems 1–8.

EXAMPLE

Chris uses $4\frac{1}{4}$ tubes of paint. Nico uses $1\frac{1}{2}$ times as much paint as Chris. How much paint did Nico use?

Look at how you can solve this problem using equations.

$$4\tfrac{1}{4} \times 1 = 4\tfrac{1}{4}$$

$$4\tfrac{1}{4} \times \tfrac{1}{2} = \left(4 \times \tfrac{1}{2}\right) + \left(\tfrac{1}{4} \times \tfrac{1}{2}\right) = 2 + \tfrac{1}{8}$$

$$4\tfrac{1}{4} + 2 + \tfrac{1}{8} = 6\tfrac{1}{4} + \tfrac{1}{8} = 6\tfrac{2}{8} + \tfrac{1}{8} = 6\tfrac{3}{8}$$

Solution $6\frac{3}{8}$ tubes of paint

Breaking apart a mixed number happens twice in this problem.

PAIR/SHARE
How does the product compare to $4\frac{1}{4}$?

APPLY IT

1 Josh exercises at the gym $3\frac{3}{4}$ hours a week. He spends $\frac{2}{5}$ of his time at the gym lifting weights. How many hours a week does Josh spend lifting weights at the gym? Show your work.

Possible student work using an equation:

$$\left(3 \times \tfrac{2}{5}\right) + \left(\tfrac{3}{4} \times \tfrac{2}{5}\right) = \tfrac{6}{5} + \tfrac{6}{20}$$

$$= \tfrac{24}{20} + \tfrac{6}{20}$$

$$= \tfrac{30}{20}$$

$$= \tfrac{3}{2}, \text{ or } 1\tfrac{1}{2}$$

Solution $\frac{3}{2}$ or $1\frac{1}{2}$ hours

How do I know what operation to use to solve this problem?

PAIR/SHARE
What is a reasonable estimate for the number of hours Josh lifts weights each week?

Error Alert

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{19}{20}$ hour	have added the fractions.	Discuss the meaning of the problem with students. Reinforce the idea that the word *of* in the phrase $\frac{1}{5}$ *of* $\frac{3}{4}$ is interpreted as multiplication.
$\frac{1}{20}$ hour	have multiplied the denominators and neglected the numerators.	Have students use a visual model to find $\frac{1}{5}$ of $\frac{3}{4}$. Students may draw a number line showing 1 whole divided into fourths and then divide each fourth into 5 equal parts. Have students note that the whole is now divided into twentieths. Have students shade 1 fifth of each of 3 fourths to see that $\frac{1}{5}$ of $\frac{3}{4}$ is $\frac{3}{20}$.

EXAMPLE

$6\frac{3}{8}$ tubes of paint; The equations shown are one way to solve the problem. Students could also solve the problem by rewriting $4\frac{1}{4}$ as $\frac{17}{4}$, rewriting $1\frac{1}{2}$ as $\frac{3}{2}$, and multiplying the two fractions. They could also draw a visual model divided into fourths, with the fourths divided into halves.

Look for The solution requires multiplying a mixed number by a mixed number. Explain that parentheses can be used to show what operation should be done first. In this case, the multiplication must be done before the addition.

APPLY IT

1. $\frac{3}{2}$, or $1\frac{1}{2}$, hours; Students could solve the problem using the expression $\left(3 \times \frac{2}{5}\right) + \left(\frac{3}{4} \times \frac{2}{5}\right)$.
   DOK 2

   Look for The solution requires finding $\frac{2}{5}$ of $3\frac{3}{4}$, so find the product $\frac{2}{5} \times 3\frac{3}{4}$.

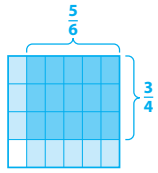
2. $\frac{15}{24}$, or $\frac{5}{8}$, square mile; Students could find the product $\frac{5}{6} \times \frac{3}{4}$ by drawing a unit square divided into sixths one way and fourths the other way, shading five of the sixths and three of the fourths and labeling the length and width of the section of overlapping shading as $\frac{5}{6}$ mile and $\frac{3}{4}$ mile.
   DOK 2

   Look for The solution requires finding the area of a rectangle.

3. **B**; Students could solve the problem by writing the equation $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.
   Explain why the other two answer choices are not correct:
   C is not correct because it is the sum of $\frac{3}{4}$ and $\frac{1}{2}$.
   D is not correct because it is the product of 3 and $\frac{1}{2}$.
   DOK 3

2. A field is in the shape of a rectangle $\frac{5}{6}$ mile long and $\frac{3}{4}$ mile wide. What is the area of the field? Show your work.

   Possible student work using an area model:



   Solution **$\frac{15}{24}$ or $\frac{5}{8}$ square mile**

What model can I use to help understand this problem?

PAIR/SHARE
Can you solve this problem in another way?

3. Ari had $\frac{3}{4}$ of a bag of popcorn. His friends ate $\frac{1}{2}$ of his popcorn. What fraction of the whole bag of popcorn did Ari's friends eat?

   Ⓐ $\frac{1}{4}$
   Ⓑ $\frac{3}{8}$
   Ⓒ $\frac{5}{4}$
   Ⓓ $\frac{3}{2}$

What equation can I write to solve this problem?

Kayla chose Ⓐ as the correct answer. How did she get that answer?

Possible answer: Kayla subtracted $\frac{1}{2}$ from $\frac{3}{4}$ instead of multiplying $\frac{1}{2}$ by $\frac{3}{4}$.

PAIR/SHARE
Does Kayla's answer make sense?

4 **C**; First, find $\frac{1}{6}$ of 24, $\frac{24}{6} = 4$. Subtract 4 from 24 to find number of bottles remaining after Monday, $24 - 4 = 20$. Then find $\frac{1}{4}$ of 20, $\frac{20}{4} = 5$. Subtract 5 from 20 to find number of bottles remaining after Tuesday, $20 - 5 = 15$. Identify the picture showing 15 bottles.
DOK 2

5 $\frac{18}{36}$, or $\frac{1}{2}$, cup; He's making only 6 servings, so multiply $\frac{3}{4}$ by $\frac{6}{9}$.
DOK 2

Error Alert Students who wrote $4\frac{2}{4}$, or $4\frac{1}{2}$, cups may not have written the number of servings Milo wants to make as a fraction of the whole recipe $\left(\frac{6}{9}\right)$ but instead multiplied $\frac{3}{4}$ by 6.

4 On Sunday, Kristen bought a carton of 24 bottles of water.

- On Monday, Kristen drank $\frac{1}{6}$ of the bottles in the carton.
- On Tuesday, Kristen drank $\frac{1}{4}$ of the bottles that remained in the carton after Monday.

Which picture represents the number of bottles of water remaining in the carton after Kristen drank her water on Tuesday?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

5 Milo's pancake recipe makes 9 servings. It calls for $\frac{3}{4}$ cup milk. Milo wants to make 6 servings. How much milk will he need?

$\frac{18}{36}$ or $\frac{1}{2}$ cup

Differentiated Instruction

RETEACH

Visual Model
Model multiplication with mixed numbers.

Students struggling with fraction multiplication involving a mixed number

Will benefit from additional work building visual models

Materials For each pair: tracing paper (2 sheets), ruler, colored pencils or crayons (2 different colors), tape

- Distribute materials to pairs. Write $\frac{2}{3} \times 1\frac{2}{5}$ on the board.
- On one sheet of paper have students draw two squares that share a common side. Have them trace the same figure onto the other sheet of paper.
- Have students draw vertical lines to divide one figure into fifths, shade it to show $1\frac{2}{5}$, draw horizontal lines to divide the other figure into thirds, and shade it a different color to show $\frac{2}{3}$. Have students tape one figure on top of the other, aligning the figures.
- Have pairs identify the intersection of the shadings to find $\frac{2}{3} \times 1\frac{2}{5}$. $\left[\frac{14}{15}\right]$
- Repeat with other fraction expressions involving mixed numbers, such as $2\frac{1}{2} \times \frac{1}{4}$.

EXTEND

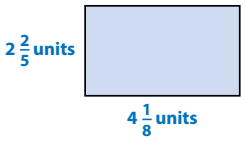
Challenge Activity
Solve a two-step problem.

Students who have achieved proficiency

Will benefit from deepening understanding of modeling problems

- Give students the following two-step problem: *Tim made a snack mix with $\frac{3}{4}$ cup of raisins and $\frac{5}{8}$ cup of nuts. He ate $\frac{1}{5}$ of the snack mix. How many cups of the mix did he eat?* $\left[\frac{3}{4} + \frac{5}{8} = \frac{11}{8}$ and $\frac{1}{5} \times \frac{11}{8} = \frac{11}{40}$; Tim ate $1\frac{1}{40}$ cups.$\right]$
- Have students solve the problem, using both visual models and equations to represent the situation. Have students compare models and equations.

6 Jillian draws a rectangle with the dimensions shown below. What is the area of Jillian's rectangle?



Solution $\frac{396}{40}$, $\frac{99}{10}$, or $9\frac{9}{10}$ square units

7 Lily paints 3 trees for a wall mural. The middle tree is $2\frac{1}{2}$ ft tall. The tree on the left is $\frac{3}{4}$ as tall as the middle tree. The tree on the right is $1\frac{3}{4}$ times as tall as the middle tree. How tall is each tree? Show your work.

Possible answer:

$2\frac{1}{2} = \frac{5}{2}$; $1\frac{3}{4} = \frac{7}{4}$

Left tree: $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$

Right tree: $\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$

Solution The left tree is $\frac{15}{8}$, or $1\frac{7}{8}$, ft tall. The middle tree is $2\frac{1}{2}$ ft tall. The right tree is $4\frac{3}{8}$ ft tall.

8 MATH JOURNAL

Write a word problem for the expression $3\frac{1}{2} \times \frac{1}{2}$. Use a visual model or an equation to show how to solve your problem.

Possible answer: Mary wants to build a fence that is $3\frac{1}{2}$ yards long. She builds $\frac{1}{2}$ of her fence on Monday. How many yards of fence did she build on Monday?

$3\frac{1}{2} = \frac{7}{2}$; $\frac{1}{2} \times \frac{7}{2} = \frac{7}{4}$

Mary builds $\frac{7}{4}$ yards of fence on Monday.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

6 $\frac{396}{40}$, $\frac{99}{10}$, or $9\frac{9}{10}$ square units; Multiply the length by the width, $\frac{12}{5} \times \frac{33}{8} = \frac{396}{40}$.
DOK 1

7 The left tree is $\frac{15}{8}$, or $1\frac{7}{8}$, feet tall. The middle tree is $2\frac{1}{2}$ feet tall. The right tree is $\frac{35}{8}$, or $4\frac{3}{8}$, feet tall; For the left tree height, multiply the height of the middle tree by $\frac{3}{4}$, $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$. The height of the middle tree is given in the problem. For the right tree height, multiply the height of the middle tree by $\frac{7}{4}$, $\frac{7}{4} \times \frac{5}{2} = \frac{35}{8}$.
DOK 2

REINFORCE

Problems 4–8

Multiply fractions.

All students will benefit from additional work with multiplying fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide children with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

8 MATH JOURNAL

Student responses should indicate understanding of problem situations that require multiplying a fraction and a mixed number, such as finding the area of a rectangle or scaling a quantity, and understanding of how to use a visual model or equations to represent and solve a problem of this type.

Error Alert **If** students multiply only the two fractions in the expression, writing the answer as $3\frac{1}{4}$, **then** review multiplying with a mixed number to clarify that the 3 wholes must be included in the multiplication. Have them rewrite the product as $\left(3 + \frac{1}{2}\right) \times \frac{1}{2}$ and use the distributive property.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener page.

Lesson Overview

LESSON 24

Divide Unit Fractions in Word Problems

Lesson Objectives

Content Objectives

- Represent and solve real-world problems involving division of unit fractions by whole numbers using visual fraction models and equations.
- Represent and solve real-world problems involving division of whole numbers by unit fractions using visual fraction models and equations.
- For a given division equation with a unit fraction and a whole number, use the inverse relationship between multiplication and division to write a related multiplication equation.

Language Objectives

- Draw visual models to represent word problems involving division with unit fractions.
- Write equations to represent word problems involving division with unit fractions.
- Describe the relationship between a visual model and an equation that both represent the same problem situation.
- Write a multiplication equation to check a quotient of a division equation.

Prerequisite Skills

- Divide whole numbers.
- Multiply whole numbers and fractions by unit fractions.
- Understand that multiplication and division are inverse operations.
- Use a visual fraction model to find the quotient of a unit fraction divided by a whole number.
- Use a visual fraction model to find the quotient of a whole number divided by a unit fraction.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

5 Use appropriate tools strategically.

8 Look for and express regularity in repeated reasoning.

*See page 305m to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key term.

- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In the previous lesson students acquired a conceptual understanding of dividing with unit fractions (fractions with a numerator of 1). They used visual models, such as area models, bar models, number lines, and pictures to understand what it means to divide with unit fractions. Students divided whole numbers by unit fractions and divided unit fractions by whole numbers. They used their understanding that $\frac{a}{b} = a \div b = a \times \frac{1}{b}$ to write multiplication equations to solve division problems with unit fractions.

In this lesson students apply their understanding of division with unit fractions to solve word problems. Students use models to help them make sense of the problems and to write equations to represent the division with unit fractions in the problems. Students find quotients to solve the problems, and they use the inverse relationship between multiplication and division to check their answers.

In Grade 6 students will solve mathematical and real-world problems involving division of fractions by fractions.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Understand Division with Unit Fractions

Dividing Unit Fractions in Word Problems
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 473–474

SESSION 2
Develop
45–60 min

Dividing a Unit Fraction by a Whole Number
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 479–480

Fluency
Dividing a Unit Fraction by a Whole Number

SESSION 3
Develop
45–60 min

Dividing a Whole Number by a Unit Fraction
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 485–486

Fluency
Dividing a Whole Number by a Unit Fraction

SESSION 4
Refine
45–60 min

Dividing Unit Fractions in Word Problems
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Activity Sheet:* Fraction Bars
Activities	*Per student:* 1 set of fraction circles or tiles, yarn $\left(\frac{1}{2}\text{ foot}\right)$, scissors, glue, ruler, sheet of paper, modeling clay (2 equal-sized portions) *Activity Sheet:* Fraction Bars
Math Toolkit	fraction tiles, fraction bars, number lines, grid paper, index cards, sticky notes, ribbon or yarn
Digital Math Tool	Fraction Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 4
- Lesson 24 Multiply Fractions by Whole Numbers

RETEACH

Tools for Instruction

Grade 4
- Lesson 24 Multiply a Whole Number and a Fraction

Grade 5
- Lesson 24 Dividing Unit Fractions to Solve Word Problems

REINFORCE

Math Center Activity

Grade 5
- Lesson 24 Find the Division Expression

EXTEND

Enrichment Activity

Grade 5
- Lesson 24 Switching Places

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 5
- Divide Unit Fractions in Word Problems

Connect to Family, Community, and Language Development

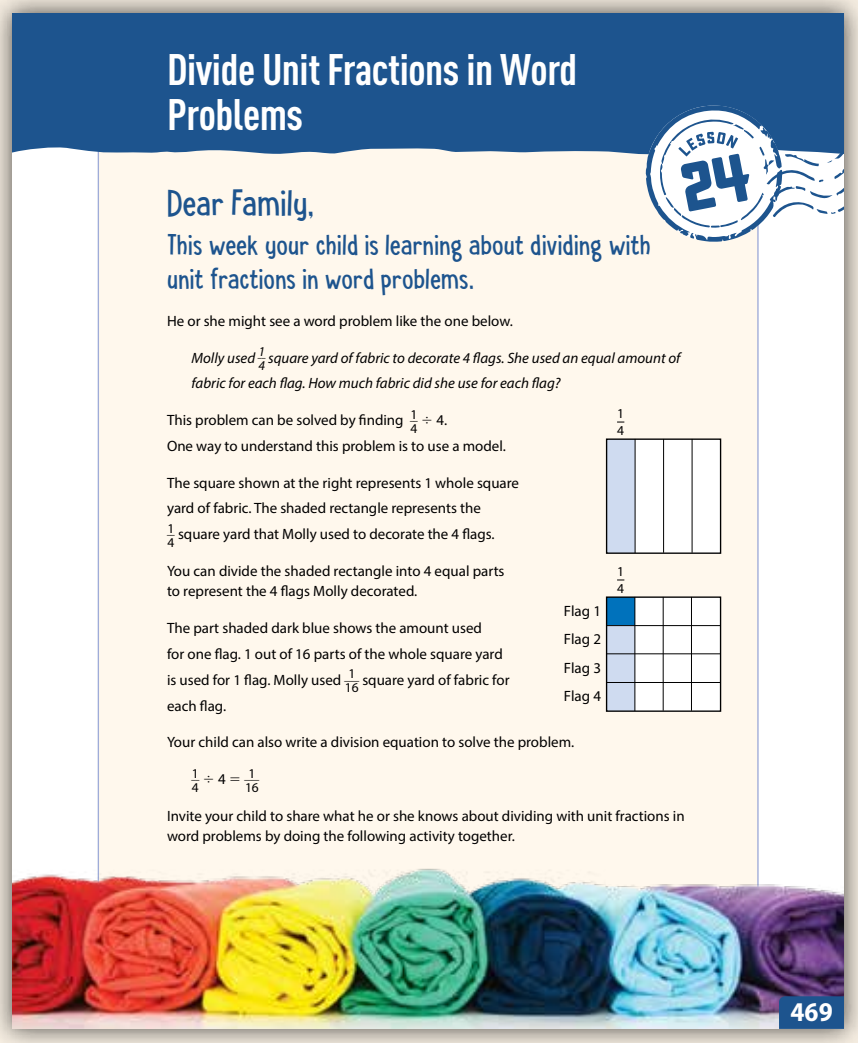
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Divide Unit Fractions in Word Problems

LESSON 24

Dear Family,

This week your child is learning about dividing with unit fractions in word problems.

He or she might see a word problem like the one below.

Molly used $\frac{1}{4}$ square yard of fabric to decorate 4 flags. She used an equal amount of fabric for each flag. How much fabric did she use for each flag?

This problem can be solved by finding $\frac{1}{4} \div 4$.
One way to understand this problem is to use a model.

The square shown at the right represents 1 whole square yard of fabric. The shaded rectangle represents the $\frac{1}{4}$ square yard that Molly used to decorate the 4 flags.

You can divide the shaded rectangle into 4 equal parts to represent the 4 flags Molly decorated.

The part shaded dark blue shows the amount used for one flag. 1 out of 16 parts of the whole square yard is used for 1 flag. Molly used $\frac{1}{16}$ square yard of fabric for each flag.

Flag 1
Flag 2
Flag 3
Flag 4

Your child can also write a division equation to solve the problem.

$\frac{1}{4} \div 4 = \frac{1}{16}$

Invite your child to share what he or she knows about dividing with unit fractions in word problems by doing the following activity together.

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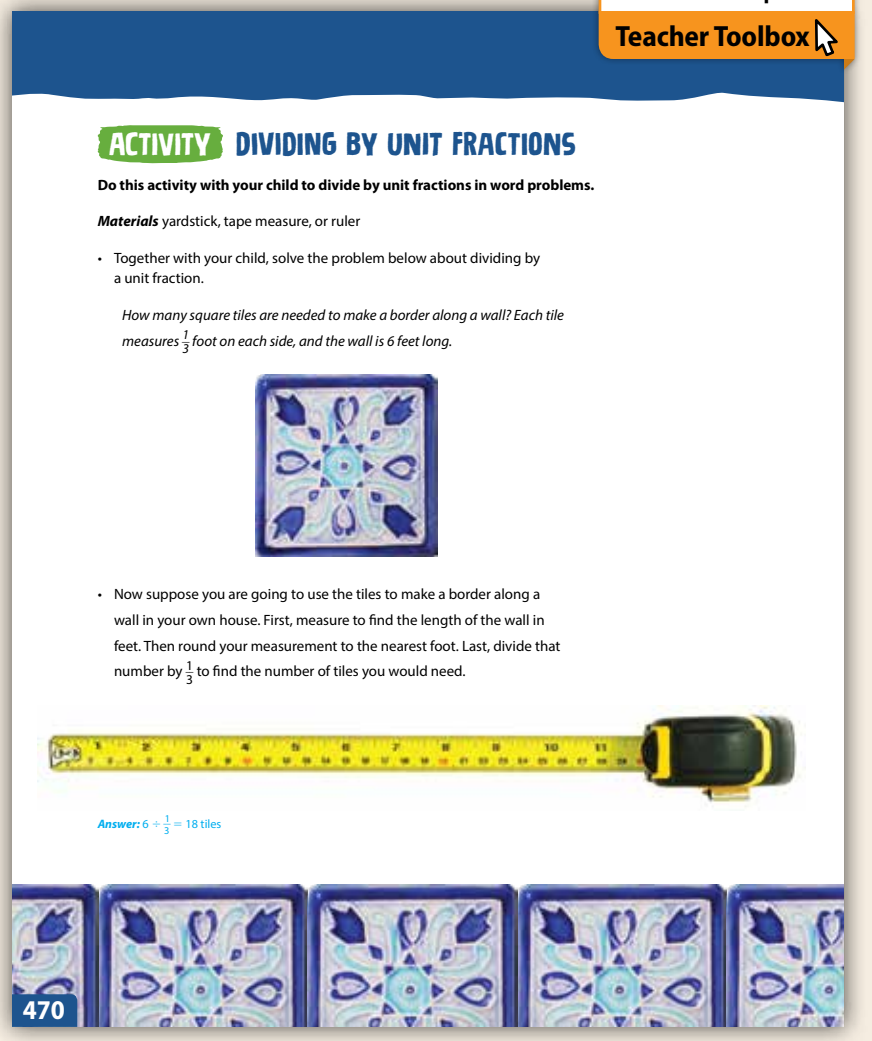

ACTIVITY DIVIDING BY UNIT FRACTIONS

Do this activity with your child to divide by unit fractions in word problems.

Materials yardstick, tape measure, or ruler

- Together with your child, solve the problem below about dividing by a unit fraction.

  How many square tiles are needed to make a border along a wall? Each tile measures $\frac{1}{3}$ foot on each side, and the wall is 6 feet long.

- Now suppose you are going to use the tiles to make a border along a wall in your own house. First, measure to find the length of the wall in feet. Then round your measurement to the nearest foot. Last, divide that number by $\frac{1}{3}$ to find the number of tiles you would need.

Answer: $6 \div \frac{1}{3} = 18$ tiles

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Goal

The goal of the Family Letter is to provide opportunities for families to help students reinforce their understanding of dividing with unit fractions in the context of word problems.

Activity

Encourage students to work with family members to solve real-world problems that involve dividing with a unit fraction. Look at the *Dividing by Unit Fractions* activity and modify it if necessary to connect with your students.

Math Talk at Home

Have students and family members talk about unit fractions at home. They can find examples as they talk about cooking, building, or sharing.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *What unit fractions are easy to show by cutting a pizza or pie into equal shares?*
- *How do you divide granola bars in halves? How do you divide them in sixths?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- Ask students to share if they have participated in races in schools or know anyone who runs. If time permits, ask students to research any local or national races for a given time period, including the distance of each race. Have students select one of the races and make a diagram that shows water stops.

Session 2 **Use with *Apply It*.**

- Tell students that some problems involving fractions use pizza. Pizza is a circular food that can be separated into parts. Ask students to share what foods in their culture have a circular shape. If time permits, have students describe their food and how they could divide it.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Speaking/Writing Read the ***Try It*** problem aloud. Help students make sense of the problem by acting it out. Use 6 equal pieces of string to represent each mile of the race. Place them end to end with no gaps or overlaps. Ask volunteers to stand on the string to represent each half mile mark and at the end to represent the water stop at the finish line. After the role-play activity, organize students into pairs to model the problem with fraction bars. Encourage them to use gestures, pictures, numbers, and words to communicate ideas. Have them write an answer using the frame: • *There are* ___12___ *water stops.*	**Speaking/Writing** Read the ***Try It*** problem with students. Organize them into pairs to model and solve the problem using fraction bars. Provide a bank of terms to support them as they think aloud to model the problem: *fraction*, *numerator*, *denominator*, *whole*, *part*, *divide*, *multiply*, *half*, and *represent*. Have students complete the sentence frames below and point out how their model supports each statement: • *The model shows* ___6___ *wholes.* • *The wholes are divided into* ___halves___ *.*	**Speaking/Writing** Have students read and discuss the ***Try It*** with a partner. Encourage partners to explore strategies together, but to work separately to create their own models. Then have them debrief and compare their work. Ask: *How are your models alike? How are they different?* Have students reflect on their solutions. Ask: *Is your answer reasonable, considering the context of the problem?* Instruct students to explain, in writing, why they think their answer is reasonable.

LESSON 24

SESSION 1 Explore

Purpose In this session students draw on their understanding of what it means to divide with unit fractions. They share models to explore how to represent problem situations with visual models. They will look ahead to think about equations they can use to represent and solve these types of problems.

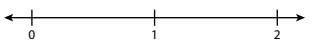
Start

Connect to Prior Knowledge

Why Support students' facility with using a visual model to represent division of a whole number by a unit fraction.

How Have students copy and then complete a number line model to represent the expression $2 \div \frac{1}{5}$. Discuss that the quotient is 10 and that this means there are 10 groups of $\frac{1}{5}$ in 2.

Copy and complete the model to show $2 \div \frac{1}{5}$.



Look for Number lines should show each whole divided into fifths, so there are 10 fifths in all.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that there are water stops at half-mile intervals along the 6-mile race route. The first water stop is at the half-mile mark and the last one is at the 6-mile mark.

DISCUSS IT

Support Partner Discussion

Encourage students to use the *Discuss It* question and sentence starter on the Student Worktext page as part of their discussion.

Look for, and prompt as necessary for, understanding of:

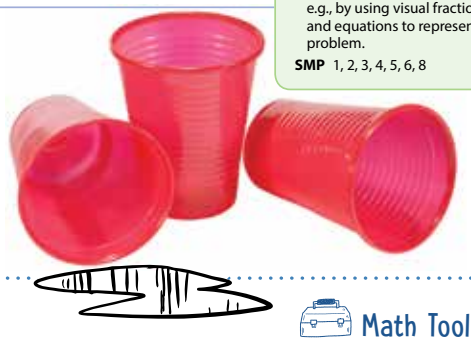
- 6 as the number of wholes
- $\frac{1}{2}$ as the equal-size parts each whole is divided into

LESSON 24 SESSION 1 ● ○ ○ ○

Explore Dividing Unit Fractions in Word Problems

Previously, you learned about what it means to divide with unit fractions. Use what you know to try to solve the problem below.

Micah is running a 6-mile race. There are water stops every $\frac{1}{2}$ mile and at the 6-mile finish line. How many water stops are there in all? Use a visual model to show your solution.



Learning Target

- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

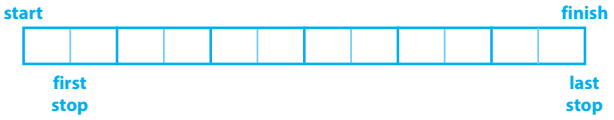
SMP 1, 2, 3, 4, 5, 6, 8

TRY IT

Possible student work:

Sample A

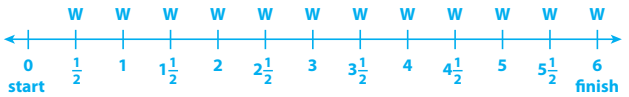
Use 6 rectangles to represent the 6 miles.



There are 12 water stops.

Sample B

6-mile race with 2 water stops each mile



There are 12 water stops in all.

Math Toolkit

- fraction bars
- fraction models
- number lines
- grid paper
- index cards
- sticky notes

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

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Common Misconception Look for students who are not comfortable interpreting the location of water stops at every half mile as including water stops at whole-number miles as well as at $\frac{1}{2}$ mile, $1\frac{1}{2}$ miles, $2\frac{1}{2}$ miles, and so on. As students present solutions, have them specify how far beyond the first water station the second water station is and where along the race route it is located.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing wholes and halves
- drawings to represent the problem
- number lines marked in wholes and halves
- division or multiplication equations showing 12 water stops in all

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the number of wholes? The number of parts in each whole?*

Listen for 6 is the number of wholes. Each part is $\frac{1}{2}$, so there are 2 in each whole.

CONNECT IT

1 LOOK BACK

Look for understanding that you can show the 6 miles of the race on the number line, mark each $\frac{1}{2}$ mile along it to represent the location of a water stop, showing there are 12 water stops in all.

Hands-On Activity

Use fraction bars for fraction division.

If . . . *students are unsure about using visual models to show division with fractions,*

Then . . . *use this activity to make a model.*

Materials For each student: Activity Sheet *Fraction Bars* (6 wholes)

- Distribute fraction bars. Explain to students that each bar represents 1 mile of the race.
- Prompt students to identify the water stops for the first mile are at $\frac{1}{2}$ mile and 1 mile. Have them fold one fraction bar in half, open it, and label one water stop on the fold and one at the end of the fraction bar.
- Guide students to see the stops for the first mile represent the location of stops for each mile. Have them fold and mark the other fraction bars accordingly.
- Have students arrange the fraction bars end-to-end and count the water stops along the route, confirming there are 12 water stops.
- Repeat activity for additional division expressions, such as $3 \div \frac{1}{2}$ and $4 \div \frac{1}{4}$.

2 LOOK AHEAD

Point out that you can also model and solve problems involving dividing whole numbers by unit fractions using division or multiplication equations.

Ask *Why in one equation is the number of miles, 5, divided by the fraction $\frac{1}{3}$, and in the other equation 5 is multiplied by 3?*

Listen for The division equation represents dividing the 5-mile race into $\frac{1}{3}$-mile segments to find how many $\frac{1}{3}$s there are in all; the multiplication equation represents multiplying the number of water stops in one mile, 3, by the number of miles in the race, 5.

CONNECT IT

1 LOOK BACK

How many water stops are there in all? Explain how you can use a number line to support your answer.

12 water stops; Possible explanation: You can use tick marks to show the locations of water stops at each $\frac{1}{2}$-mile point of the 6-mile race. Then count how many water stops there are in all.

2 LOOK AHEAD

On the previous page, you used a visual model to solve a word problem involving dividing a whole number by a unit fraction. You can also use equations to represent and solve these types of problems. Consider this word problem.

Micah now runs in a 5-mile race. There are water stops every $\frac{1}{3}$ mile and at the 5-mile finish line in this new race. How many water stops are there in all?

a. Complete the division equation below.

miles in race: 5 ÷ $\frac{1}{3}$ = w (number of water stops)

fraction of a mile for each water stop

b. Complete the multiplication equation below.

miles in race: 5 × 3 = w (number of water stops)

number of water stops in each mile

c. How many water stops are in this race? Explain how you know.

15; Possible explanation: There are 5 miles and 3 water stops in each mile.

3 REFLECT

Explain what it means to divide 5 by $\frac{1}{3}$, or $5 \div \frac{1}{3}$.

Possible answer: It means finding out how many thirds are in 5 wholes.

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Close: Exit Ticket

3 REFLECT

Look for understanding of what it means to divide a whole number by a unit fraction. Student responses should include references to finding how many parts equal in size to the unit fraction there are in a given number of wholes.

Common Misconception **If** students think that the quotient cannot be greater than the dividend, **then** have them write the quotients for the following expressions: $6 \div 3$, $6 \div 2$, $6 \div 1$ and discuss the patterns they see. Have them extrapolate from the pattern how the quotient compares to the dividend when the divisor is less than 1.

Real-World Connection

Encourage students to think about everyday places or situations where people might need to divide with unit fractions. Have volunteers share ideas. Examples: food preparation, construction, physical training.

Solutions

Support Vocabulary Development

1 Have students discuss models they know. Then ask them to share what they know about fractions. Ask: *How can you represent a fraction?* Have students think of shapes they can divide into equal parts. Then ask them to draw the shape and show how to divide it into equal parts.

As students write examples in the graphic organizer, ask them to identify how the whole and the fraction are represented.

2 Read the problem. Ask: *How many wholes do you need to divide?* Have students draw a model to show the 4 wholes. Then ask: *How do you divide each whole into parts of size* $\frac{1}{2}$*?*

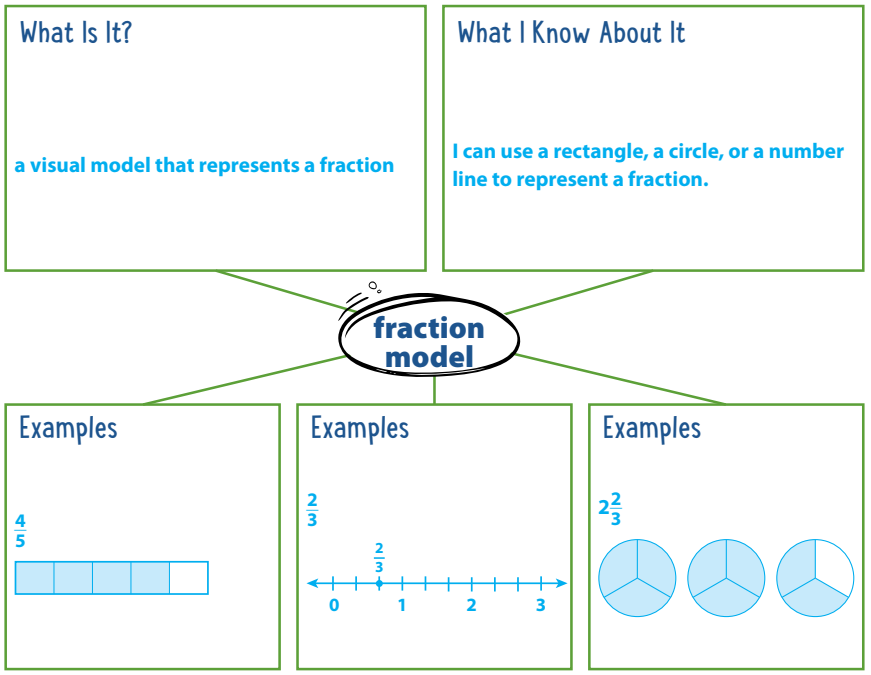
Supplemental Math Vocabulary

- *bar diagram*
- *number line*

Name: _____ LESSON 24 SESSION 1

Prepare for Dividing Unit Fractions in Word Problems

1 Think about what you know about fraction models. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Draw a fraction model to show the expression $4 \div \frac{1}{2}$.

Possible answer:

3 Assign problem 3 to provide another look at dividing with unit fractions in word problems.

This problem is very similar to the problem about Micah running a 6-mile race. In both problems, students are asked to draw a visual model to help them to divide a whole number by a unit fraction. The question asks how many pieces of ribbon are there in all.

Students may want to use fraction bars, number lines, string, or ribbon.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

$2 \div \frac{1}{4} = 2 \times 4 = 8$. Check that models show 8 pieces.

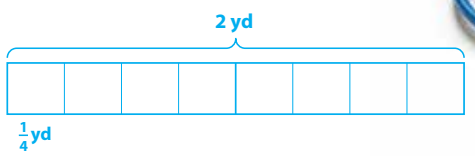
Medium

4 Have students solve the problem a different way to check their answer.

3 Solve the problem. Show your work.

Adela has a ribbon that is 2 yards long. She cuts the ribbon into pieces that are $\frac{1}{4}$ yard long. How many pieces of ribbon are there in all? Use a visual model to show your solution.

Possible student work using a picture:



$2 \div \frac{1}{4} = 2 \times 4 = 8$

Solution **There are 8 pieces of ribbon in all.**

4 Check your answer. Show your work.

Possible student work:

2-yard ribbon with 4 pieces for each yard

$2 \times 4 = 8$

There are 8 pieces of ribbon in all.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3

Listening/Speaking Have students chorally read ***Apply It*** problem 6. In partners, ask them to talk about the problem. Use questions and sentence frames to guide them:

- *What food does Felipe want to share?*
- *How much of that does Felipe have?*
- *How will Felipe share it?*
- *How much will each person get?*

- *Felipe wants to share _____.*
- *He has _____ of the _____.*
- *Felipe will share it _____ with _____.*
- *Each person will get _____.*

Levels 2–4

Listening/Speaking Have students chorally read ***Apply It*** problem 6. In partners, ask them to talk about the problem. Use questions to guide them:

- *What food does Felipe want to share?*
- *How much of that does Felipe have?*
- *How will Felipe share it?*
- *How much will each person get?*

Levels 3–5

Speaking/Writing Ask students to read ***Apply It*** problem 6 and to write the steps they will take to solve the problem.

Have students trade the steps with a partner. Partners review the steps and revise as needed. Once partners agree on a set of steps, have them solve the problem and talk about the answer.

Ask: *Did you and your partner agree on all the steps? How much pizza does each friend get?*

LESSON 24

SESSION 2 Develop

Purpose In this session students solve a problem that requires dividing $\frac{1}{6}$ by 3. Students model the numbers in the word problem either on paper or with manipulatives to develop strategies for solving word problems that involve dividing a unit fraction by a whole number.

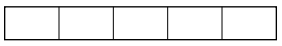
Start

Connect to Prior Knowledge

Why Support students' facility with using a visual model to represent division of a unit fraction by a whole number.

How Have students copy and then complete a fraction bar model to represent the quotient $\frac{1}{5} \div 2$. Discuss that the quotient is $\frac{1}{10}$.

Copy and complete the model to show $\frac{1}{5} \div 2$.



Look for Models should show each fifth divided into halves, so there are 10 parts in all, to show $\frac{1}{10}$ as the quotient.

Develop Language

Why Clarify understanding of the word *border* as it relates to the perimeter or length around an object.

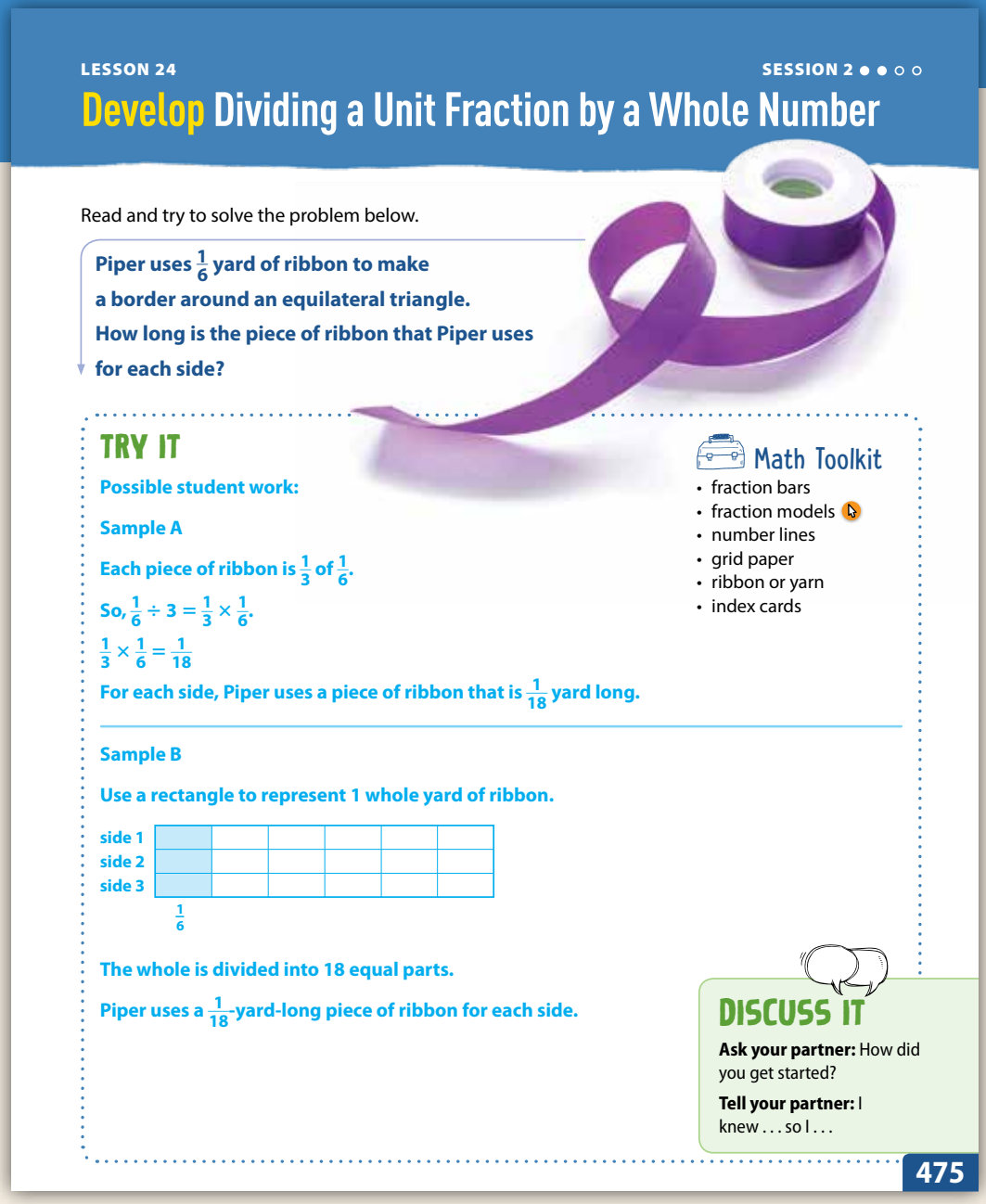
How Explain that to *make a border around* something means to put something around its edge. Model with a ribbon, yarn, or a measuring tape and a shape or classroom object. Have students find the word in the *Try It* problem. Ask: *What is Piper using to make a border around the triangle?*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify an equilateral triangle as one that has equal-length sides.

Ask *What is the whole length of ribbon Piper has? How many pieces does she need?*



LESSON 24 SESSION 2 ● ● ○ ○

Develop Dividing a Unit Fraction by a Whole Number

Read and try to solve the problem below.

Piper uses $\frac{1}{6}$ yard of ribbon to make a border around an equilateral triangle. How long is the piece of ribbon that Piper uses for each side?

TRY IT

Possible student work:

Sample A

Each piece of ribbon is $\frac{1}{3}$ of $\frac{1}{6}$.

So, $\frac{1}{6} \div 3 = \frac{1}{3} \times \frac{1}{6}$.

$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

For each side, Piper uses a piece of ribbon that is $\frac{1}{18}$ yard long.

Sample B

Use a rectangle to represent 1 whole yard of ribbon.

The whole is divided into 18 equal parts.

Piper uses a $\frac{1}{18}$-yard-long piece of ribbon for each side.

Math Toolkit

- fraction bars
- fraction models
- number lines
- grid paper
- ribbon or yarn
- index cards

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I knew . . . so I . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to name the model or strategy they used as they discuss.

Support as needed with questions such as:

- *How would you describe your model?*
- *How is your model similar to your partner's?*

Common Misconception Look for students who confuse the dividend and divisor and find $3 \div \frac{1}{6}$ instead of $\frac{1}{6} \div 3$. As students present solutions, have them specify the quantity being divided and the number of parts it is divided into.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing fifths, thirds, and fifteenths
- drawings to represent the problem
- number lines marked to show the fractions
- division or multiplication equations showing a ribbon length of $\frac{1}{18}$ yard

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the total length of ribbon Piper has? Where does it represent the number of pieces she needs? How does it show the length of each piece?*

Listen for Students should recognize that representations should show $\frac{1}{6}$ of a whole yard divided into three equal parts. Responses may include $\frac{1}{6}$ yard as the length, 3 as the number of pieces, and $\frac{1}{18}$ yard as the length of each piece.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the ribbon length Piper has to start
- the number of equal parts needed
- the length of ribbon for each side

Ask *How are the ribbon pieces represented in the picture? In the equations? Is it the same or different?*

Listen for Each representation shows the pieces as 1 of 3 equal parts of $\frac{1}{6}$.

For a picture of the ribbon, prompt students to identify how the ribbon is labeled to represent the problem.

- *How is it helpful that a whole of 1 yard is represented?*
- *Why is the entire yard of ribbon divided into thirds, and not just one sixth of the yard?*

For an equation model, prompt students to identify how both equations represent the situation.

- *Why is the length of ribbon, $\frac{1}{6}$ yard divided by 3 in one equation and multiplied by $\frac{1}{3}$ in the other?*

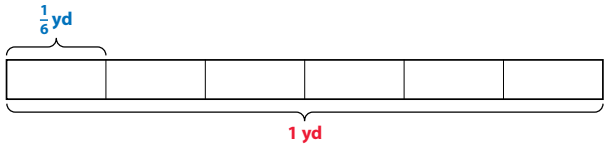
Explore different ways to understand dividing a unit fraction by a whole number to solve word problems.

Piper uses $\frac{1}{6}$ yard of ribbon to make a border around an equilateral triangle. How long is the piece of ribbon that Piper uses for each side?

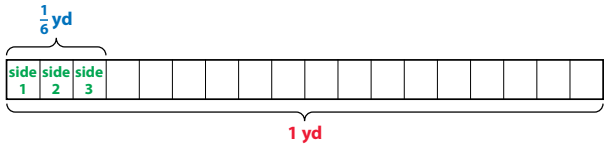
PICTURE IT

You can draw a picture to help understand the problem.

Draw a **1-yard** length of ribbon and then draw and label a $\frac{1}{6}$**-yard** length.



Divide the $\frac{1}{6}$**-yard** length into **3 equal parts**, one for each side of the equilateral triangle.



MODEL IT

You can use equations to model the problem.

Write a division equation.

$\frac{1}{6} \div 3 = s$ — $\frac{1}{6}$: length of ribbon; 3: number of equal parts; s: length of each part

Write a multiplication equation.

$\frac{1}{3} \times \frac{1}{6} = s$ — $\frac{1}{3}$: fraction of the ribbon; $\frac{1}{6}$: length of ribbon; s: length of each part

Deepen Understanding

Equation Model

SMP 2 Reason abstractly and quantitatively.

Prompt students to consider how the equations in *Model It* reflect different ways to represent the relationship between the quantities in the problem.

Ask *How does the equation $\frac{1}{6} \div 3 = s$ represent the ribbon in the problem? Explain the meaning of each number, $\frac{1}{6}$ and 3, and why the operation is division.*

Listen for $\frac{1}{6}$ represents the $\frac{1}{6}$ yard of ribbon that Piper uses. The triangle is an equilateral triangle, so each side is the same length. That means Piper needs to divide the $\frac{1}{6}$ yard of ribbon into 3 equal parts, or $\frac{1}{6} \div 3$.

Ask *How does the equation $\frac{1}{3} \times \frac{1}{6} = s$ represent the ribbon in the problem? Explain the meaning of the number $\frac{1}{3}$ and why the operation is multiplication.*

Listen for The triangle has 3 equal sides, so each piece of ribbon will be $\frac{1}{3}$ of Piper's $\frac{1}{6}$-yard ribbon. To find $\frac{1}{3}$ of $\frac{1}{6}$, you multiply.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page students will use what they know about multiplying fractions to help them find quotients when they divide a unit fraction by a whole number.

Monitor and Confirm

1 and 3 Check for understanding that:

- a ribbon that is $\frac{1}{6}$ yard long is $\frac{1}{6}$ of 1 whole yard
- the $\frac{1}{6}$-yard length of ribbon needs to be divided into 3 equal pieces
- $\frac{1}{18}$ yard is the length of each piece of ribbon

Support Whole Class Discussion

2 In discussing problem 2, check that students understand how the division expression $\frac{1}{6} \div 3$ and the multiplication expression $\frac{1}{3} \times \frac{1}{6}$ each describe one way to interpret the second diagram in *Picture It*. Then prepare them for problem 4, in which they will think about a different multiplication equation related to the quotient $\frac{1}{6} \div 3$.

Ask *Consider the whole number division problem 48 ÷ 12 = n. How do you use the inverse relationship between multiplication and division to help you divide, or to check your answer?*

Listen for You can think about the related multiplication equation $12 \times n = 48$ and find the missing factor, which is 4. To use multiplication to check that $48 \div 4 = 12$, you find the product 12×4, which is 48.

4 Look for the idea that the inverse relationship between multiplication and division shows that $\frac{1}{6} \div 3 = \frac{1}{18}$ because $\frac{1}{18} \times 3 = \frac{3}{18}$, or $\frac{1}{6}$.

5 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to divide a unit fraction by a whole number.

1 Look at **Picture It**. What does the first diagram show? What whole is being divided?

Possible answer: It shows 1 yard divided into 6 sections. Each section is $\frac{1}{6}$ yard, which is the length of ribbon Piper uses.

Why does the second diagram show each $\frac{1}{6}$-yard section divided into 3 equal parts?

Possible answer: The $\frac{1}{6}$-yard section of ribbon has to go around 3 sides of a triangle. The sides have equal lengths.

2 Look at **Model It**. How does each equation relate to the second diagram in **Picture It**?

Possible answer: The division equation shows the $\frac{1}{6}$-yard ribbon divided into 3 equal parts just like in Picture It. The multiplication equation shows that you are finding $\frac{1}{3}$ of $\frac{1}{6}$ yard. Each part in Picture It also is $\frac{1}{3}$ of $\frac{1}{6}$ yard.

3 How long is the piece of ribbon Piper uses for each side of the triangle?

$\frac{1}{18}$ yard

4 What is $\frac{1}{6} \div 3$? How can you use a multiplication equation different from the one shown in **Model It** to check that your answer is correct?

$\frac{1}{18}$; Possible answer: You can use the equation $\frac{1}{18} \times 3 = \frac{3}{18}$, or $\frac{1}{6}$. Because $\frac{1}{18} \times 3 = \frac{1}{6}$, $\frac{1}{6} \div 3 = \frac{1}{18}$.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for dividing a unit fraction by a whole number to solve word problems? Explain.

Some students may like using a drawing because it helps them reason through the problem. Others may say that using equations and what they know about multiplying fractions is a quick way to solve a division problem.

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Hands-On Activity

Act out the problem.

If . . . *students are unsure about representing the division symbolically,*

Then . . . *use this activity to connect a concrete model to an abstract equation.*

Materials For each student: yarn $\left(\frac{1}{2} \text{ foot}\right)$, scissors, glue, ruler, sheet of paper

- Tell students they will use the $\frac{1}{2}$ foot of yarn to make an equilateral triangle.
- Have students use whatever strategy they choose to make 3 equal-length pieces from the $\frac{1}{2}$-foot piece of yarn. Prompt students to write a division expression to describe what they have done so far. $\left[\frac{1}{2} \div 3\right]$
- Tell students to glue the yarn pieces onto the paper to make an equilateral triangle, have them measure the length of each side as 2 inches, and guide them in writing this length as a fraction of a foot: 2 inches $= \frac{1}{6}$ foot.
- Ask: *What equation can you write to find the length of each piece when you divide a $\frac{1}{2}$-foot piece of yarn into 3 equal parts?* $\left[\frac{1}{2} \div 3 = \frac{1}{6}\right]$ Discuss how to use the related multiplication equation to check the answer. $\left[\frac{1}{6} \times 3 = \frac{3}{6}, \text{ or } \frac{1}{2}\right]$

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing equal parts in fraction models can be difficult.

6 Each friend gets $\frac{1}{8}$ of the pizza; See possible work on the Student Worktext page. Students may also interpret the problem as finding $\frac{1}{2}$ of $\frac{1}{4}$ and use the multiplication equation $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

7 $\frac{1}{12}$ of the whole garden has red roses; See possible model and division equation on the Student Worktext page.

Close: Exit Ticket

8 **A**; You can think of the problem situation as finding $\frac{1}{4}$ of $\frac{1}{3}$, which can be represented by the expression $\frac{1}{4} \times \frac{1}{3}$.

C; The related multiplication equation for the division equation $\frac{1}{3} \div 4 = \frac{1}{12}$ is $\frac{1}{12} \times 4 = \frac{1}{3}$.

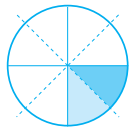
Error Alert **If** students choose B, D, or E, **then** review using the inverse relationship between multiplication and division to write related multiplication and division equations, starting with whole numbers for both factors. Also have them analyze the models they drew and guide them to see how the models can be interpreted as representing $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{4} \times \frac{1}{3}$.

APPLY IT

Use what you just learned to solve these problems.

6 Felipe has $\frac{1}{4}$ of a pizza. He wants to share it equally with a friend. How much of the original whole pizza will each of them get? Show your work.

Possible student work:



$\frac{1}{4} \div 2 = \frac{1}{8}$

Solution Each friend gets $\frac{1}{8}$ of the pizza.

7 Angela uses $\frac{1}{3}$ of her rectangular flower garden for roses. She plants equal rectangular areas of red, white, pink, and orange roses in this part of the garden. What fraction of the whole garden has red roses? Draw a model and write a division equation to represent and solve the problem. **Possible student work:**

Red roses		
White roses		
Pink roses		
Orange roses		

$\frac{1}{3} \div 4 = \frac{1}{12}$

Solution $\frac{1}{12}$ of the whole garden has red roses.

8 Look at problem 7. Which multiplication expressions can be used to represent the situation or check the division equation?

Ⓐ $\frac{1}{4} \times \frac{1}{3}$ Ⓑ $4 \times \frac{1}{3}$

Ⓒ $\frac{1}{12} \times 4$ Ⓓ 3×4

Ⓔ $3 \times \frac{1}{4}$

Solutions

1. $\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$
Basic

2. Less than; Students may reason that a lesser amount of punch is poured into the same number of glasses, so the amount in each glass will be less. See possible description for how the model would change on the student page.
Medium

Name: _____ LESSON 24 SESSION 2

Practice Dividing a Unit Fraction by a Whole Number

Study the Example showing one way to solve a word problem involving dividing a fraction by a whole number. Then solve problems 1–5.

EXAMPLE

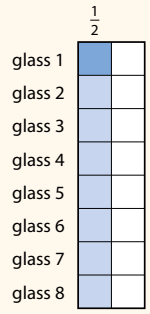
Felicia makes $\frac{1}{2}$ gallon of fruit punch. She pours an equal amount into 8 glasses. What fraction of a gallon of fruit punch is in each glass?

Find $\frac{1}{2} \div 8$.

The model shows a rectangle divided into halves and then divided into 8 equal parts. There are a total of 16 parts, and one part is the amount of fruit punch in 1 glass.

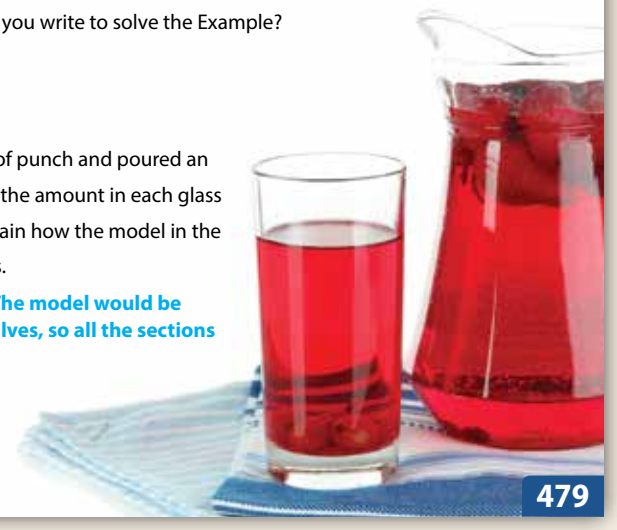
$\frac{1}{2} \div 8 = \frac{1}{16}$

The amount in 1 glass is $\frac{1}{16}$ gallon.



1. What multiplication equation could you write to solve the Example?
$\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

2. Suppose Felicia had made $\frac{1}{4}$ gallon of punch and poured an equal amount into 8 glasses. Would the amount in each glass be more or less than $\frac{1}{16}$ gallon? Explain how the model in the Example would change to show this.
Less than; Possible explanation: The model would be divided into fourths instead of halves, so all the sections would be smaller.



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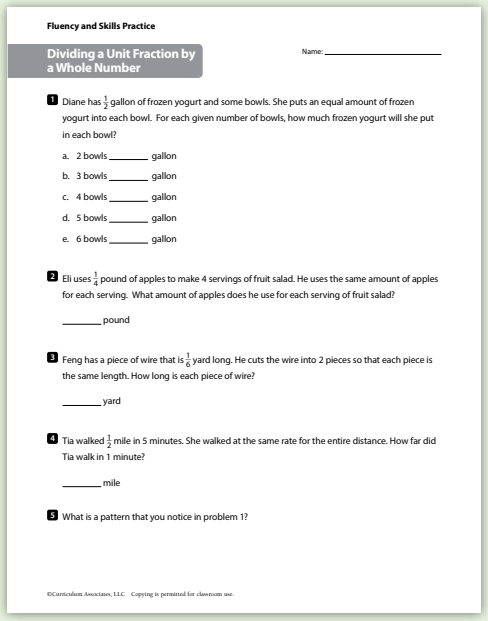
Fluency & Skills Practice Teacher Toolbox

Assign *Dividing a Unit Fraction by a Whole Number*

In this activity students divide unit fractions by whole numbers. This skill is useful in everyday situations involving fractional measurements. One such situation might involve distributing $\frac{3}{4}$ gallon of juice equally among 5 children. Another would be dividing $\frac{4}{5}$ cup of cooking oil into 4 equal amounts.



Fluency and Skills Practice

Dividing a Unit Fraction by a Whole Number

Name: _____

1. Diane has $\frac{1}{2}$ gallon of frozen yogurt and some bowls. She puts an equal amount of frozen yogurt into each bowl. For each given number of bowls, how much frozen yogurt will she put in each bowl?
   a. 2 bowls _____ gallon
   b. 3 bowls _____ gallon
   c. 4 bowls _____ gallon
   d. 5 bowls _____ gallon
   e. 6 bowls _____ gallon

2. Eli uses $\frac{1}{4}$ pound of apples to make 4 servings of fruit salad. He uses the same amount of apples for each serving. What amount of apples does he use for each serving of fruit salad?
   _____ pound

3. Feng has a piece of wire that is $\frac{1}{6}$ yard long. He cuts the wire into 2 pieces so that each piece is the same length. How long is each piece of wire?
   _____ yard

4. Tia walked $\frac{1}{2}$ mile in 5 minutes. She walked at the same rate for the entire distance. How far did Tia walk in 1 minute?
   _____ mile

5. What is a pattern that you notice in problem 1?

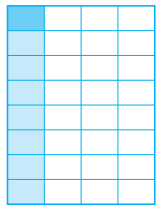
3 $\frac{1}{32}$ pound; See possible model and equations on the student page.
Medium

4 $\frac{1}{6}$ of the original stack; Students may show an area model divided in half one way and thirds the other way, with overlapping shading in $\frac{1}{6}$ of the model. They may also use the expression $\frac{1}{2} \div 3$ or $\frac{1}{3} \times \frac{1}{2}$ to solve the problem.
Medium

5 $\frac{1}{8}$ of the original stack; See possible division equation on the student page.
Medium

3 Donal buys a $\frac{1}{4}$-pound package of cheese. There are 8 slices of cheese in the package. Each slice has the same weight. What fraction of a pound is each slice? Draw a model and write a division equation to represent and solve the problem.

Possible student work:



$\frac{1}{4} \div 8 = \frac{1}{8} \times \frac{1}{4}$

$\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$

Solution $\frac{1}{32}$ pound

4 Student volunteers are getting ready to hand out programs at a talent show. Leah and Tomas are each given $\frac{1}{2}$ of a stack of programs to hand out. Leah divides her $\frac{1}{2}$ equally among herself and 2 friends. What fraction of the original stack of programs do Leah and her 2 friends each have? Show your work.

Students may use area models, equations, or some other method to find $\frac{1}{2} \div 3$.

Solution $\frac{1}{6}$ of the original stack

5 Look at problem 4. If Tomas divides his stack of programs between himself and his 3 friends, what fraction of the original stack will each of his friends have? Write a division equation to represent and solve the problem.

Possible work:

$\frac{1}{2} \div 4 = \frac{1}{4} \times \frac{1}{2}$

$= \frac{1}{8}$

Solution $\frac{1}{8}$ of the original stack

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Speaking/Writing Read ***Apply It*** problem 11 aloud. Ask students to identify context clues for the word *submarine* (sandwich). Have them label the picture in the Student Worktext "submarine sandwich." Then say: *Dylan cut the sandwich into sixths.* Ask: *What fraction shows the size of each piece?* $\left[\frac{1}{6}\right]$

Ask: *How many pieces does Dylan get when he divides 3 sandwiches into pieces of size* $\frac{1}{6}$*?*

Have students complete the sentence frame:

- *Dylan gets _____ pieces.*

Levels 2–4

Listening/Speaking Have students chorally read ***Apply It*** problem 11. Say: *Dylan cut the sandwich into sixths.* Ask: *What fraction shows the size of each piece?* $\left[\frac{1}{6}\right]$

Ask: *How can you represent the action with numbers?* Provide support as needed to help students state the division. Then ask: *How many pieces does Dylan get when he divides 3 sandwiches into pieces of size* $\frac{1}{6}$*?*

Have students complete the sentence frame:

- *Dylan gets _____ pieces.*

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 11 with a partner. Encourage them to discuss how they can represent the information they know and use that model to find the solution to the problem.

Have students use complete sentences to explain the model. Provide the following sentence starters:

- *My partner and I know that _____.*
- *First, we show _____.*
- *Then we show _____.*
- *Dylan gets _____.*

LESSON 24

SESSION 3 Develop

Purpose In this session students solve a problem that requires dividing 2 by $\frac{1}{4}$. Students model the numbers in the word problem either on paper or with manipulatives to develop strategies for solving word problems that involve dividing a whole number by a unit fraction.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *Fraction Bars* (6 fifths bars)

Why Support students' facility with writing whole numbers as fractions.

How Have students use fraction bars to model the whole numbers 1–6 as numbers of fifths and then write each whole number as a fraction.

Use fraction bars to show each whole number as fifths.

$3 = \frac{\square}{5}$ $5 = \frac{\square}{5}$

$1 = \frac{\square}{5}$ $6 = \frac{\square}{5}$

Solutions

$3 = \frac{15}{5}$ $5 = \frac{25}{5}$

$1 = \frac{5}{5}$ $6 = \frac{30}{5}$

Look for Models should show the appropriate number of fifths fraction bars for that number of wholes.

Develop Language

Why Reinforce understanding of the word *split* in the context of fractions.

How Write the word *split* on the board. Explain that to split means to divide, separate, or break into parts. Have students circle the word in the *Try It* problem. Ask: *What does Alex do when he splits the dough?*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the amount of dough Alex makes and the amount of dough each loaf needs.

Ask *What size parts does Alex split the dough into?*

LESSON 24 SESSION 3 ● ● ● ○

Develop Dividing a Whole Number by a Unit Fraction

Read and try to solve the problem below.

Alex makes 2 pounds of bread dough. He splits the dough into $\frac{1}{4}$-pound loaves before baking them in the oven. How many loaves does he make?

TRY IT

Possible student work:

Sample A

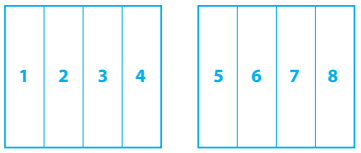
There are four $\frac{1}{4}$-pound loaves in each whole pound.

$2 \times 4 = 8$

Alex made 8 loaves.

Sample B

2 pounds are divided into fourths.



$2 \div \frac{1}{4} = 8$

Alex made 8 loaves.

Math Toolkit

- fraction tiles
- fraction bars
- fraction models
- number lines
- grid paper
- index cards

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: The strategy I used to find the answer was . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *fourths* and *pounds* as they discuss their work.

Support as needed with questions such as:

- *How did you get started?*
- *Why did you choose that strategy?*

Common Misconception Look for students who are not comfortable counting the number of equal parts and instead count the vertical lines dividing their model into equal parts, arriving at a solution of more than 8 loaves. As students present solutions, have them point out and count the loaves.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing wholes and fourths
- drawings to represent the problem
- number lines marked to show wholes and fourths
- division or multiplication equations showing that Alex makes 8 loaves

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does your model show the whole amount of dough? the equal parts each whole is divided into?*

Listen for Students should recognize that accurate representations show 2 wholes, each representing 1 pound of dough, divided into 4 equal parts, each representing one $\frac{1}{4}$-pound loaf.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the 2 pounds of dough
- the equal parts each pound is divided into

Ask *In the second* Model It, *the number 2 is rewritten as the equivalent fraction $\frac{8}{4}$. How does the number line in the first* Model It *show 2 is equal to $\frac{8}{4}$?*

Listen for The number line is divided into 8 fourths between 0 and 2.

For a number line model, prompt students to identify the greatest number on the number line and the number of divisions.

- *How is the number line divided?*
- *Why is it done that way?*

For the equation model, prompt students to compare the two equations and tell how they are alike and how they are different.

- *Why was 4 chosen as the common denominator?*
- *How is it helpful to think of 2 as $\frac{8}{4}$?*

Explore different ways to understand how to divide a whole number by a unit fraction in order to solve word problems.

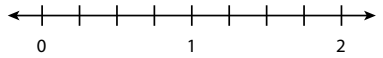
Alex makes 2 pounds of bread dough. He splits the dough into $\frac{1}{4}$-pound loaves before baking them in the oven. How many loaves does he make?

MODEL IT

You can use a number line to help understand the problem.

Draw a number line and label it to show the 2 pounds of bread dough.

Mark the number line to divide each whole into fourths.



MODEL IT

You can use what you know about equations, equivalent fractions, and common denominators to solve the problem.

The equation $2 \div \frac{1}{4} = n$ models the problem with n being the number of loaves Alex makes.

Write the numbers in the equation with a common denominator.

$\frac{8}{4} \div \frac{1}{4} = n$

Now you can divide $\frac{8}{4}$ into equal groups of $\frac{1}{4}$.

Deepen Understanding

Equivalent Fractions

SMP 8 Use repeated reasoning.

When discussing the equation method shown in the second *Model It*, prompt students to consider where else they have used renaming as a strategy.

Ask *In your own words, how do you describe the strategy shown for dividing a whole number by a unit fraction?*

Listen for You rename the whole number, 2, as the fraction $\frac{8}{4}$ so that the dividend and the divisor describe the same-size parts of a whole. You can then think of dividing 8 fourths into equal groups of 1 fourth.

Ask *Where else have you used a strategy of renaming? What are examples?*

Listen for Students should recognize that they have used a renaming strategy in many situations. Examples may include regrouping ones as tens, or tens as hundreds, to add whole numbers; writing decimals in tenths as decimals in hundredths to subtract decimals; and writing fractions with common denominators to add fractions.

CONNECT IT

- Remind students that one thing that is alike about all the representations is that they show dividing 2 wholes into same-size fractional parts.
- Explain that on this page students will use what they know about equivalent fractions and the relationship between division and multiplication to explain and check the quotient.

Monitor and Confirm

1 – 3 Check for understanding that:

- 2 is the number of whole pounds of dough
- 4 fourths are in one whole; 8 fourths are in two wholes

Support Whole Class Discussion

4 – 6 Tell students that these problems show how to use equivalent fractions to find the number of $\frac{1}{4}$s in 2. In problem 7, they will think about how to use multiplication to check the answer.

Be sure students understand that the reason for writing 2 as a fraction with a denominator of 4 is so both the dividend and the divisor are expressed as a number of equal-size parts of 1 whole. The numerator shows the number of those parts in 2 wholes.

Ask *How do the strategies discussed in problems 4 and 6 help you identify the quotient* $2 \div \frac{1}{4}$*?*

Listen for They help you think about how many $\frac{1}{4}$ parts are in 2 wholes. They both show that 2 has 8 one-fourth parts.

7 Look for the idea that to check the quotient of a division equation you can use the inverse relationship between multiplication and division to write the related multiplication equation.

8 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to divide a whole number by a unit fraction.

1 In the first **Model It** number line, how are the 2 pounds of bread dough represented?

The pounds of dough are the numbered points 1 and 2.

2 How are the $\frac{1}{4}$-pound loaves represented on the number line?

Each whole number, or pound, is divided into fourths using tick marks.

3 How many fourths are in one whole? **4** In two wholes? **8**

4 Look at the second **Model It**. How was the equation $2 \div \frac{1}{4} = n$ changed to an equation involving fractions with common denominators?

Possible answer: The number 2 was changed to a fraction in fourths, so it has the same denominator as $\frac{1}{4}$. $2 = \frac{8}{4}$.

5 How many groups of $\frac{1}{4}$ are in $\frac{8}{4}$? What is $\frac{8}{4} \div \frac{1}{4}$? Explain.

8, 8; Possible explanation: There are eight $\frac{1}{4}$s in $\frac{8}{4}$ because $8 \times \frac{1}{4} = \frac{8}{4}$. So, $\frac{8}{4} \div \frac{1}{4} = 8$.

6 How many loaves does Alex make? How are the first **Model It** and second **Model It** alike in showing how to find the solution?

8 loaves; Possible explanation: The first Model It shows eight $\frac{1}{4}$ tick marks in 2 on the number line. The second Model It shows eight $\frac{1}{4}$s in $\frac{8}{4}$ using an equation.

7 What multiplication equation can you write to check your answer to $2 \div \frac{1}{4}$? Explain.

$8 \times \frac{1}{4} = \frac{8}{4}$ or 2. Because $8 \times \frac{1}{4} = 2$, $2 \div \frac{1}{4} = 8$.

8 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for dividing a whole number by a unit fraction? Explain.

Some students may say they like using a visual model to reason through the problem. Others may choose using equations because they know how to use the relationship between multiplication and division.

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Hands-On Activity

Act out the problem.

If . . . *students are unsure about using multiplication to check division,*

Then . . . *use this activity to connect a concrete model to both equations.*

Materials For each student: modeling clay (2 equal-sized portions)

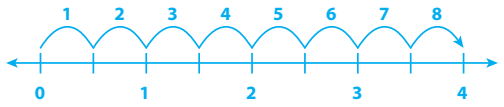
- Tell students they will act out the problem of splitting up the bread dough. Explain that each portion of modeling clay represents 1 pound of dough.
- Guide students to divide each of the 2 pounds of bread dough into $\frac{1}{4}$-pound parts for the individual loaves. Discuss the division represented, $2 \div \frac{1}{4}$. Have students identify the quotient and write the division equation. $\left[2 \div \frac{1}{4} = 8\right]$
- Remind students division and multiplication are inverse operations—that multiplication undoes division. Have students put the $\frac{1}{4}$-pound portions back together to make whole pounds, reforming the original 2 portions. Discuss the multiplication represented, $8 \times \frac{1}{4}$. Have students write the multiplication equation that is related to the division equation above. $\left[8 \times \frac{1}{4} = 2\right]$

APPLY IT

Use what you just learned to solve these problems.

9 Stacy has 4 sheets of paper to make cards. Each card requires $\frac{1}{2}$ sheet of paper. How many cards can Stacy make? Draw a model and write a division equation to represent and solve the problem.

Possible student work using a number line:



Solution $4 \div \frac{1}{2} = 8$; 8 cards

10 Look at problem 9 above. Which multiplication expressions can be used to represent the situation or check the division equation?

Ⓐ 8×2

Ⓑ $4 \times \frac{1}{2}$

Ⓒ $16 \times \frac{1}{2}$

Ⓓ $8 \times \frac{1}{2}$

Ⓔ 4×2

11 Dylan makes 3 submarine sandwiches. He cuts each sandwich into sixths to share. He stacks all the sandwich pieces on a plate. How many sandwich pieces does Dylan stack on the plate? Show your work.

Possible student work:

$3 \div \frac{1}{6} = p$

Write an equation with a common denominator.

$\frac{18}{6} \div \frac{1}{6} = 18$

Solution Dylan stacks 18 sandwich pieces on the plate.

APPLY IT

For all problems, encourage students to draw a model to support their thinking. Allow some leeway in precision; dividing fraction models into equal parts can be difficult.

9 $4 \div \frac{1}{2} = 8$; 8 cards; See possible model on the Student Worktext page. Students may also draw 4 rectangles, each divided in half, with the halves numbered consecutively from 1 to 8. They may also write the division equation $\frac{8}{2} \div \frac{1}{2} = 8$, reasoning there are 8 groups of $\frac{1}{2}$ in $\frac{8}{2}$.

10 **D**; The multiplication expression $8 \times \frac{1}{2}$ can be used to check the equation $4 \div \frac{1}{2} = 8$.
E; 2 cards can be made from each sheet of paper. There are 4 sheets, so 4×2 represents the situation.

Close: Exit Ticket

11 Dylan stacks 18 sandwich pieces on the plate; Students may write the division equation $\frac{18}{6} \div \frac{1}{6} = 18$, reasoning there are 18 groups of $\frac{1}{6}$ in $\frac{18}{6}$. They may also draw a fraction model, such as 3 rectangles, each divided in sixths.

Students' solutions should indicate understanding of:

- 3 wholes divided into equal parts of size $\frac{1}{6}$
- There are 6 one sixths in each whole, so there are 3×6, or 18 one sixths in 3 wholes.

Error Alert **If** students find a quotient other than 18, **then** have them write their division equation and then use multiplication to check their answer. For example, if they find the quotient to be $\frac{1}{2}$, have them write $3 \div \frac{1}{6} = \frac{1}{2}$ (the incorrect division equation) and then use $\frac{1}{2} \times \frac{1}{6}$ to check. Point out that because $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$, not 3, the quotient for $3 \div \frac{1}{6}$ is not $\frac{1}{2}$. Review that $3 \div \frac{1}{6}$ means to divide 3 wholes into parts, each of which is $\frac{1}{6}$ of 1 whole.

Solutions

1. $2 \times 5 = 10$; Each whole is divided into 5 fifths, so 2 wholes show 2×5, or 10 fifths.
Basic

2. 45 times; Students may write and solve the division equation $9 \div \frac{1}{5} = n$ or the multiplication expression $9 \times 5 = n$. They may also extend the number line in the example to show the number of $\frac{1}{5}$s in 9.
Medium

3. See possible explanation on the student page. Students may also explain how they counted the number of $\frac{1}{5}$s in a model they drew that showed 9 wholes.
Medium

Name: _____ LESSON 24 SESSION 3

Practice Dividing a Whole Number by a Unit Fraction

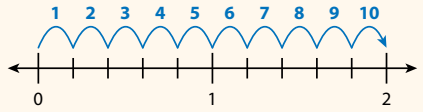
Study the Example showing one way to solve a word problem involving dividing a whole number by a fraction. Then solve problems 1–6.

EXAMPLE

Darius walks dogs at an animal shelter. He walks each dog for $\frac{1}{5}$ hour. He walks the dogs one at a time. How many dogs can Darius walk in 2 hours?

Find $2 \div \frac{1}{5}$.

The number line shows two hours. Each hour is divided into fifths.



There are 10 fifths in 2.

$2 \div \frac{1}{5} = 10$

Darius can walk 10 dogs in 2 hours.

1. What multiplication equation could you write to solve the Example?
$2 \times 5 = 10$

2. Use the information from the Example. In one month, Darius spends 9 hours walking dogs. How many times does he walk a dog in one month?
45 times

3. Explain how you got your answer to problem 2.
Answers will vary. Possible explanation: I know Darius can do 5 dog walks in 1 hour, so in 9 hours he can do 9×5, or 45, dog walks.

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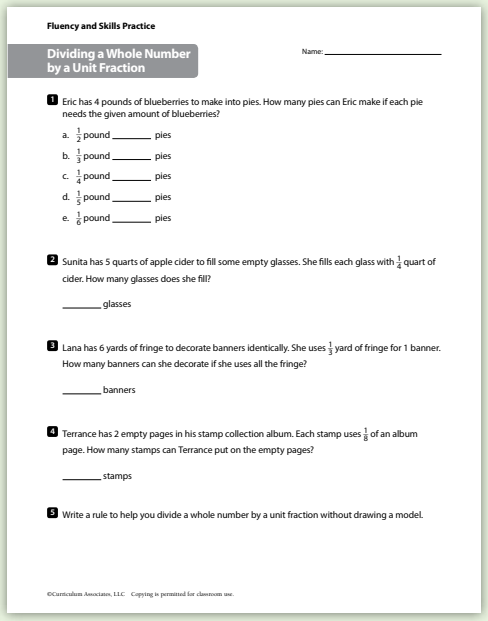
Fluency & Skills Practice Teacher Toolbox

Assign *Dividing a Whole Number by a Unit Fraction*

In this activity students divide whole numbers by unit fractions. This skill can be used in everyday activities such as cooking. For example, suppose a person is making cheese-stuffed potatoes. Each potato requires $\frac{1}{4}$ cup of milk, and the cook has 2 cups of milk. By dividing 2 by $\frac{1}{4}$, the cook finds that he or she has enough milk to make 8 cheese-stuffed potatoes.



Fluency and Skills Practice

Dividing a Whole Number by a Unit Fraction

Name: _____

1. Eric has 4 pounds of blueberries to make into pies. How many pies can Eric make if each pie needs the given amount of blueberries?
   a. $\frac{1}{2}$ pound _____ pies
   b. $\frac{1}{3}$ pound _____ pies
   c. $\frac{1}{4}$ pound _____ pies
   d. $\frac{1}{5}$ pound _____ pies
   e. $\frac{1}{6}$ pound _____ pies

2. Sunita has 5 quarts of apple cider to fill some empty glasses. She fills each glass with $\frac{1}{4}$ quart of cider. How many glasses does she fill?
   _____ glasses

3. Lana has 6 yards of fringe to decorate banners identically. She uses $\frac{1}{3}$ yard of fringe for 1 banner. How many banners can she decorate if she uses all the fringe?
   _____ banners

4. Terrance has 2 empty pages in his stamp collection album. Each stamp uses $\frac{1}{8}$ of an album page. How many stamps can Terrance put on the empty pages?
   _____ stamps

5. Write a rule to help you divide a whole number by a unit fraction without drawing a model.

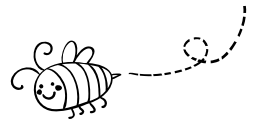
4 20 pieces of tape; Students may use the expression $5 \div \frac{1}{4}$ or $\frac{20}{4} \div \frac{1}{4}$ to solve the problem. They may also draw a fraction model showing 5 wholes divided into fourths.
Medium

5 14 vases; See possible model and division equation on the student page. Students may also write the division equation $\frac{14}{2} \div \frac{1}{2} = 14$.
Medium

6 14 vases; Students should show a different way of solving the problem than in problem 5. They may use the equation $14 \times \frac{1}{2} = 7$ to check their answer.
Challenge

4 Mrs. Wing will tape up posters made by her students on the wall. She cuts tape into $\frac{1}{4}$-foot pieces. How many $\frac{1}{4}$-foot pieces can she cut from 5 feet of tape? Show your work.

Students may use area models, equations, or some other method to find $5 \div \frac{1}{4}$.



Solution **20 pieces**

5 Taylor is helping decorate tables with flowers for a graduation celebration. She has 7 bunches of tulips. She will put $\frac{1}{2}$ of each bunch in a vase. How many vases does she need? Draw a model and write a division equation to represent and solve the problem.

$7 \div \frac{1}{2} = 14$

Solution **14 vases**

6 Look at how you solved problem 5. Use a different way to solve the problem and show how a multiplication equation can be used to check the answer.

Students may use area models, equations, or some other method to find $7 \div \frac{1}{2}$. It should be different than the way shown in problem 5. Students may use the equation $14 \times \frac{1}{2} = 7$ to check their answer.

Solution **14 vases**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Speaking/Listening Have students chorally read ***Apply It*** problem 3. Ask them to state what is known and unknown about the problem:

I know: Devonte uses _____ for each event. Devonte fills _____ of one sheet of paper.

I need to know: how many _____ Devonte makes notes for.

Have partners make a model and write a division expression.

When complete, ask: *Did Barry get the correct answer?*

Levels 2–4

Speaking/Listening Have students chorally read ***Apply It*** problem 3. Ask them to state what is known and unknown about the problem:

I know: Devonte uses _____ and _____.

I need to know: _____.

Have partners make a model and write a division expression.

When complete, ask: *Did Barry get the correct answer? Why?*

Levels 3–5

Reading/Writing Have students chorally read ***Apply It*** problem 3. In partners, have students discuss what is known and unknown about the problem. Ask partners to make a model to show the number of events Devonte makes notes for.

Have students use sequence terms (*first*, *then*, *last*) to write the steps they took to write a division expression.

Then have them compare their answer with another group. When complete, have students work together to explain how Barry got D as an answer.

LESSON 24

SESSION 4 Refine

Purpose In this session students solve word problems involving dividing a whole number by a unit fraction or a unit fraction by a whole number. They then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of solving a word problem that involves dividing a unit fraction by a whole number.

How Have students draw a visual model and write a division equation to solve the problem.

Draw a visual model and write a division equation to solve.

A pitcher contains $\frac{1}{5}$ gallon of juice. If 5 friends share the juice equally, how much does each person get?

Solution

$\frac{1}{25}$ gallon;
$\frac{1}{5} \div 5 = \frac{1}{25}$

Look for Models show $\frac{1}{5}$ divided into 5 equal parts of a whole divided into 25 equal parts.

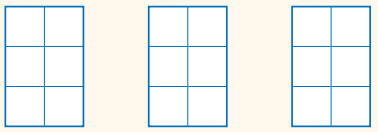
Refine Dividing Unit Fractions in Word Problems

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Sierra has a photo album with 3 empty pages. Each photo uses $\frac{1}{6}$ of an album page. How many photos can Sierra put on the empty pages?

Look at how you could show your work using rectangles.



6 photos will fit on each of the 3 pages.

Solution 18 photos

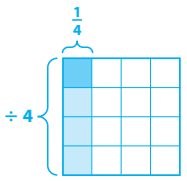
The student used a model to visualize the problem.

PAIR/SHARE
What related equations can you write to represent the problem?

APPLY IT

1 Corrine picked $\frac{1}{4}$ gallon of blackberries. She poured equal amounts of berries into 4 containers. What fraction of a gallon is in each container? Show your work.

Possible student work using a model:



Solution $\frac{1}{16}$ gallon

Can you draw a model to help understand the problem?

PAIR/SHARE
How will the answer compare to $\frac{1}{4}$ gallon?

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
25	have thought that the answer is the number of parts in the whole after dividing.	Have students draw an area model or a number line and identify $\frac{1}{5}$. Then, have students divide the fifth into 5 equal-sized pieces. Point out that each friend gets one of these pieces, which is $\frac{1}{5}$ of $\frac{1}{5}$, or $\frac{1}{5} \times \frac{1}{5}$.
1	have multiplied instead of dividing.	Write "$\frac{1}{5} \times 5$" and "$\frac{1}{5} \div 5$" and discuss what each expression means in the context of the problem. $\frac{1}{5} \times 5$ could mean "$\frac{1}{5}$ of 5 friends" or "each of 5 friends has $\frac{1}{5}$ gallon." These do not describe the problem situation. $\frac{1}{5} \div 5$ means "$\frac{1}{5}$ gallon divided (or shared) among 5 friends," which describes the problem situation exactly.

EXAMPLE

18 photos; the fraction model shown is one way to solve the problem. Students could also solve the problem using the expression $3 \div \frac{1}{6}$ or 6×3. They may also use a different kind of fraction model, such as a number line.

Look for You find the number of $\frac{1}{6}$s in 3.

APPLY IT

1. $\frac{1}{16}$ gallon; Students could solve the problem by drawing a model divided to show fourths one way and divided into 4 equal parts the other way, with the overlapping shading showing $\frac{1}{16}$.
   DOK 2

   Look for Find one of 4 equal parts of $\frac{1}{4}$ gallon.

2. $\frac{1}{10}$ of the drive; Students could solve the problem by representing it with the equation $\frac{1}{2} \div 5 = f$, using reasoning to think of it as finding $\frac{1}{5}$ of $\frac{1}{2}$, or $\frac{1}{5} \times \frac{1}{2} = f$.
   DOK 2

   Look for Find one of 5 equal parts of $\frac{1}{2}$.

3. **B**; Students could solve the problem by reasoning that the starting amount, 2 sides, is divided into groups that are $\frac{1}{8}$ page in size. Explain why the other two answer choices are not correct:

   A is not correct because it represents 2 groups that are each $\frac{1}{8}$ of a page.

   C is not correct because it represents $\frac{1}{8}$ of 2 pages.

   DOK 3

2 Cooper's USB drive is $\frac{1}{2}$ full with 5 video files. Each video file is the same size. What fraction of the USB drive does 1 video file use? Show your work.

How could I represent this problem using an equation?

Possible student work using an equation:

$\frac{1}{2} \div 5 = f$

f is $\frac{1}{5}$ of $\frac{1}{2}$.

$\frac{1}{5} \times \frac{1}{2} = f$

$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

Solution $\frac{1}{10}$ **of the drive**

PAIR/SHARE
How can you check your answer?

3 Devonte is studying for a history test. He uses $\frac{1}{8}$ of a side of one sheet of paper to write notes for each historical event. He fills 2 full sides of one sheet of paper. Which expression could be used to find how many events Devonte makes notes for?

Is this problem like one you have seen before?

Ⓐ $2 \times \frac{1}{8}$

Ⓑ $2 \div \frac{1}{8}$

Ⓒ $\frac{1}{8} \times 2$

Ⓓ $\frac{1}{8} \div 2$

Barry chose Ⓓ as the correct answer. How did he get that answer?

Possible answer: Barry knew that he should write a division expression with 2 and $\frac{1}{8}$, but he thought that the order of the numbers didn't matter in division.

PAIR/SHARE
Does Barry's answer make sense?

APPLY IT

4 **A**; Each pound has two halves, so each pound can make 2 containers of applesauce. Elise has 6 pounds. $6 \times 2 = 12$.
DOK 2

5 9; Divide each mile shown on the number line in thirds, to show the distance each team member will run. Count the number of $\frac{1}{3}$s.
DOK 2

6 Each piece is $\frac{1}{18}$ of the cake; Represent cutting the $\frac{1}{3}$ cake into 6 equal pieces with the expression $\frac{1}{3} \div 6$. Think of that as finding $\frac{1}{6}$ of $\frac{1}{3}$ and write it as the expression $\frac{1}{6} \times \frac{1}{3}$.
DOK 2

Error Alert Students may write the whole number 6 instead of the fraction $\frac{1}{6}$ when writing the related expression for $\frac{1}{3} \div 6$; writing instead $6 \times \frac{1}{3}$ and showing the answer $\frac{6}{3}$.

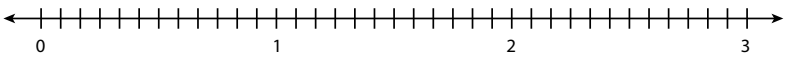
4 Elise picks 6 pounds of apples. She uses $\frac{1}{2}$ pound of apples to make 1 container of applesauce. How many containers of applesauce can Elise make with all the apples?

Ⓐ 12 containers
Ⓑ $6\frac{1}{2}$ containers
Ⓒ $5\frac{1}{2}$ containers
Ⓓ 3 containers

5 Students are running in a relay race. Each team will run a total of 3 miles. Each member of a team will run $\frac{1}{3}$ mile. How many students will a team need to complete the race? Circle the correct number below.

$\frac{1}{9}$ 3 9 12 36

You may use the number line to help find your answer.



6 Tanya has $\frac{1}{3}$ of a cake left over from a party. She cuts the leftover cake into 6 equal pieces to store in the freezer. What fraction of the original cake is each piece? Show your work.

Possible student work:

$\frac{1}{3} \div 6 = \frac{1}{6} \times \frac{1}{3}$
$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$

Solution Each piece is $\frac{1}{18}$ of the cake.

Differentiated Instruction

RETEACH

Hands-On Activity
Use fraction circles to divide a whole number by a unit fraction.

Students struggling with understanding dividing by a unit fraction

Will benefit from additional work with concrete representations

Materials For each student: 1 set of fraction circles or tiles

- Distribute materials. Pose the following problem: *A painter can paint $\frac{1}{3}$ of a room in an hour. How many painters are needed to paint 6 rooms in an hour?*
- Have students trace the whole fraction circle six times to represent 6 rooms. Ask: *How much of a room can 1 painter paint in 1 hour?* [$\frac{1}{3}$ of a room] Ask: *How many painters are needed to paint an entire room in 1 hour?* [3 painters] Have students place one-third fraction pieces in one circle to show this.
- Have students divide the remaining whole circles into thirds to show the painters needed for each room. Ask students to write a division equation for the problem and a solution. [$6 \div \frac{1}{3} = 18$; 18 painters] Repeat the activity for other numbers of rooms, such as 3 or 5.

EXTEND

Challenge Activity
Write and solve equations.

Students who have achieved proficiency

Will benefit from deepening understanding of division with unit fractions

- Challenge students to write a division equation showing a whole number divided by a unit fraction that has a quotient of 12. [Possible answer: $3 \div \frac{1}{4} = 12$]
- Challenge students to solve a division equation involving a mixed number. Ask them to solve $4\frac{1}{2} \div 3 = n$. [$\frac{3}{2}$]

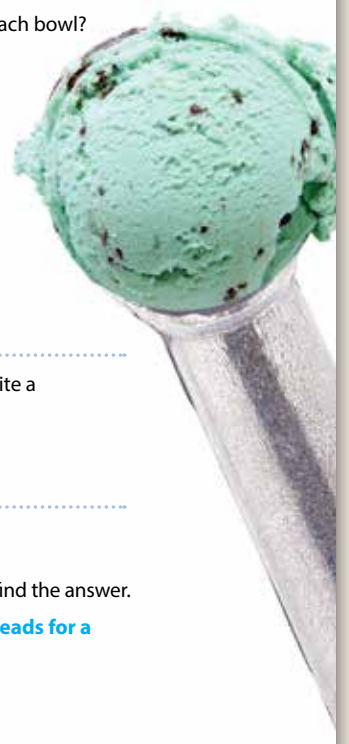
7 4, 8, 12, 16; See completed table on the Student Worktext page. Each bow requires $\frac{1}{4}$ yard of ribbon. Each yard makes 4 bows.
DOK 2

8 **Part A**
Ted puts $\frac{1}{24}$ gallon of ice cream in each bowl. See possible model on the Student Worktext page. $\frac{1}{4}$ of $\frac{1}{6}$ is $\frac{1}{24}$.
Part B
$\frac{1}{6} \div 4 = \frac{1}{24}$; $\frac{1}{24} \times 4 = \frac{4}{24}$, or $\frac{1}{6}$.
DOK 2

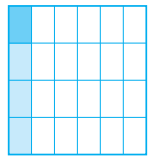
7 Marina has a pattern to make bows that requires $\frac{1}{4}$ yard of ribbon for each bow. Fill in the table to show how many bows she can make from a given length of ribbon.

Ribbon Length (yards)	Number of Bows
1	4
2	8
3	12
4	16

8 **Part A** Ted serves $\frac{1}{6}$ gallon of ice cream. He puts an equal amount of ice cream in each of 4 bowls. How many gallons of ice cream does Ted put in each bowl? Use a visual model to support your answer.



Possible student model:



$\frac{1}{4}$ of $\frac{1}{6}$ is $\frac{1}{24}$.

Solution Ted puts $\frac{1}{24}$ gallon of ice cream in each bowl.

Part B Write a division equation to represent this situation. Then write a multiplication equation you can use to check your answer.

Solution $\frac{1}{6} \div 4 = \frac{1}{24}$ and $\frac{1}{24} \times 4 = \frac{1}{6}$

9 MATH JOURNAL
Write a word problem represented by $\frac{1}{5} \div 4$. Explain or show how to find the answer.
Possible problem: Willa uses $\frac{1}{5}$ lb of clay to make 4 equal-sized beads for a necklace. How heavy is each bead?
Possible student work: $\frac{1}{5} \div 4 = \frac{1}{4} \times \frac{1}{5}$; Each bead is $\frac{1}{20}$ lb.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

REINFORCE

Problems 4–9
Divide unit fractions.

All students will benefit from additional work with dividing unit fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL
Student responses should indicate understanding of problem situations that require dividing a unit fraction by a whole number, such as dividing a fractional measure of a quantity into equal portions, as well as an understanding of how to use a visual model or equations to solve a problem of this type.

Error Alert **If** students confuse the dividend and the divisor when representing the expression with a word problem, **then** have them draw a visual model for the expression and use it to make up a word problem. Have them identify the numbers that are the dividend and divisor in each representation.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener page.