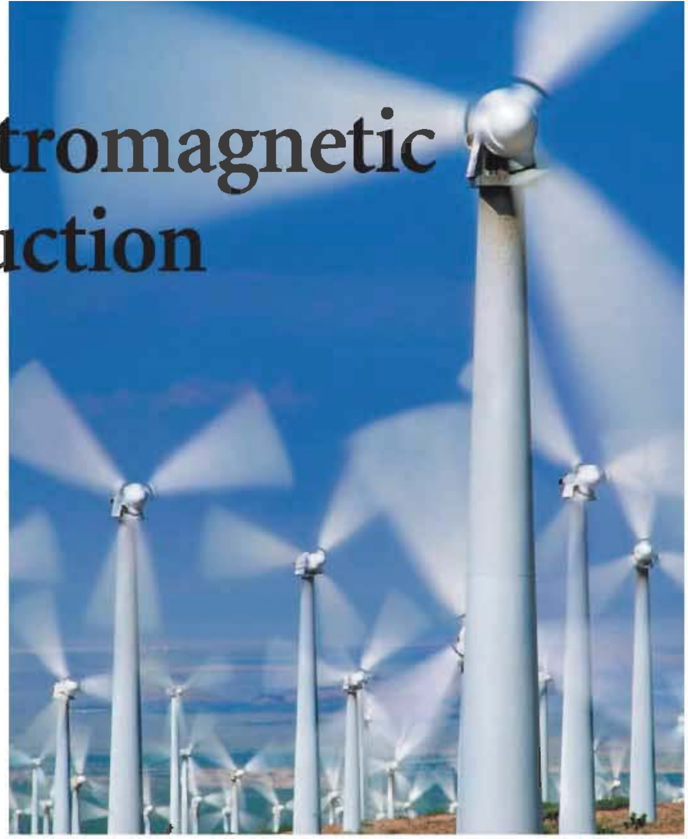
34 Electromagnetic Induction



Electromagnetic induction is the scientific principle that underlies many modern technologies, from the generation of electricity to communications and data storage.

▶ Looking Ahead

The goal of Chapter 34 is to understand and apply electromagnetic induction.
In this chapter you will learn to:

- Calculate induced current.
- Calculate magnetic flux.
- Use Lenz's law and Faraday's law to determine the direction and size of induced currents.
- Understand how induced electric and magnetic fields lead to electromagnetic waves.
- Analyze circuits with inductors.

◀ Looking Back

This chapter will join together ideas about magnetic fields and electric potential. Please review:

- Section 11.3 The vector dot product.
- Section 30.2 Sources of electric potential.
- Sections 33.4–33.8 Magnetic fields and magnetic forces.

What do windmills, metal detectors, video recorders, computer hard disks, and cell phones have in common? Surprisingly, these diverse technologies all stem from a single scientific principle, electromagnetic induction. **Electromagnetic induction** is the process of generating an electric current by varying the magnetic field that passes through a circuit.

The many applications of electromagnetic induction make it an important topic for study. More fundamentally, electromagnetic induction establishes an important link between electricity and magnetism, a link with important implications for understanding light as an electromagnetic wave.

Electromagnetic induction is a subtle topic, so we will build up to it gradually. We'll first examine different aspects of induction and become familiar with its basic characteristics. Section 34.5 will then introduce Faraday's law, a new law of physics not derivable from any previous laws you have studied. The remainder of the chapter will explore its implications and applications.

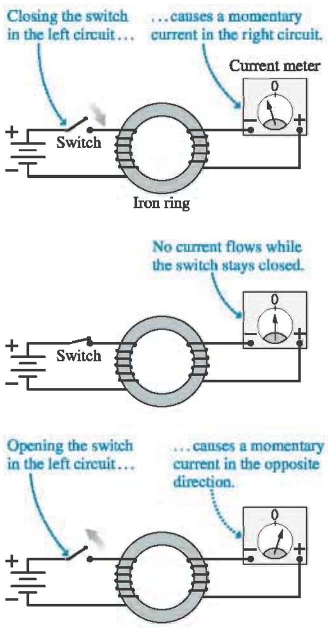
34.1 Induced Currents

Oersted's 1820 discovery that a current creates a magnetic field generated enormous excitement. Dozens of scientists immediately began to explore the implications of this discovery. One question they hoped to answer was whether the converse of Oersted's discovery was true. That is, can a magnet be used to create a current? Many experiments were reported in which wires and coils were placed in or around magnets, but no one was able to generate a current.

The breakthrough came in 1831. In America, science teacher Joseph Henry was the first to discover how to produce a current from magnetism, a process we now call *electromagnetic induction*. But Henry had no time for follow-up studies, and he was not able to publish his discovery until later. At about the same time, in England, Michael Faraday made the same discovery and immediately published his findings. You met Faraday in Chapter 26 as the inventor of the concept of a *field*.

Credit in science usually goes to the first to publish, so today we study *Faraday's law* rather than *Henry's law*. The situation is not entirely unjust. Even if Faraday did not have priority of discovery, it was Faraday who established the properties of electromagnetic induction and realized he had discovered a new law of nature.

FIGURE 34.1 Faraday's discovery of electromagnetic induction.



Faraday's Discovery

Faraday's 1831 discovery, like Oersted's, was a happy combination of an unplanned event and a mind that was ready to recognize its significance. Faraday was experimenting with two coils of wire wrapped around an iron ring, as shown in **FIGURE 34.1**. He had hoped that the magnetic field generated in the coil on the left would induce a magnetic field in the iron, and that the magnetic field in the iron might then somehow create a current in the circuit on the right.

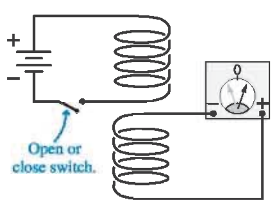
Like all his previous attempts, this technique failed to generate a current. But Faraday happened to notice that the needle of the current meter jumped ever so slightly at the instant when he closed the switch in the circuit on the left. After the switch was closed, the needle immediately returned to zero. The needle again jumped when he later opened the switch, but this time in the opposite direction. Faraday recognized that the motion of the needle indicated a very slight current in the circuit on the right. But the effect happened only during the very brief interval when the current on the left was starting or stopping, not while it was steady.

Faraday applied his mental picture of field lines to this discovery. The current on the left first magnetizes the iron ring, then the field of the iron ring passes through the coil on the right. Faraday's observation that the current-meter needle jumped only when the switch was opened and closed suggested that a current was generated only if the magnetic field was *changing* as it passed through the coil. This would explain why all the previous attempts to generate a current were unsuccessful: they had used only steady, unchanging magnetic fields.

Faraday set out to test this hypothesis. If the critical issue was *changing* the magnetic field through the coil, then the iron ring should not be necessary. That is, any method that changes the magnetic field should work. Faraday began a series of experiments to find out if this was true.

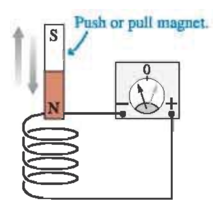
Faraday investigates electromagnetic induction

Faraday placed one coil directly above the other, without the iron ring. There was no current in the lower circuit while the switch was in the closed position, but a momentary current appeared whenever the switch was opened or closed.



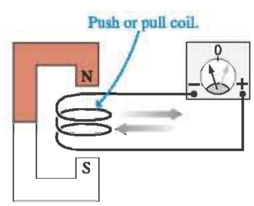
Opening or closing the switch creates a momentary current.

He pushed a bar magnet into a coil of wire. This action caused a momentary deflection of the current-meter needle, although *holding* the magnet inside the coil had no effect. A quick withdrawal of the magnet deflected the needle in the other direction.



Pushing the magnet into the coil or pulling it out creates a momentary current.

Must the magnet move? Faraday created a momentary current by rapidly pulling a coil of wire out of a magnetic field. Pushing the coil *into* the magnet caused the needle to deflect in the opposite direction.



Pushing the coil into the magnet or pulling it out creates a momentary current.

To summarize

Faraday found that there is a current in a coil of wire if and only if the magnetic field passing through the coil is *changing*. This is an informal statement of what we'll soon call *Faraday's law*.

It makes no difference what causes the magnetic field to change: current stopping or starting in a nearby circuit, moving a magnet through the coil, or moving the coil in and out of a magnet. The effect is the same in all cases. There is no current if the field through the coil is not changing, so it's not the magnetic field itself that is responsible for the current but, instead, it is the *changing of the magnetic field*.

The current in a circuit due to a changing magnetic field is called an **induced current.** Opening the switch or moving the magnet *induces* a current in a nearby circuit. An induced current is not caused by a battery. It is a completely new way to generate a current, and we will have to discover how it is similar to and how it is different from currents we have studied previously.

Magnetic data storage, whether it's the magnetic stripe on a credit card or a 20 GB hard disk, encodes information in a pattern of alternating magnetic fields. When these fields move past a small *pick-up coil,* the changing magnetic field creates an induced current in the coil. This current is amplified into a sequence of voltage pulses that represent the 0s and 1s of digital data. Magnetic data storage is just one of countless applications of electromagnetic induction.

34.2 Motional emf

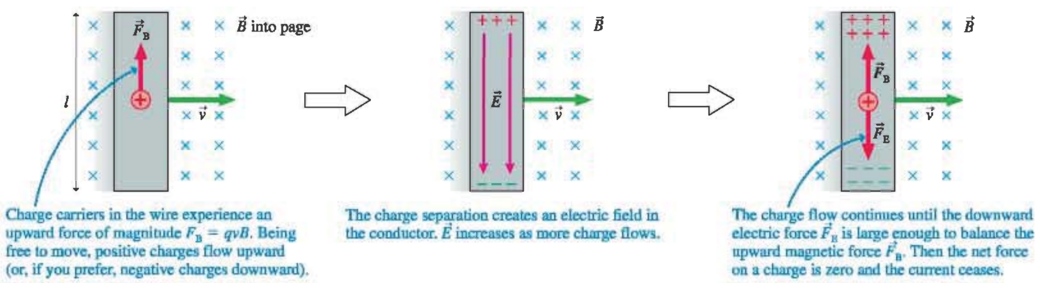
An induced current can be created two different ways:

1. By changing the size or orientation of a circuit in a stationary magnetic field, or
2. By changing the magnetic field through a stationary circuit.

Although the effects are the same, the causes turn out to be different. We'll start our investigation of electromagnetic induction by looking at situations in which the magnetic field is fixed while the circuit moves or changes.

To begin, consider a conductor of length l that moves with velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$, as shown in FIGURE 34.2. The charge carriers inside the wire also move with velocity $\vec{v}$, so they each experience a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$. For simplicity, we will assume that $\vec{v}$ is perpendicular to $\vec{B}$, in which case the strength of the force is $F_B = qvB$. This force causes the charge carriers to move, separating the positive and negative charges. The separated charges then create an electric field inside the conductor.

FIGURE 34.2 The magnetic force on the charge carriers in a moving conductor creates an electric field inside the conductor.



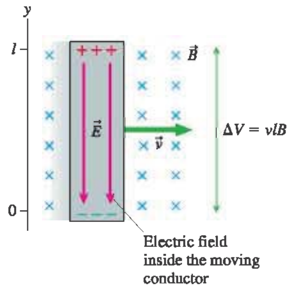
The charge carriers continue to move until the electric force $F_E = qE$ exactly balances the magnetic force $F_B = qvB$. This balance happens when the electric field strength is

$$E = vB \tag{34.1}$$

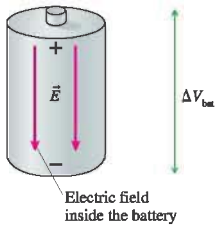
In other words, **the magnetic force on the charge carriers in a moving conductor creates an electric field $E = vB$ inside the conductor.**

FIGURE 34.3 Two different ways to generate an emf.

(a) Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.



(b) Chemical reactions separate the charges and cause a potential difference between the ends. This is a chemical emf.



The electric field, in turn, creates an electric potential difference between the two ends of the moving conductor. **FIGURE 34.3a** defines a coordinate system in which $\vec{E} = -vB\hat{j}$. Using the connection between the electric field and the electric potential that we found in Chapter 30,

$$\Delta V = V_{top} - V_{bottom} = -\int_0^l E_y\,dy = -\int_0^l (-vB)\,dy = vlB \qquad (34.2)$$

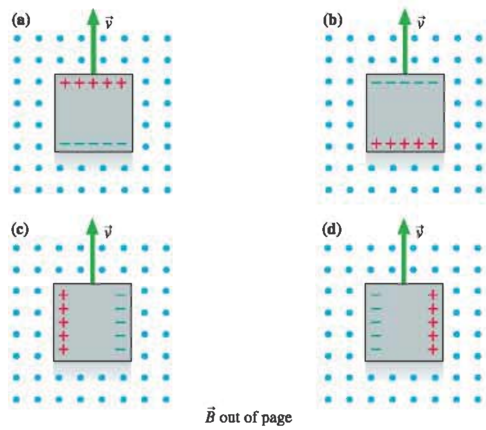
Thus the motion of the wire through a magnetic field *induces* a potential difference vlB between the ends of the conductor. The potential difference depends on the strength of the magnetic field and on the wire's speed through the field.

There's an important analogy between this potential difference and the potential difference of a battery. **FIGURE 34.3b** reminds you that a battery uses a nonelectric force—the charge escalator—to separate positive and negative charges. The emf $\mathcal{E}$ of the battery was defined as the work performed per charge (W/q) to separate the charges. An isolated battery, with no current, has a potential difference $\Delta V_{bat} = \mathcal{E}$. We could refer to a battery, where the charges are separated by chemical reactions, as a source of *chemical emf.*

The moving conductor develops a potential difference because of the work done by magnetic forces to separate the charges. You can think of the moving conductor as a "battery" that stays charged only as long as it keeps moving but "runs down" if it stops. The emf of the conductor is due to its motion, rather than to chemical reactions inside, so we can define the **motional emf** of a conductor moving with velocity $\vec{v}$ perpendicular to a magnetic field $\vec{B}$ to be

$$\mathcal{E} = vlB \qquad (34.3)$$

STOP TO THINK 34.1 A square conductor moves through a uniform magnetic field. Which of the figures shows the correct charge distribution on the conductor?



EXAMPLE 34.1 Measuring the earth's magnetic field

It is known that the earth's magnetic field over northern Canada points straight down. The crew of a Boeing 747 aircraft flying at 260 m/s over northern Canada finds a 0.95 V potential difference between the wing tips. The wing span of a Boeing 747 is 65 m. What is the magnetic field strength there?

MODEL The wing is a conductor moving through a magnetic field, so there is a motional emf.

SOLVE The magnetic field is perpendicular to the velocity, so we can use Equation 34.3 to find

$$B = \frac{\mathcal{E}}{vL} = \frac{0.95\ \text{V}}{(260\ \text{m/s})(65\ \text{m})} = 5.6 \times 10^{-5}\ \text{T}$$

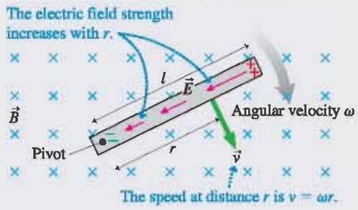
ASSESS Chapter 33 noted that the earth's magnetic field is roughly 5×10^{-5} T. The field is somewhat stronger than this near the magnetic poles, somewhat weaker near the equator.

EXAMPLE 34.2 Potential difference along a rotating bar

A metal bar of length l rotates with angular velocity ω about a pivot at one end of the bar. A uniform magnetic field $\vec{B}$ is perpendicular to the plane of rotation. What is the potential difference between the ends of the bar?

VISUALIZE FIGURE 34.4 is a pictorial representation of the bar. The magnetic forces on the charge carriers will cause the outer end to be positive with respect to the pivot.

FIGURE 34.4 Pictorial representation of a metal bar rotating in a magnetic field.



SOLVE Even though the bar is rotating, rather than moving in a straight line, the velocity of each charge carrier is perpendicular to $\vec{B}$. Consequently, the electric field created inside the bar is exactly that given in Equation 34.1, $E = vB$. But v, the speed of the charge carrier, now depends on its distance from the pivot. Recall that in rotational motion the tangential speed at radius r from the center of rotation is $v = \omega r$. Thus the electric field at distance r from the pivot is $E = \omega r B$. The electric field increases in strength as you move outward along the bar.

The electric field $\vec{E}$ points toward the pivot, so its radial component is $E_r = -\omega r B$. If we integrate outward from the center, the potential difference between the ends of the bar is

$$\Delta V = V_{\text{tip}} - V_{\text{pivot}} = -\int_0^l E_r\, dr$$

$$= -\int_0^l (-\omega r B)\, dr = \omega B \int_0^l r\, dr = \frac{1}{2}\omega l^2 B$$

ASSESS $\frac{1}{2}\omega l$ is the speed at the midpoint of the bar. Thus ΔV is $v_{\text{mid}} l B$, which seems reasonable.

Induced Current in a Circuit

The moving conductor of Figure 34.2 had an emf, but it couldn't sustain a current because the charges had nowhere to go. It's like a battery that is disconnected from a circuit. We can change this by including the moving conductor in a circuit.

FIGURE 34.5 shows a conducting wire sliding with speed v along a U-shaped conducting rail. We'll assume that the rail is attached to a table and cannot move. The wire and the rail together form a closed conducting loop—a circuit.

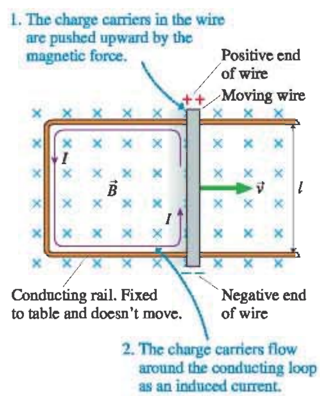
Suppose a magnetic field $\vec{B}$ is perpendicular to the plane of the circuit. Charges in the moving wire will be pushed to the ends of the wire by the magnetic force, just as they were in Figure 34.2, but now the charges can continue to flow around the circuit. That is, the moving wire acts like a battery in a circuit.

The current in the circuit is an *induced current*. In this example, the induced current is counterclockwise (ccw). If the total resistance of the circuit is R, the induced current is given by Ohm's law as

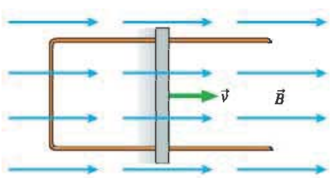
$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R} \tag{34.4}$$

In this situation, the induced current is due to magnetic forces on moving charges.

FIGURE 34.5 A current is induced in the circuit as the wire moves through a magnetic field.



STOP TO THINK 34.2 Is there an induced current in this circuit? If so, what is its direction?



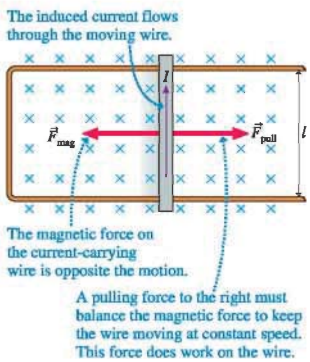


FIGURE 34.6 A pulling force is needed to move the wire to the right.

We've assumed that the wire is moving along the rail at constant speed. It turns out that we must apply a continuous pulling force $\vec{F}_{pull}$ to make this happen. **FIGURE 34.6** shows why. The moving wire, which now carries induced current I, is in a magnetic field. You learned in Chapter 33 that a magnetic field exerts a force on a current-carrying wire. According to the right-hand rule, the magnetic force $\vec{F}_{mag}$ on the moving wire points to the left. This "magnetic drag" will cause the wire to slow down and stop *unless* we exert an equal but opposite pulling force $\vec{F}_{pull}$ to keep the wire moving.

NOTE ▶ Think about this carefully. As the wire moves to the right, the magnetic force $\vec{F}_B$ pushes the charge carriers *parallel* to the wire. Their motion, as they continue around the circuit, is the induced current I. Now, because we have a current, a second magnetic force $\vec{F}_{mag}$ enters the picture. This force on the current is *perpendicular* to the wire and acts to slow the wire's motion. ◀

The magnitude of the magnetic force on a current-carrying wire was found in Chapter 33 to be $F_{mag} = IlB$. Using that result, along with Equation 34.4 for the induced current, we find that the force required to pull the wire with a constant speed v is

$$F_{pull} = F_{mag} = IlB = \left(\frac{vlB}{R}\right)lB = \frac{vl^2B^2}{R} \tag{34.5}$$

Energy Considerations

The environment must do work on the wire to pull it. What happens to the energy transferred to the wire by this work? Is energy conserved as the wire moves along the rail? It will be easier to answer this question if we think about power rather than work. Power is the *rate* at which work is done on the wire. You learned in Chapter 11 that the power exerted by a force pushing or pulling an object with velocity v is $P = Fv$. The power provided to the circuit by pulling on the wire is

$$P_{input} = F_{pull}v = \frac{v^2l^2B^2}{R} \tag{34.6}$$

This is the rate at which energy is added to the circuit by the pulling force.

But the circuit also dissipates energy by transforming electric energy into the thermal energy of the wires and components, heating them up. As we found in Chapter 32, the power dissipated by current I as it passes through resistance R is $P = I^2R$. Equation 34.4 for the induced current I gives us the power dissipated by the circuit of Figure 34.5:

$$P_{dissipated} = I^2R = \frac{v^2l^2B^2}{R} \tag{34.7}$$

You can see that Equations 34.6 and 34.7 are identical. **The rate at which work is done on the circuit exactly balances the rate at which energy is dissipated.** Thus *energy is conserved.*

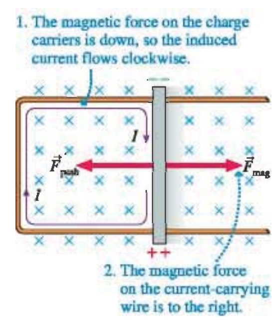


FIGURE 34.7 A pushing force is needed to move the wire to the left.

If you have to *pull* on the wire to get it to move to the right, you might think that it would spring back to the left on its own. **FIGURE 34.7** shows the same circuit with the wire moving to the left. In this case, you must *push* the wire to the left to keep it moving. The magnetic force is always opposite to the wire's direction of motion.

In both Figure 34.6, where the wire is pulled, and Figure 34.7, where it is pushed, a mechanical force is used to create a current. In other words, we have a conversion of *mechanical* energy to *electric* energy. A device that converts mechanical energy to electric energy is called a **generator.** The slide-wire circuits of Figure 34.6 and 34.7 are simple examples of a generator. We will look at more practical examples of generators later in the chapter.

We can summarize our analysis as follows:

1. Pulling or pushing the wire through the magnetic field at speed v creates a motional emf $\mathcal{E}$ in the wire and induces a current $I = \mathcal{E}/R$ in the circuit.

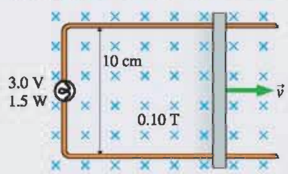
2. To keep the wire moving at constant speed, a pulling or pushing force must balance the magnetic force on the wire. This force does work on the circuit.
3. The work done by the pulling or pushing force exactly balances the energy dissipated by the current as it passes through the resistance of the circuit.

EXAMPLE 34.3 Lighting a bulb

FIGURE 34.8 shows a circuit consisting of a flashlight bulb, rated 3.0 V/1.5 W, and ideal wires with no resistance. The right wire of the circuit, which is 10 cm long, is pulled at constant speed v through a perpendicular magnetic field of strength 0.10 T.

a. What speed must the wire have to light the bulb to full brightness?
b. What force is needed to keep the wire moving?

FIGURE 34.8 Circuit of Example 34.3.



MODEL Treat the moving wire as a source of motional emf.

VISUALIZE The direction of the magnetic force on the charge carriers, $\vec{F}_B = q\vec{v} \times \vec{B}$, will cause a counterclockwise (ccw) induced current.

SOLVE a. The bulb's rating of 3.0 V/1.5 W means that at full brightness it will dissipate 1.5 W at a potential difference of 3.0 V. Because the power is related to the voltage and current by $P = I\Delta V$, the current causing full brightness is

$$I = \frac{P}{\Delta V} = \frac{1.5\ \text{W}}{3.0\ \text{V}} = 0.50\ \text{A}$$

The bulb's resistance—the total resistance of the circuit—is

$$R = \frac{\Delta V}{I} = \frac{3.0\ \text{V}}{0.50\ \text{A}} = 6.0\ \Omega$$

Equation 34.4 gives the speed needed to induce this current:

$$v = \frac{IR}{lB} = \frac{(0.50\ \text{A})(6.0\ \Omega)}{(0.10\ \text{m})(0.10\ \text{T})} = 300\ \text{m/s}$$

You can confirm from Equation 34.6 that the input power at this speed is 1.5 W.

b. From Equation 34.5, the pulling force must be

$$F_{pull} = \frac{vl^2B^2}{R} = 5.0 \times 10^{-3}\ \text{N}$$

You can also obtain this result from $F_{pull} = P/v$.

ASSESS Example 34.1 showed that high speeds are needed to produce significant potential difference. Thus 300 m/s is not surprising. The pulling force is not very large, but even a small force can deliver large amounts of power $P = Fv$ when v is large.

Eddy Currents

FIGURE 34.9 shows a *rigid* square loop of wire between the poles of a magnet. The magnetic field points downward and is confined to the region between the poles. The magnetic field in Figure 34.9a passes through the loop, but the wires are not in the field. None of the charge carriers in the wire experience a magnetic force, so there is no induced current and it takes no force to pull the loop to the right.

But when the left edge of the loop enters the field, as shown in Figure 34.9b, the magnetic force on the charge carriers induces a current in the loop. The magnetic field then exerts a retarding magnetic force on this current, so **a pulling force must be exerted to pull the loop out of the magnetic field.** Note that the wire, typically copper, is *not* a magnetic material. A piece of the wire held near the magnet would feel no force. Nor would a force be required to pull the wire out if there were a gap in the loop, breaking the circuit and preventing a current. It is the *induced current* in the complete loop that causes the wire to experience a retarding force.

These ideas have interesting implications. Consider pulling a *sheet* of metal through a magnetic field, as shown in FIGURE 34.10a on the next page. The metal, we will assume, is not a magnetic material, so it experiences no magnetic force if it is at rest. The charge carriers in the metal experience a magnetic force as the sheet is dragged between the pole tips of the magnet. A current is induced, just as in the loop of wire, but here the currents do not have wires to define their path. As a consequence, two "whirlpools" of current begin to circulate in the metal. These spread-out current whirlpools in a solid metal are called **eddy currents.**

FIGURE 34.9 Pulling a loop of wire out of a magnetic field.

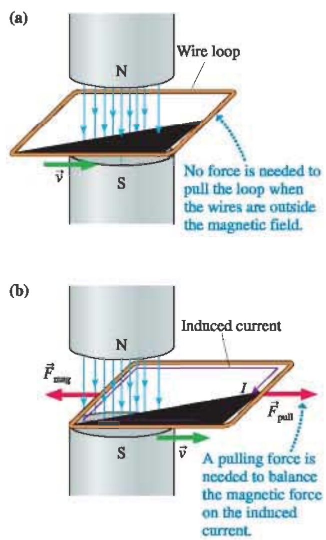


FIGURE 34.10 Eddy currents.

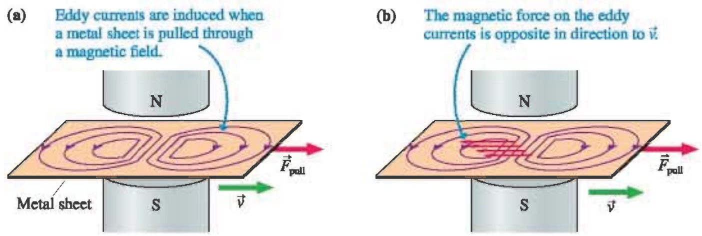


FIGURE 34.11 Magnetic braking systems are an application of eddy currents.

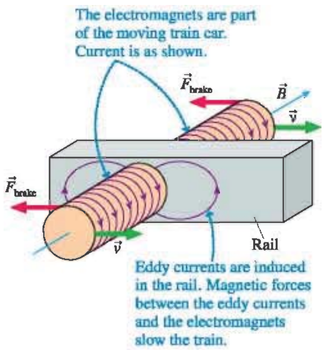
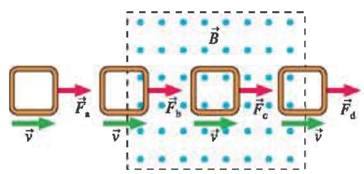


FIGURE 34.10b shows the magnetic force on the eddy current as it passes between the pole tips. This force is to the left, acting as a retarding force. Thus **an external force is required to pull a metal through a magnetic field.** If the pulling force ceases, the retarding magnetic force quickly causes the metal to decelerate until it stops. Similarly, a force is required to push a sheet of metal *into* a magnetic field.

Eddy currents are often undesirable. The power dissipation of eddy currents can cause unwanted heating, and the magnetic forces on eddy currents mean that extra energy must be expended to move metals in magnetic fields. But eddy currents also have important useful applications. A good example is magnetic braking, which is used in some trains and transit-system vehicles.

The moving train car has an electromagnet that straddles the rail, as shown in FIGURE 34.11. During normal travel, there is no current through the electromagnet and no magnetic field. To stop the car, a current is switched into the electromagnet. The current creates a strong magnetic field that passes *through* the rail, and the motion of the rail relative to the magnet induces eddy currents in the rail. The magnetic force between the electromagnet and the eddy currents acts as a braking force on the magnet and, thus, on the car. Magnetic braking systems are very efficient, and they have the added advantage that they heat the rail rather than the brakes.

STOP TO THINK 34.3 A square loop of copper wire is pulled through a region of magnetic field. Rank in order, from strongest to weakest, the pulling forces $\vec{F}_a$, $\vec{F}_b$, $\vec{F}_c$, and $\vec{F}_d$ that must be applied to keep the loop moving at constant speed.



34.3 Magnetic Flux

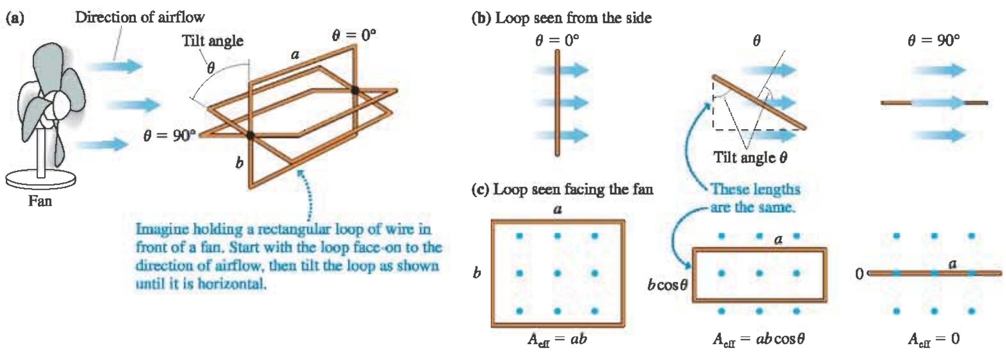
Faraday found that a current is induced when the amount of magnetic field passing through a coil or a loop of wire changes. And that's exactly what happens as the slide wire moves down the rail in Figure 34.5! As the circuit expands, more magnetic field passes through. It's time to define more clearly what we mean by "the amount of field passing through a loop."

Imagine holding a rectangular loop of wire in front of a fan, as shown in FIGURE 34.12. The amount of air that flows through the loop depends on the effective area of the loop as seen along the direction of flow. You can see from the figure that the effective area (i.e., as seen facing the fan) is

$$A_{\text{eff}} = ab\cos\theta = A\cos\theta \tag{34.8}$$

where $A = ab$ is the area of the loop and θ is the tilt angle of the loop. A loop perpendicular to the flow, with $\theta = 0°$, has $A_{\text{eff}} = A$, the full area of the loop. No air at all flows through the loop if it is tilted 90°, and you can see that $A_{\text{eff}} = 0$ in this case.

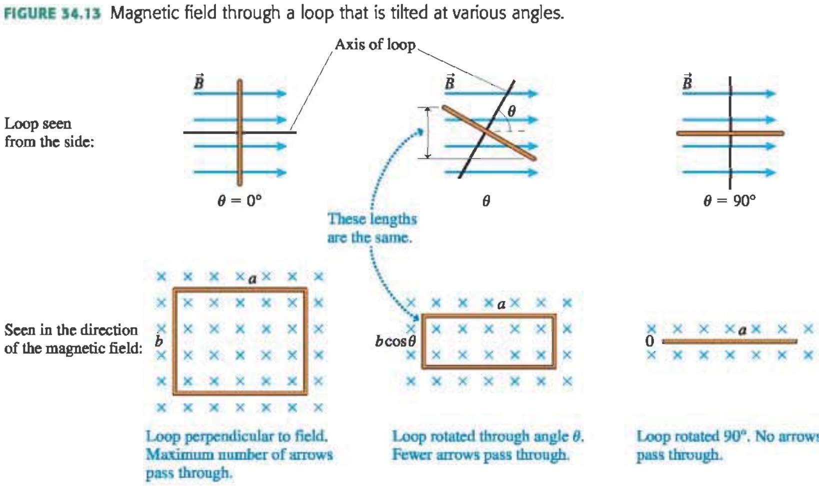
FIGURE 34.12 The amount of air flowing through a loop depends on the effective area of the loop.



We can apply this idea to a magnetic field passing through a loop. **FIGURE 34.13** shows a loop of area $A = ab$ in a uniform magnetic field. Think of the field vectors, seen here from behind, as if they were arrows shot into the page. The density of arrows (arrows per m^2) is proportional to the strength B of the magnetic field; a stronger field would be represented by arrows spaced closer together. The number of arrows passing through a loop of wire depends on two factors:

1. The density of arrows, which is proportional to B, and
2. The effective area $A_{\text{eff}} = A\cos\theta$ of the loop.

FIGURE 34.13 Magnetic field through a loop that is tilted at various angles.



The angle θ is the angle between the magnetic field and the axis of the loop. The maximum number of arrows passes through the loop when it is perpendicular to the magnetic field ($\theta = 0°$). No arrows pass through the loop if it is tilted 90°.

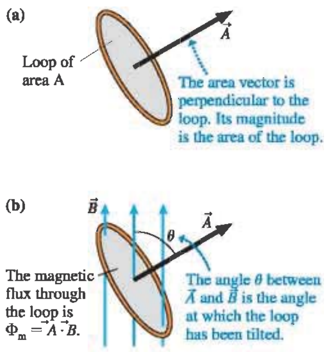
With this in mind, let's define the **magnetic flux** Φ_m as

$$\Phi_m = A_{eff}B = AB\cos\theta \qquad (34.9)$$

The magnetic flux measures the amount of magnetic field passing through a loop of area A if the loop is tilted at angle θ from the field. The SI unit of magnetic flux is the **weber.** From Equation 34.9 you can see that

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T m}^2$$

FIGURE 34.14 Magnetic flux can be defined in terms of an area vector $\vec{A}$.



Equation 34.9 is reminiscent of the vector dot product: $\vec{A}\cdot\vec{B} = AB\cos\theta$. With that in mind, let's define an **area vector** $\vec{A}$ to be a vector *perpendicular* to the loop magnitude equal to the area A of the loop. Vector $\vec{A}$ has units of m^2. FIGURE 34.14a shows the area vector $\vec{A}$ for a circular loop of area A.

FIGURE 34.14b shows a magnetic field passing through a loop. The angle between vectors $\vec{A}$ and $\vec{B}$ is the same angle used in Equations 34.8 and 34.9 to define the effective area and the magnetic flux. So Equation 34.9 really is a dot product, and we can define the magnetic flux more concisely as

$$\Phi_m = \vec{A}\cdot\vec{B} \qquad (34.10)$$

Writing the flux as a dot product helps make clear how angle θ is defined: θ is the angle between the magnetic field and the axis of the loop.

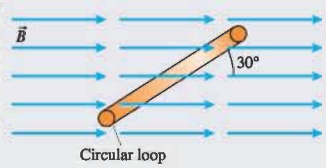
EXAMPLE 34.4 A circular loop in a magnetic field

FIGURE 34.15 is an edge view of a 10-cm-diameter circular loop in a uniform 0.050 T magnetic field. What is the magnetic flux through the loop?

SOLVE Angle θ is the angle between the loop's area vector $\vec{A}$, which is perpendicular to the plane of the loop, and the magnetic field $\vec{B}$. In this case, $\theta = 60°$, not the 30° angle shown in the figure. Vector $\vec{A}$ has magnitude $A = \pi r^2 = 7.85 \times 10^{-3}\ \text{m}^2$. Thus the magnetic flux is

$$\Phi_m = \vec{A}\cdot\vec{B} = AB\cos\theta = 2.0 \times 10^{-4}\ \text{Wb}$$

FIGURE 34.15 A circular loop in a magnetic field.



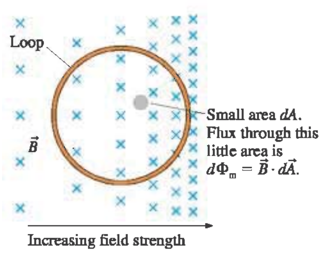
Magnetic Flux in a Nonuniform Field

Equation 34.10 for the magnetic flux assumes that the field is uniform over the area of the loop. We can calculate the flux in a nonuniform field, one where the field strength changes from one edge of the loop to the other, but we'll need to use calculus.

FIGURE 34.16 shows a loop in a nonuniform magnetic field. Imagine dividing the loop into many small pieces of area dA. The infinitesimal flux $d\Phi_m$ through one such area, where the magnetic field is $\vec{B}$, is

$$d\Phi_m = \vec{B}\cdot d\vec{A} \qquad (34.11)$$

FIGURE 34.16 A loop in a nonuniform magnetic field.



The total magnetic flux through the loop is the sum of the fluxes through each of the small areas. We find that sum by integrating. Thus the total magnetic flux through the loop is

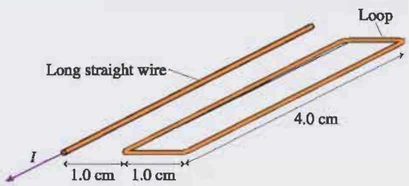
$$\Phi_m = \int_{\text{area of loop}} \vec{B}\cdot d\vec{A} \qquad (34.12)$$

Equation 34.12 is a more general definition of magnetic flux. It may look rather formidable, so we'll illustrate its use with an example.

EXAMPLE 34.5 Magnetic flux from the current in a long straight wire

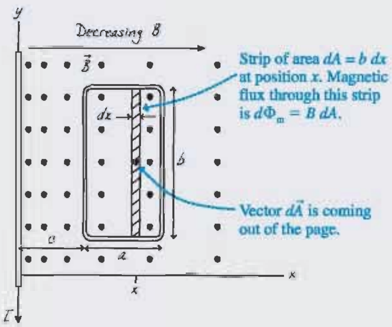
The 1.0 cm × 4.0 cm rectangular loop of FIGURE 34.17 is 1.0 cm away from a long straight wire. The wire carries a current of 1.0 A. What is the magnetic flux through the loop?

FIGURE 34.17 A loop next to a current carrying wire.



MODEL We'll treat the wire as if it were infinitely long. The magnetic field strength of a wire decreases with distance from the wire, so the field is *not* uniform over the area of the loop.

FIGURE 34.18 Calculating the magnetic flux through the loop.



VISUALIZE Using the right-hand rule, we see that the field, as it circles the wire, is perpendicular to the plane of the loop. FIGURE 34.18 redraws the loop with the field coming out of the page and establishes a coordinate system.

SOLVE Let the loop have dimensions a and b, as shown, with the near edge at distance c from the wire. The magnetic field varies with distance x from the wire, but the field is constant along a line parallel to the wire. This suggests dividing the loop into many narrow rectangular strips of length b and width dx, each forming a small area $dA = b\,dx$. The magnetic field has the same strength at all points within this small area. One such strip is shown in the figure at position x.

The area vector $d\vec{A}$ is perpendicular to the strip (coming out of the page), which makes it parallel to $\vec{B}$ ($\theta = 0°$). Thus the infinitesimal flux through this little area is

$$d\Phi_m = \vec{B}\cdot d\vec{A} = B\,dA = Bb\,dx = \frac{\mu_0 Ib}{2\pi x}\,dx$$

where, from Chapter 33, we've used $B = \mu_0 I/2\pi x$ as the magnetic field at distance x from a long straight wire. Integrating "over the area of the loop" means to integrate from the near edge of the loop at $x = c$ to the far edge at $x = c + a$. Thus

$$\Phi_m = \frac{\mu_0 Ib}{2\pi}\int_c^{c+a}\frac{dx}{x} = \frac{\mu_0 Ib}{2\pi}\ln x\Big|_c^{c+a} = \frac{\mu_0 Ib}{2\pi}\ln\left(\frac{c+a}{c}\right)$$

Evaluating for $a = c = 0.010$ m, $b = 0.040$ m, and $I = 1.0$ A gives

$$\Phi_m = 5.5 \times 10^{-9}\ \text{Wb}$$

ASSESS The flux measures how much of the wire's magnetic field passes through the loop, but we had to integrate, rather than simply using Equation 34.10, because the field is stronger at the near edge of the loop than at the far edge.

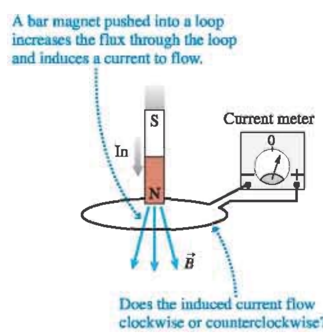
34.4 Lenz's Law

We started out by looking at a situation in which a moving wire caused a loop to expand in a magnetic field. This is one way to change the magnetic flux through the loop. But Faraday found that a current can be induced by any change in the magnetic flux, no matter how it's accomplished.

For example, a momentary current is induced in the loop of FIGURE 34.19 as the bar magnet is pushed toward the loop, increasing the flux through the loop. Pulling the magnet back out of the loop causes the current meter to deflect in the opposite direction. The conducting wires aren't moving, so this is not a motional emf. Nonetheless, the induced current is very real.

The German physicist Heinrich Lenz began to study electromagnetic induction after learning of Faraday's discovery. Three years later, in 1834, Lenz announced a rule for determining the direction of the induced current. We now call his rule **Lenz's law**, and it can be stated as follows:

FIGURE 34.19 Pushing a bar magnet toward the loop induces a current in the loop.



Lenz's law There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

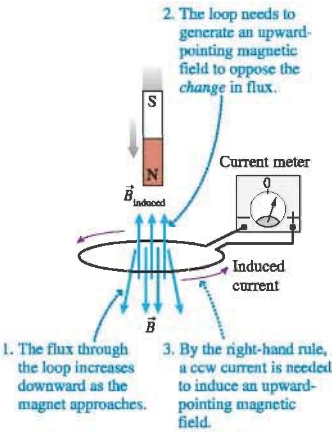
Lenz's law is rather subtle, and it takes some practice to see how to apply it.

NOTE ► One difficulty with Lenz's law is the term *flux*. In everyday language, the word *flux* already implies that something is changing. Think of the phrase, "The situation is in flux." Not so in physics, where *flux* means "passes through." A steady magnetic field through a loop creates a steady, *un*changing magnetic flux. ◄

Lenz's law tells us to look for situations where the flux is *changing*. This can happen in three ways.

1. The magnetic field through the loop changes (increases or decreases),
2. The loop changes in area or angle, or
3. The loop moves into or out of a magnetic field.

FIGURE 34.20 The induced current is ccw.

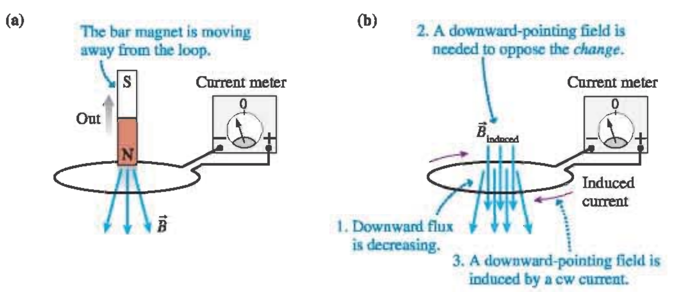


Lenz's law depends on an idea that we hinted at in our discussion of eddy currents. If a current is induced in a loop, that current generates its own magnetic field $\vec{B}_{\text{induced}}$. This is the *induced magnetic field* of Lenz's law. You learned in Chapter 33 how to use the right-hand rule to determine the direction of this induced magnetic field.

In Figure 34.19, pushing the bar magnet into the loop causes the magnetic flux to *increase* in the downward direction. To oppose the *change* in flux, which is what Lenz's law requires, the loop itself needs to generate the *upward*-pointing magnetic field of FIGURE 34.20. The induced magnetic field at the center of the loop will point upward if the current is ccw. Thus pushing the north end of a bar magnet toward the loop induces a ccw current around the loop. The induced current ceases as soon as the magnet stops moving.

Now suppose the bar magnet is pulled back away from the loop, as shown in FIGURE 34.21a. There is a downward magnetic flux through the loop, but the flux *decreases* as the magnet moves away. According to Lenz's law, the induced magnetic field of the loop *opposes this decrease*. To do so, the induced field needs to point in the *downward* direction, as shown in FIGURE 34.21b. Thus as the magnet is withdrawn, the induced current is clockwise (cw), opposite to the induced current of Figure 34.20.

FIGURE 34.21 Pulling the magnet away induces a cw current.

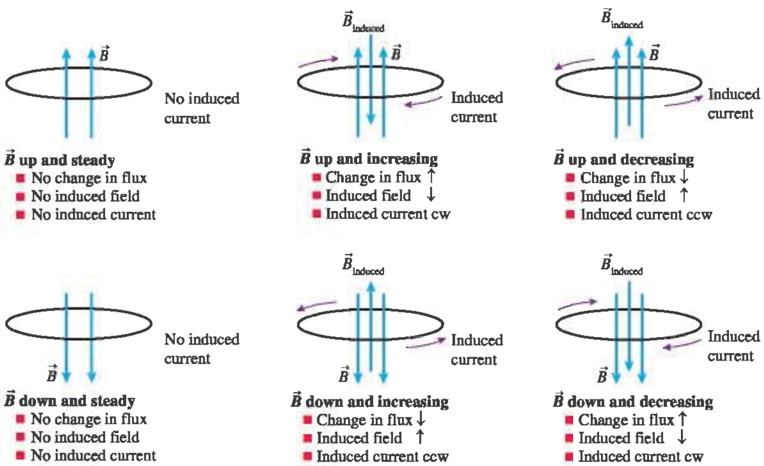


NOTE ► Notice that the magnetic field of the bar magnet is pointing downward in both Figures 34.20 and 34.21. It is not the *flux* due to the magnet that the induced current opposes, but the *change* in the flux. This is a subtle but critical distinction.

If the induced current opposed the flux itself, the current in both Figures 34.20 and 34.21 would be ccw to generate an upward magnetic field. But that's not what happens. When the field of the magnet points down and is increasing, the induced current opposes the increase by generating an upward field. When the field of the magnet points down but is decreasing, the induced current opposes the decrease by generating a downward field. ◀

FIGURE 34.22 shows six basic situations. The magnetic field can point either up or down through the loop. For each, the flux can either increase, hold steady, or decrease in strength. These observations form the basis for a set of rules about using Lenz's law.

FIGURE 34.22 The induced current for six different situations.



TACTICS BOX 34.1 **Using Lenz's law** MP

❶ **Determine the direction of the applied magnetic field.** The field must pass through the loop.

❷ **Determine how the flux is changing.** Is it increasing, decreasing, or staying the same?

❸ **Determine the direction of an induced magnetic field that will oppose the *change* in the flux.**

- Increasing flux: the induced magnetic field points opposite the applied magnetic field.
- Decreasing flux: the induced magnetic field points in the same direction as the applied magnetic field.
- Steady flux: there is no induced magnetic field.

❹ **Determine the direction of the induced current.** Use the right-hand rule to determine the current direction in the loop that generates the induced magnetic field you found in step 3.

Exercises 10–14

Let's look at some examples.

EXAMPLE 34.6 Lenz's law 1

The switch in the circuit of FIGURE 34.23 has been closed for a long time. What happens in the lower loop when the switch is opened?

MODEL We'll use the right-hand rule to find the magnetic fields of current loops.

SOLVE FIGURE 34.24 shows the four steps of using Lenz's law. Opening the switch induces a ccw current in the lower loop. This is a momentary current, lasting only until the magnetic field of the upper loop drops to zero.

ASSESS The conclusion is consistent with Figure 34.22.

FIGURE 34.23 Circuits of Example 34.6.

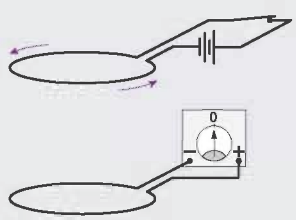
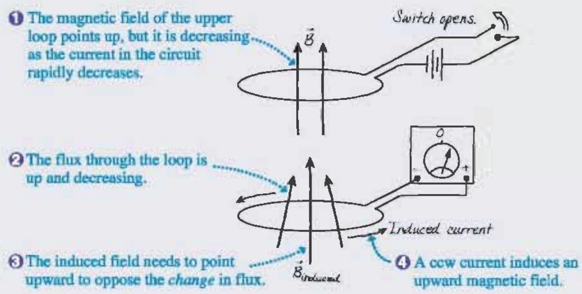


FIGURE 34.24 Applying Lenz's law.



EXAMPLE 34.7 Lenz's law 2

FIGURE 34.25 shows two solenoids facing each other. When the switch for coil 1 is closed, does the induced current in coil 2 pass from right to left or from left to right through the current meter?

MODEL We'll use the right-hand rule to find the magnetic fields of solenoids.

VISUALIZE It is very important to look at the *direction* in which a solenoid is wound around the cylinder. Notice that the two solenoids in Figure 34.25 are wound in opposite directions.

SOLVE FIGURE 34.26 shows the four steps of using Lenz's law. Closing the switch induces a current that passes from right to left through the current meter. The induced current is only momentary. It lasts only until the field from coil 1 reaches full strength and is no longer changing.

ASSESS The conclusion is consistent with Figure 34.22.

FIGURE 34.25 The two solenoids of Example 34.7.

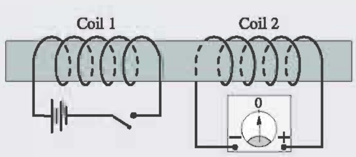
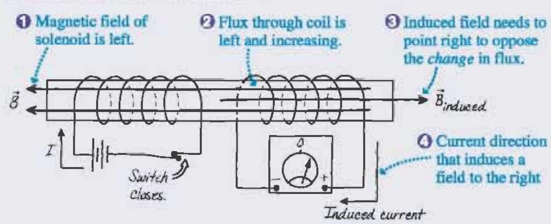
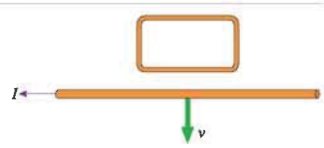


FIGURE 34.26 Applying Lenz's law.



STOP TO THINK 34.4 A current-carrying wire is pulled away from a conducting loop in the direction shown. As the wire is moving, is there a cw current around the loop, a ccw current, or no current?



34.5 Faraday's Law

Faraday discovered that a current is induced when the magnetic flux through a conducting loop changes. Lenz's law allows us to find the direction of the induced current. To put electromagnetic induction to practical use, we also need to know the *size* of the induced current.

ActivPhysics ONLINE 13.9, 13.10

Charges don't start moving spontaneously. A current requires an emf to provide the energy. We started our analysis of induced currents with circuits in which a *motional emf* can be understood in terms of magnetic forces on moving charges. But we've also seen that a current can be induced by changing the magnetic field through a stationary circuit, a circuit in which there is no motion. There *must* be an emf in this circuit, even though the mechanism for this emf is not yet clear.

The emf associated with a changing magnetic flux, regardless of what causes the change, is called an **induced emf** $\mathcal{E}$. Then, if there is a complete circuit having resistance R, a current

$$I_{\text{induced}} = \frac{\mathcal{E}}{R} \tag{34.13}$$

is established in the wire as a *consequence* of the induced emf. The direction of the current is given by Lenz's law. The last piece of information we need is the size of the induced emf $\mathcal{E}$.

The research of Faraday and others eventually led to the discovery of the basic law of electromagnetic induction, which we now call **Faraday's law.** Faraday's law is a new law of physics, not derivable from any previous laws you have studied. It states:

Faraday's law An emf $\mathcal{E}$ is induced around a closed loop if the magnetic flux through the loop changes. The magnitude of the emf is

$$\mathcal{E} = \left|\frac{d\Phi_{\text{m}}}{dt}\right| \tag{34.14}$$

and the direction of the emf is such as to drive an induced current in the direction given by Lenz's law.

In other words, the induced emf is the *rate of change* of the magnetic flux through the loop.

As a corollary to Faraday's law, a coil of wire consisting of N turns in a changing magnetic field acts like N batteries in series. The induced emf of each of the coils adds, so the induced emf of the entire coil is

$$\mathcal{E}_{\text{coil}} = N\left|\frac{d\Phi_{\text{per coil}}}{dt}\right| \quad \text{(Faraday's law for an } N\text{-turn coil)} \tag{34.15}$$

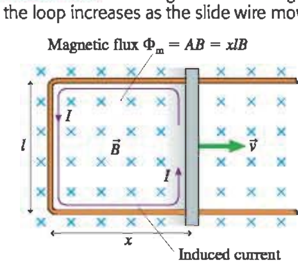
As a first example of using Faraday's law, return to the situation of Figure 34.5, where a wire moves through a magnetic field by sliding on a U-shaped conducting rail. **FIGURE 34.27** shows the circuit again. The magnetic field $\vec{B}$ is perpendicular to the plane of the conducting loop, so $\theta = 0°$ and the magnetic flux is $\Phi = AB$, where A is the area of the loop. If the slide wire is distance x from the end, the area is $A = xl$ and the flux at that instant of time is

$$\Phi_{\text{m}} = AB = xlB \tag{34.16}$$

The flux through the loop increases as the wire moves. According to Faraday's law, the induced emf is

$$\mathcal{E} = \left|\frac{d\Phi_{\text{m}}}{dt}\right| = \frac{d}{dt}(xlB) = \frac{dx}{dt}lB = vlB \tag{34.17}$$

FIGURE 34.27 The magnetic flux through the loop increases as the slide wire moves.



where the wire's velocity is $v = dx/dt$. We can now use Equation 34.13 to find that the induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R} \quad (34.18)$$

The flux is increasing into the loop, so the induced magnetic field opposes this increase by pointing out of the loop. This requires a ccw induced current in the loop. Faraday's law leads us to the conclusion that the loop will have a ccw induced current $I = vlB/R$. This is exactly the conclusion we reached in Section 34.2, where we analyzed the situation from the perspective of magnetic forces on moving charge carriers. Faraday's law confirms what we already knew but, at least in this case, doesn't seem to offer anything new.

Using Faraday's Law

Most electromagnetic induction problems can be solved with a four-step strategy.

PROBLEM-SOLVING STRATEGY 34.1 Electromagnetic induction

MP

MODEL Make simplifying assumptions about wires and magnetic fields.

VISUALIZE Draw a picture or a circuit diagram. Use Lenz's law to determine the direction of the induced current.

SOLVE The mathematical representation is based on Faraday's law

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right|$$

For an N-turn coil, multiply by N. The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

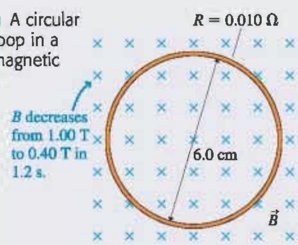
EXAMPLE 34.8 Electromagnetic induction in a loop

A patient having an MRI scan has neglected to remove a copper bracelet. The bracelet is 6.0 cm in diameter and has a resistance of 0.010 Ω. The magnetic field in the MRI solenoid is directed along the person's body from head to foot; the bracelet is perpendicular to $\vec{B}$. As a scan is taken, the magnetic field in the solenoid decreases from 1.00 T to 0.40 T in 1.2 s. What are the magnitude and direction of the current induced in the bracelet?

MODEL Assume that B decreases linearly with time.

VISUALIZE FIGURE 34.28 shows the bracelet and the applied field looking down along the patient's body. As the applied field decreases, the flux into the loop decreases. To oppose the decreasing flux, the field from the induced current must be in the direction of the applied field. Thus, from the right-hand rule, the induced current in the bracelet must be clockwise.

FIGURE 34.28 A circular conducting loop in a decreasing magnetic field.



SOLVE The magnetic field is perpendicular to the plane of the loop, hence $\theta = 0°$ and the magnetic flux is $\Phi_m = AB = \pi r^2 B$. The radius of the loop doesn't change with time, but B does. According to Faraday's law, the magnitude of the induced emf is

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$$

The rate at which the magnetic field changes is

$$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = \frac{-0.60\ \text{T}}{1.2\ \text{s}} = -0.50\ \text{T/s}$$

dB/dt is negative because the field is decreasing, but all we need for Faraday's law is the absolute value. Thus

$$\mathcal{E} = \pi r^2 \left|\frac{dB}{dt}\right| = \pi(0.030\ \text{m})^2(0.50\ \text{T/s}) = 0.0014\ \text{V}$$

The current induced by this emf is

$$I = \frac{\mathcal{E}}{R} = \frac{0.0014\ \text{V}}{0.010\ \Omega} = 0.14\ \text{A}$$

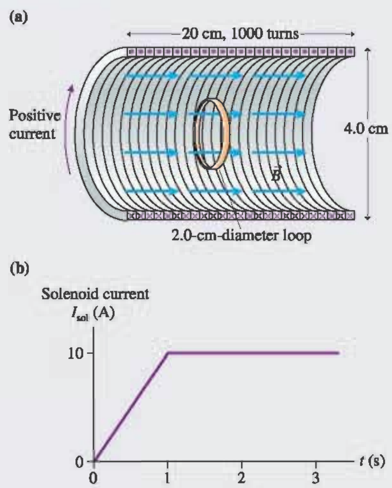
The decreasing magnetic field causes a 0.14 A clockwise current lasting for 1.2 s.

ASSESS The emf is quite small, but, because the resistance of a metal bracelet is also very small, the current is respectable. We know that electromagnetic induction produces currents large enough for practical applications, so this result seems plausible. The magnetic field of the induced current could easily distort the readings of the MRI machine. Consequently, operators are careful to have patients remove all metal before an MRI.

EXAMPLE 34.9 Electromagnetic induction in a solenoid

A 2.0-cm-diameter loop of wire with a resistance of 0.010 Ω is placed in the center of the solenoid seen in **FIGURE 34.29a**. The solenoid is 4.0 cm in diameter, 20 cm long, and wrapped with 1000 turns of wire. **FIGURE 34.29b** shows the current through the solenoid as a function of time as the solenoid is "powered up." A positive current is defined to be cw when seen from the left. Find the current in the loop as a function of time and show the result as a graph.

FIGURE 34.29 A loop inside a solenoid.



MODEL The solenoid's length is much greater than its diameter, so the field near the center should be nearly uniform.

VISUALIZE The magnetic field of the solenoid creates a magnetic flux through the loop of wire. The solenoid current is always positive, meaning that it is cw as seen from the left. Consequently, from the right-hand rule, the magnetic field inside the solenoid always points to the right. During the first second, while the solenoid current is increasing, the flux through the loop is to the right and increasing. To oppose the *change* in the flux, the loop's induced magnetic field must point to the left. Thus, again using the right-hand rule, the induced current must flow ccw as seen from the left. This is a *negative* current. There's no *change* in the flux for $t > 1$ s, so the induced current is zero.

SOLVE Now we're ready to use Faraday's law to find the magnitude of the current. Because the field is uniform inside the solenoid and perpendicular to the loop ($\theta = 0°$), the flux is $\Phi_m = AB$, where $A = \pi r^2 = 3.14 \times 10^{-4}\ \text{m}^2$ is the area of the loop (*not* the area of the solenoid). The field of a long solenoid of length l was found in Chapter 33 to be

$$B = \frac{\mu_0 N I_{sol}}{l}$$

The flux when the solenoid current is I_{sol} is thus

$$\Phi_m = \frac{\mu_0 A N I_{sol}}{l}$$

The changing flux creates an induced emf $\mathcal{E}$ that is given by Faraday's law:

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \frac{\mu_0 A N}{l}\left|\frac{dI_{sol}}{dt}\right| = 2.0 \times 10^{-6}\left|\frac{dI_{sol}}{dt}\right|$$

From the slope of the graph, we find

$$\left|\frac{dI_{sol}}{dt}\right| = \begin{cases} 10\ \text{A/s} & 0.0\ \text{s} < t < 1.0\ \text{s} \\ 0 & 1.0\ \text{s} < t < 3.0\ \text{s} \end{cases}$$

Thus the induced emf is

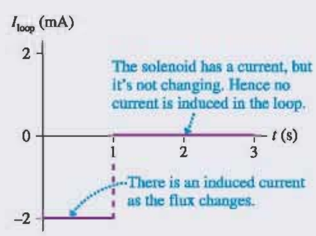
$$\mathcal{E} = \begin{cases} 2.0 \times 10^{-5}\ \text{V} & 0.0\ \text{s} < t < 1.0\ \text{s} \\ 0\ \text{V} & 1.0\ \text{s} < t < 3.0\ \text{s} \end{cases}$$

Finally, the current induced in the loop is

$$I_{loop} = \frac{\mathcal{E}}{R} = \begin{cases} -2.0\ \text{mA} & 0.0\ \text{s} < t < 1.0\ \text{s} \\ 0\ \text{mA} & 1.0\ \text{s} < t < 3.0\ \text{s} \end{cases}$$

where the negative sign comes from Lenz's law. This result is shown in **FIGURE 34.30**.

FIGURE 34.30 The induced current in the loop.



What Does Faraday's Law Tell Us?

The induced current in the slide-wire circuit of Figure 34.27 can be understood as a motional emf due to magnetic forces on moving charges. We had not anticipated this kind of current in Chapter 33, but it takes no new laws of physics to understand it.

The induced currents in Examples 34.8 and 34.9 are different. We cannot explain or predict these induced currents on the basis of previous laws or principles. This is new physics.

Faraday recognized that all induced currents are associated with a changing magnetic flux. There are two fundamentally different ways to change the magnetic flux through a conducting loop:

1. The loop can move or expand or rotate, creating a motional emf.
2. The magnetic field can change.

We can see both of these if we write Faraday's law as

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B}\cdot\frac{d\vec{A}}{dt} + \vec{A}\cdot\frac{d\vec{B}}{dt}\right| \qquad (34.19)$$

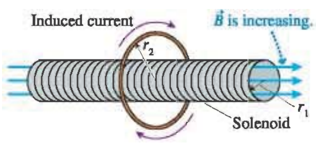
The first term on the right side represents a motional emf. The magnetic flux changes because the loop itself is changing. This term includes not only situations like the slide-wire circuit, where the area A changes, but also loops that rotate in a magnetic field. The physical area of a rotating loop does not change, but the area *vector* $\vec{A}$ does. The loop's motion causes magnetic forces on the charge carriers in the loop.

The second term on the right side is the new physics in Faraday's law. It says that an emf can also be created simply by changing a magnetic field, even if nothing is moving. This was the case in Examples 34.8 and 34.9.

Faraday's law tells us that the induced emf is simply the rate of change of the magnetic flux through the loop, *regardless* of what causes the flux to change. The "old physics" of motional emf is included within Faraday's law as one way of changing the flux, but Faraday's law then goes on to say that any other way of changing the flux will have the same result.

An Unanswered Question

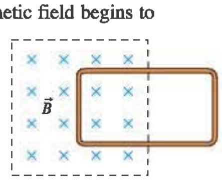
FIGURE 34.31 A changing current in the solenoid induces a current in the loop.



As a final example in this section, consider the loop shown in FIGURE 34.31. A long, tightly wound solenoid of radius r_1 passes through the center of a conducting loop having a larger radius r_2. Even though the loop is completely outside the magnetic field of the solenoid, changing the current through the solenoid causes an induced current around the loop.

How can the charge carriers in the conducting loop possibly know that the magnetic field inside the solenoid is changing? How do they know which way to move? In the case of a motional emf, the *mechanism* that causes an induced current is the magnetic force on the moving charges. But here, where there's no motion, what is the mechanism that creates a current when the magnetic flux changes? This is an important question, one that we will answer in the next section.

STOP TO THINK 34.5 A conducting loop is halfway into a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



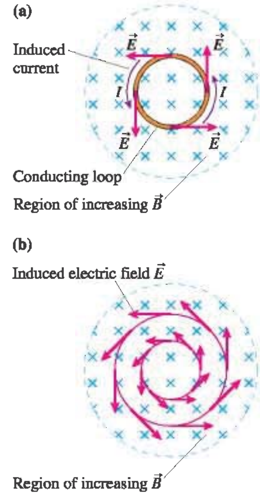
a. The loop is pushed upward, toward the top of the page.
b. The loop is pushed downward, toward the bottom of the page.
c. The loop is pulled to the left, into the magnetic field.
d. The loop is pushed to the right, out of the magnetic field.
e. The tension in the wires increases but the loop does not move.

34.6 Induced Fields

Faraday's law is a tool for calculating the strength of an induced current, but one important piece of the puzzle is still missing. What *causes* the current? That is, what *force* pushes the charges around the loop against the resistive forces of the metal? The agents that exert forces on charges are electric fields and magnetic fields. Magnetic forces are responsible for motional emfs, but magnetic forces cannot explain the current induced in a *stationary* loop by a changing magnetic field.

FIGURE 34.32a shows a conducting loop in an increasing magnetic field. According to Lenz's law, there is an induced current in the ccw direction. Something has to act on the charge carriers to make them move, so we infer that there must be an *electric* field tangent to the loop at all points. This electric field is *caused* by the changing magnetic field and is called an **induced electric field.** The induced electric field is the *mechanism* that creates a current inside a stationary loop when there's a changing magnetic field.

FIGURE 34.32 An induced electric field creates a current in the loop.



The conducting loop isn't necessary. The space in which the magnetic field is changing is filled with the pinwheel pattern of induced electric fields shown in FIGURE 34.32b. Charges will move if a conducting path is present, but the induced electric field is there as a direct consequence of the changing magnetic field.

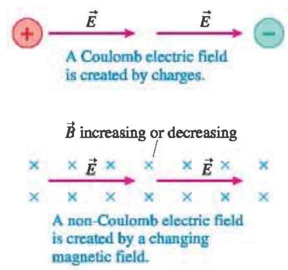
But this is a rather peculiar electric field. All the electric fields we have examined until now have been created by charges. Electric field vectors pointed away from positive charges and toward negative charges. An electric field created by charges is called a **Coulomb electric field.** The induced electric field of Figure 34.32b is caused not by charges but by a changing magnetic field. It is called a **non-Coulomb electric field.**

So it appears that there are two different ways to create an electric field:

1. A Coulomb electric field is created by positive and negative charges.
2. A non-Coulomb electric field is created by a changing magnetic field.

Both exert a force $\vec{F} = q\vec{E}$ on a charge, and both create a current in a conductor. However, the origins of the fields are very different. FIGURE 34.33 is a quick summary of the two ways to create an electric field.

FIGURE 34.33 Two ways to create an electric field.



We first introduced the idea of a field as a way of thinking about how two charges exert long-range forces on each other through the emptiness of space. The field may have seemed like a useful pictorial representation of charge interactions, but we had little evidence that fields are *real,* that they actually exist. Now we do. The electric field has shown up in a completely different context, independent of charges, as the explanation of the very real existence of induced currents.

The electric field is not just a pictorial representation; it is real.

Calculating the Induced Field

The induced electric field is peculiar in another way: It is nonconservative. Recall that a force is conservative if it does no net work on a particle moving around a closed path. "Uphills" are balanced by "downhills." We can associate a potential energy with a conservative force, hence we have gravitational potential energy for the conservative gravitational force and electric potential energy for the conservative electric force of charges (a Coulomb electric field).

But a charge moving around a closed path in the induced electric field of Figure 34.32 is always being pushed *in the same direction* by the electric force $\vec{F} = q\vec{E}$. There's never any negative work to balance the positive work, so the net work done in going around a closed path is not zero. Because it's nonconservative, we cannot associate an electric potential with an induced electric field. Only the Coulomb field of charges has an electric potential.

However, we can associate the induced field with the emf of Faraday's law. The emf was defined as the work required per unit charge to separate the charge. That is,

$$\mathcal{E} = \frac{W}{q} \tag{34.20}$$

In batteries, a familiar source of emf, this work is done by chemical forces. But the emf that appears in Faraday's law arises when work is done by the force of an induced electric field.

If a charge q moves through a small displacement $d\vec{s}$, the small amount of work done by the electric field is $dW = \vec{F} \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$. The emf of Faraday's law is an emf around a *closed curve* through which the magnetic flux Φ_m is changing. The work done by the induced electric field as charge q moves around a closed curve is

$$W_{\text{closed curve}} = q\oint \vec{E} \cdot d\vec{s} \tag{34.21}$$

where the integration symbol with the circle is the same as the one we used in Ampère's law to indicate an integral around a closed curve. If we use this work in Equation 34.20, we find that the emf around a closed loop is

$$\mathcal{E} = \frac{W_{\text{closed curve}}}{q} = \oint \vec{E} \cdot d\vec{s} \tag{34.22}$$

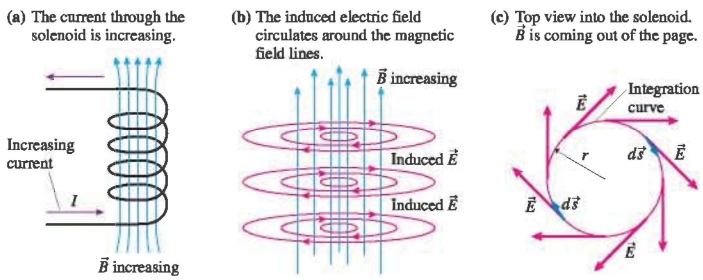
If we restrict ourselves to situations such as Figure 34.32 where the loop is perpendicular to the magnetic field and only the field is changing, we can write Faraday's law as $\mathcal{E} = |d\Phi_m/dt| = A|dB/dt|$. Consequently

$$\oint \vec{E} \cdot d\vec{s} = A\left|\frac{dB}{dt}\right| \tag{34.23}$$

Equation 34.23 is an alternative statement of Faraday's law that relates the induced electric field to the changing magnetic field.

The solenoid in FIGURE 34.34a provides a good example of the connection between $\vec{E}$ and $\vec{B}$. If there were a conducting loop inside the solenoid, we could use Lenz's law to determine that the direction of the induced current would be clockwise. But Faraday's law, in the form of Equation 34.23, tells us that **an induced electric field is present whether there's a conducting loop or not.** The electric field is induced simply due to the fact that $\vec{B}$ is changing.

FIGURE 34.34 The induced electric field circulates around the changing magnetic field inside a solenoid.



The shape and direction of the induced electric field have to be such that it *could* drive a current around a conducting loop, if one were present, and it has to be consistent with the cylindrical symmetry of the solenoid. The only possible choice, shown in FIGURE 34.34b, is an electric field that circulates clockwise around the magnetic field lines.

NOTE ▶ Circular electric field lines violate the Chapter 27 rule that electric field lines have to start and stop on charges. However, that rule applied only to Coulomb fields created by source charges. An induced electric field is a non-Coulomb field created not by source charges but by a changing magnetic field. Without source charges, induced electric field lines *must* form closed loops. ◀

To use Faraday's law, choose a *clockwise* circle of radius r as the closed curve for evaluating the integral. FIGURE 34.34c shows that the electric field vectors are everywhere tangent to the curve, so the line integral of $\vec{E}$ is

$$\oint \vec{E} \cdot d\vec{s} = El = 2\pi r E \qquad (34.24)$$

where $l = 2\pi r$ is the length of the closed curve. This is exactly like the integrals we did for Ampère's law in Chapter 33.

If we stay inside the solenoid ($r < R$), the flux passes through area $A = \pi r^2$ and Equation 34.23 becomes

$$\oint \vec{E} \cdot d\vec{s} = 2\pi r E = A\left|\frac{dB}{dt}\right| = \pi r^2\left|\frac{dB}{dt}\right| \qquad (34.25)$$

Thus the strength of the induced electric field inside the solenoid is

$$E_{\text{inside}} = \frac{r}{2}\left|\frac{dB}{dt}\right| \qquad (34.26)$$

This result shows very directly that the induced electric field is created by a *changing* magnetic field. A constant $\vec{B}$, with $dB/dt = 0$, would give $E = 0$.

EXAMPLE 34.10 An induced electric field

A 4.0-cm-diameter solenoid is wound with 2000 turns per meter. The current through the solenoid oscillates at 60 Hz with an amplitude of 2.0 A. What is the maximum strength of the induced electric field inside the solenoid?

MODEL Assume that the magnetic field inside the solenoid is uniform.

VISUALIZE The electric field lines are concentric circles around the magnetic field lines, as was shown in Figure 34.34b. They reverse direction twice every period as the current oscillates.

SOLVE You learned in Chapter 33 that the magnetic field strength inside a solenoid with n turns per meter is $B = \mu_0 n I$. In this case, the current through the solenoid is $I = I_0 \sin\omega t$, where $I_0 = 2.0$ A is the peak current and $\omega = 2\pi(60 \text{ Hz}) = 377$ rad/s. Thus the induced electric field strength at radius r is

$$E = \frac{r}{2}\left|\frac{dB}{dt}\right| = \frac{r}{2}\frac{d}{dt}(\mu_0 n I_0 \sin\omega t) = \frac{1}{2}\mu_0 n r \omega I_0 \cos\omega t$$

The field strength is maximum at maximum radius ($r = R$) *and* at the instant when $\cos\omega t = 1$. That is,

$$E_{\max} = \frac{1}{2}\mu_0 n R \omega I_0 = 0.019 \text{ V/m}$$

ASSESS This field strength, although not large, is similar to the field strength that the emf of a battery creates in a wire. Hence this induced electric field can drive a substantial induced current through a conducting loop *if* a loop is present. But the induced electric field exists inside the solenoid whether or not there is a conducting loop.

Occasionally it is useful to have a version of Faraday's law without the absolute value signs. The essence of Lenz's law is that the emf $\mathcal{E}$ opposes the *change* in Φ_m. Mathematically, this means that $\mathcal{E}$ must be opposite in sign to dB/dt. Consequently, we can write Faraday's law as

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \qquad (34.27)$$

For practical applications, it's always easier to calculate just the magnitude of the emf with Faraday's law and to use Lenz's law to find the direction of the emf or the induced current. However, the mathematically rigorous version of Faraday's law in Equation 34.27 will prove to be useful when we combine it with other equations, in Chapter 35, to predict the existence of electromagnetic waves.

Maxwell's Theory of Electromagnetic Waves

In 1855, less than two years after receiving his undergraduate degree, the Scottish physicist James Clerk Maxwell presented a paper titled "On Faraday's Lines of Force." In this paper, he began to sketch out how Faraday's pictorial ideas about fields could be given a rigorous mathematical basis. Maxwell was troubled by a certain lack of symmetry. Faraday had found that a changing magnetic field creates an induced electric field, a non-Coulomb electric field not tied to charges. But what, Maxwell began to wonder, about a changing *electric* field?

To complete the symmetry, Maxwell proposed that a changing electric field creates an **induced magnetic field,** a new kind of magnetic field not tied to the existence of currents. FIGURE 34.35 shows a region of space where the *electric* field is increasing. This region of space, according to Maxwell, is filled with a pinwheel pattern of induced magnetic fields. The induced magnetic field looks like the induced electric field, with $\vec{E}$ and $\vec{B}$ interchanged, except that—for technical reasons explored in the next chapter—the induced $\vec{B}$ points the opposite way from the induced $\vec{E}$. Although there was no experimental evidence that induced magnetic fields existed, Maxwell went ahead and included them in his electromagnetic field theory. This was an inspired hunch, soon to be vindicated.

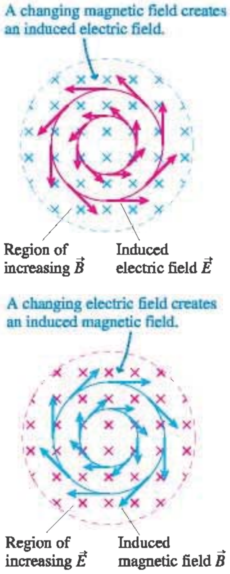


FIGURE 34.35 Maxwell hypothesized the existence of induced magnetic fields.

Maxwell soon realized that it might be possible to establish self-sustaining electric and magnetic fields that would be entirely independent of any charges or currents. That is, a changing electric field $\vec{E}$ creates a magnetic field $\vec{B}$, which then changes in just the right way to recreate the electric field, which then changes in just the right way to again recreate the magnetic field, and so on. The fields are continually recreated through electromagnetic induction without any reliance on charges or currents.

Maxwell was able to predict that electric and magnetic fields would be able to sustain themselves, free from charges and currents, if they took the form of an **electromagnetic wave.** The wave would have to have a very specific geometry, shown in FIGURE 34.36, in which $\vec{E}$ and $\vec{B}$ are perpendicular to each other as well as perpendicular to the direction of travel. That is, an electromagnetic wave would be a *transverse* wave.

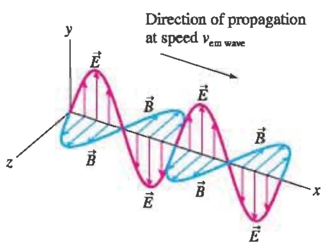


FIGURE 34.36 A self-sustaining electromagnetic wave.

Furthermore, Maxwell's theory predicted that the wave would travel with speed

$$v_{\text{em wave}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

where ϵ_0 is the permittivity constant from Coulomb's law and μ_0 is the permeability constant from the law of Biot and Savart. Maxwell computed that an electromagnetic wave, if it existed, would travel with speed $v_{\text{em wave}} = 3.00 \times 10^8$ m/s.

We don't know Maxwell's immediate reaction, but it must have been both shock and excitement. His predicted speed for electromagnetic waves, a prediction that came directly from his theory, was none other than the speed of light! This agreement could be just a coincidence, but Maxwell didn't think so. Making a bold leap of imagination, Maxwell concluded that **light is an electromagnetic wave.**

It took 25 more years for Maxwell's predictions to be tested. In 1886, the German physicist Heinrich Hertz discovered how to generate and transmit radio waves. Two years later, in 1888, he was able to show that radio waves travel at the speed of light. Maxwell, unfortunately, did not live to see his triumph. He had died in 1879, at the age of 48.

Chapter 35 will develop some of the mathematical details of Maxwell's theory and show how the ideas contained in Faraday's law lead to electromagnetic waves.

The velocity of transverse undulations in our hypothetical medium, calculated from the electromagnetic experiments of Kohlrausch and Weber [who had measured ϵ_0 and μ_0], agrees so exactly with the velocity of light calculated from the optical experiments of Fizeau that we can scarcely avoid the inference that light consists of the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.

James Clerk Maxwell

34.7 Induced Currents: Three Applications

There are many applications of Faraday's law and induced currents in modern technology. In this section we will look at three: generators, transformers, and metal detectors.

Generators

We noted in Section 34.2 that a slide wire pulled through a magnetic field on a U-shaped track is a simple generator because it transforms mechanical energy into electric energy. FIGURE 34.37 shows a more practical generator. Here a coil of wire, perhaps spun by a windmill, rotates in a magnetic field. Both the field and the area of the loop are constant, but the magnetic flux through the loop changes continuously as the loop rotates. The induced current is removed from the rotating loop by *brushes* that press up against rotating *slip rings.*

The flux through the coil is

$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos\omega t \qquad (34.28)$$

where ω is the angular frequency ($\omega = 2\pi f$) with which the coil rotates. The induced emf is given by Faraday's law,

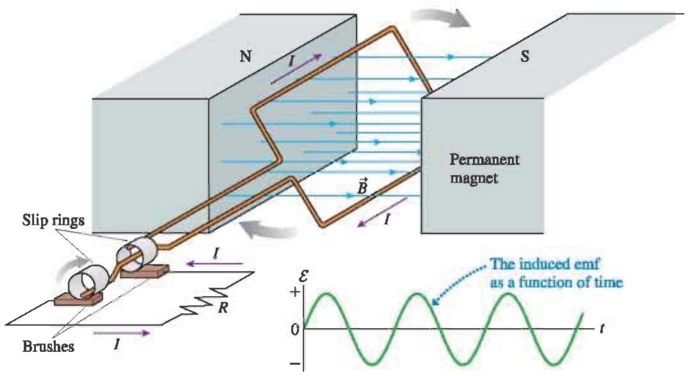
$$\mathcal{E}_{coil} = -N\frac{d\Phi_m}{dt} = -ABN\frac{d}{dt}(\cos\omega t) = \omega ABN\sin\omega t \qquad (34.29)$$

where N is the number of turns on the coil. Here it's best to use the signed version of Faraday's law, Equation 34.27, to see how the sign of $\mathcal{E}_{coil}$ alternates between positive and negative.

Because the emf alternates in sign, the current through resistor R alternates back and forth in direction. Hence the generator of Figure 34.37 is an alternating-current generator, producing what we call an *AC voltage.*

A generator inside a hydroelectric dam uses electromagnetic induction to convert the mechanical energy of a spinning turbine into electric energy.

FIGURE 34.37 An alternating-current generator.



EXAMPLE 34.11 An AC generator

A coil with area 2.0 m^2 rotates in a 0.010 T magnetic field at a frequency of 60 Hz. How many turns are needed to generate a peak voltage of 160 V?

SOLVE The coil's maximum voltage is found from Equation 34.29:

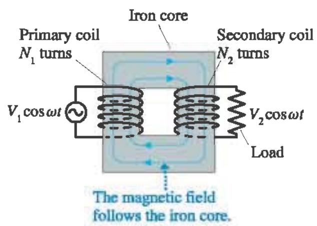
$$\mathcal{E}_{max} = \omega ABN = 2\pi fABN$$

The number of turns needed to generate $\mathcal{E}_{max} = 160$ V is

$$N = \frac{\mathcal{E}_{max}}{2\pi fAB} = \frac{160\text{ V}}{2\pi(60\text{ Hz})(2.0\text{ m}^2)(0.010\text{ T})} = 21\text{ turns}$$

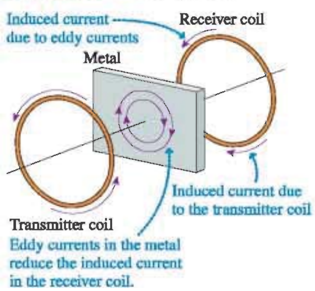
ASSESS A 0.010 T field is modest, so you can see that generating large voltages is not difficult with large (2 m^2) coils. Commercial generators use water flowing through a dam, rotating windmill blades, or turbines spun by expanding steam to rotate the generator coils. Work is required to rotate the coil, just as work was required to pull the slide wire in Section 34.2, because the magnetic field exerts retarding forces on the currents in the coil. Thus a generator is a device that turns motion (mechanical energy) into a current (electric energy). A generator is the opposite of a motor, which turns a current into motion.

FIGURE 34.38 A transformer.



Transformers are essential for transporting electric energy from the power plant to cities and homes.

FIGURE 34.39 A metal detector.



Transformers

FIGURE 34.38 shows two coils wrapped on an iron core. The left coil is called the **primary coil.** It has N_1 turns and is driven by an oscillating voltage $V_1 \cos\omega t$. The magnetic field of the primary follows the iron core and passes through the right coil, which has N_2 turns and is called the **secondary coil.** The alternating current through the primary coil causes an oscillating magnetic flux through the secondary coil and, hence, an induced emf. The induced emf of the secondary coil is delivered to the load as the oscillating voltage $V_2 \cos\omega t$.

The changing magnetic field inside the iron core is inversely proportional to the number of turns on the primary coil: $B \propto 1/N_1$. (This relation is a consequence of the coil's inductance, an idea discussed in the next section.) According to Faraday's law, the emf induced in the secondary coil is directly proportional to its number of turns: $\mathcal{E}_{sec} \propto N_2$. Combining these two proportionalities, the secondary voltage of an ideal transformer is related to the primary voltage by

$$V_2 = \frac{N_2}{N_1} V_1 \tag{34.30}$$

Depending on the ratio N_2/N_1, the voltage V_2 across the load can be *transformed* to a higher or a lower voltage than V_1. Consequently, this device is called a **transformer.** Transformers are widely used in the commercial generation and transmission of electricity. A *step-up transformer,* with $N_2 \gg N_1$, boosts the voltage of a generator up to several hundred thousand volts. Delivering power with smaller currents at higher voltages reduces losses due to the resistance of the wires. High-voltage transmission lines carry electric power to urban areas, where *step-down transformers* ($N_2 \ll N_1$) lower the voltage to 120 V.

Metal Detectors

Metal detectors, such as those used in airports for security, seem fairly mysterious. How can they detect the presence of *any* metal—not just magnetic materials such as iron—but not detect plastic or other materials? Metal detectors work because of induced currents.

A metal detector, shown in **FIGURE 34.39**, consist of two coils: a *transmitter coil* and a *receiver coil.* A high-frequency alternating current in the transmitter coil generates an alternating magnetic field along the axis. This magnetic field creates a changing flux through the receiver coil and causes an alternating induced current. The transmitter and receiver are similar to a transformer.

Suppose a piece of metal is placed between the transmitter and the receiver. The alternating magnetic field through the metal induces eddy currents in a plane parallel to the transmitter and receiver coils. The receiver coil then responds to the *superposition* of the transmitter's magnetic field and the magnetic field of the eddy currents. Because the eddy currents attempt to prevent the flux from changing, in accordance with Lenz's law, the net field at the receiver *decreases* when a piece of metal is inserted between the coils. Electronic circuits detect the current decrease in the receiver coil and set off an alarm. Eddy currents can't flow in an insulator, so this device detects only metals.

34.8 Inductors

Capacitors were first introduced as devices that produce a uniform electric field. The capacitance (i.e., the *capacity* to store charge) was defined as the charge-to-voltage ratio $C = Q/\Delta V$. We later found that a capacitor stores potential energy $U_C = \frac{1}{2}C(\Delta V)^2$ and that this energy is released when the capacitor is discharged.

A coil of wire in the form of a solenoid is a device that produces a uniform magnetic field. Do solenoids in circuits have practical uses, as capacitors do? As a starting point to answering this question, notice that the charge on a capacitor is analogous to the magnetic flux through a solenoid. That is, a larger diameter capacitor plate holds more charge just as a larger diameter solenoid contains more flux. Using the definition of capacitance $C = Q/\Delta V$ as an analog, let's define the **inductance** L of a solenoid as its flux-to-current ratio

$$L = \frac{\Phi_m}{I} \tag{34.31}$$

Strictly speaking, this is called *self-inductance* because the flux we're considering is the magnetic flux the solenoid creates in itself when there is a current.

The units of inductance are Wb/A. Recalling that 1 Wb = 1 T m^2, this is equivalent to T m^2/A. It's convenient to define an SI unit of inductance called the **henry,** in honor of Joseph Henry, as

$$1 \text{ henry} = 1 \text{ H} \equiv 1 \text{ T m}^2/\text{A}$$

Practical inductances are usually in the range of millihenries (mH) or microhenries (μH).

A coil of wire used in a circuit for the purpose of providing inductance is called an **inductor.** An *ideal inductor* is one for which the wire forming the coil has no electric resistance. The circuit symbol for an inductor is .

It's not hard to find the inductance of a solenoid. In Chapter 33 we found that the magnetic field inside a solenoid having N turns and length l is

$$B = \frac{\mu_0 N I}{l}$$

The magnetic flux through *each* coil is $\Phi_{\text{per coil}} = AB$, where A is the cross-section area of the solenoid. The total flux through all N coils is

$$\Phi_m = N\Phi_{\text{per coil}} = \frac{\mu_0 N^2 A}{l} I \tag{34.32}$$

Thus the inductance of the solenoid, using the definition of Equation 34.31, is

$$L_{\text{solenoid}} = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 A}{l} \tag{34.33}$$

The inductance of a solenoid depends only on its geometry, not at all on the current. You may recall that the capacitance of two parallel plates depends only on their geometry, not at all on their potential difference.

EXAMPLE 34.12 The length of an inductor

An inductor is made by tightly wrapping 0.30-mm-diameter wire around a 4.0-mm-diameter cylinder. What length cylinder has an inductance of 10 μH?

SOLVE The cross-section area of the solenoid is $A = \pi r^2$. If the wire diameter is d, the number of turns of wire on a cylinder of length l is $N = l/d$. Thus the inductance is

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 (l/d)^2 \pi r^2}{l} = \frac{\mu_0 \pi r^2 l}{d^2}$$

The length needed to give inductance $L = 1.0 \times 10^{-5}$ H is

$$l = \frac{d^2 L}{\mu_0 \pi r^2} = \frac{(0.00030 \text{ m})^2 (1.0 \times 10^{-5} \text{ H})}{(4\pi \times 10^{-7} \text{ T m/A})\pi (0.0020 \text{ m})^2}$$

$$= 0.057 \text{ m} = 5.7 \text{ cm}$$

The Potential Difference Across an Inductor

An inductor is not very interesting when the current through it is steady. If the inductor is ideal, with $R = 0\ \Omega$, the potential difference due to a steady current is zero. Inductors become important circuit elements when currents are changing.

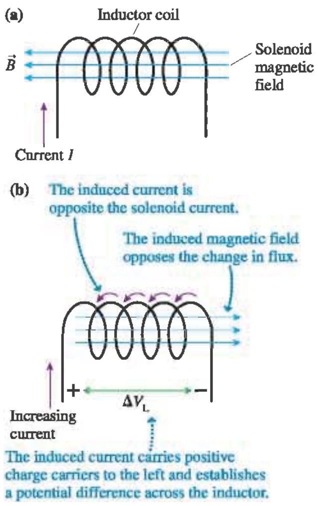


FIGURE 34.40 Increasing the current through an inductor.

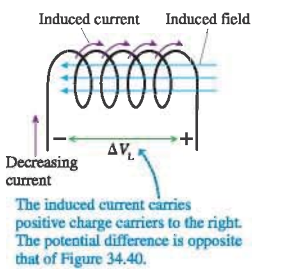


FIGURE 34.41 Decreasing the current through an inductor.

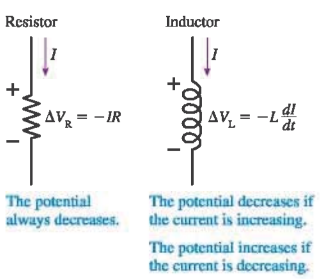


FIGURE 34.42 The potential difference across a resistor and an inductor.

FIGURE 34.40a shows a steady current into the left side of an inductor. The solenoid's magnetic field passes through the coils of the solenoid, establishing a flux.

In **FIGURE 34.40b**, the current into the solenoid is increasing. This creates an increasing flux to the left. According to Lenz's law, an induced current in the coils will oppose this increase by creating an induced magnetic field pointing to the right. This requires the induced current to be *opposite* the current into the solenoid. This induced current will carry positive charge carriers to the left until a potential difference is established across the solenoid.

You saw a similar situation in Section 34.2. The induced current in a conductor moving through a magnetic field carried positive charge carriers to the top of the wire and established a potential difference across the conductor. The induced current in the moving wire was due to magnetic forces on the moving charges. Now, in **FIGURE 34.40b**, the induced current is due to the non-Coulomb electric field induced by the changing magnetic field. Nonetheless, the outcome is the same: a potential difference across the conductor.

We can use Faraday's law to find the potential difference. The emf induced in a coil is

$$\mathcal{E}_{\text{coil}} = N\left|\frac{d\Phi_{\text{per coil}}}{dt}\right| = \left|\frac{d\Phi_{\text{m}}}{dt}\right| \tag{34.34}$$

where $\Phi = N\Phi_{\text{per coil}}$ is the total flux through all the coils. The inductance was defined such that $\Phi_{\text{m}} = LI$, so Equation 34.34 becomes

$$\mathcal{E}_{\text{coil}} = L\left|\frac{dI}{dt}\right| \tag{34.35}$$

The induced emf is directly proportional to the *rate of change* of current through the coil. We'll consider the appropriate sign in a moment, but Equation 34.35 gives us the size of the potential difference that is developed across a coil as the current through the coil changes. Note that $\mathcal{E}_{\text{coil}} = 0$ for a steady, unchanging current.

FIGURE 34.41 shows the same inductor, but now the current (still *in* to the left side) is decreasing. To oppose the decrease in flux, the induced current is in the *same* direction as the input current. The induced current carries charge to the right and establishes a potential difference opposite that in Figure 34.40b.

NOTE ▶ Notice that the induced current does not oppose the current through the inductor, which is from left to right in both Figures 34.40 and 34.41. Instead, in accordance with Lenz's law, the induced current opposes the *change* in the current in the solenoid. The practical result is that it is hard to change the current through an inductor. Any effort to increase or decrease the current is met with opposition in the form of an opposing induced current. You can think of the current in an inductor as having inertia, trying to continue what it was doing without change. ◀

Before we can use inductors in a circuit we need to establish a rule about signs that is consistent with our earlier circuit analysis. **FIGURE 34.42** first shows current I passing through a resistor. You learned in Chapter 32 that the potential difference across a resistor is $\Delta V_R = -IR$, where the minus sign indicates that the potential *decreases* in the direction of the current.

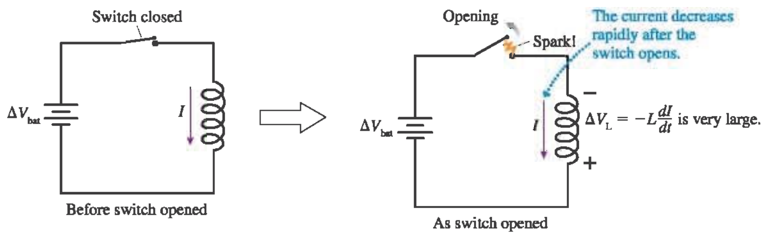
We'll use the same convention for an inductor. The potential difference across an inductor, measured along the direction of the current, is

$$\Delta V_L = -L\frac{dI}{dt} \tag{34.36}$$

If the current is increasing ($dI/dt > 0$), the input side of the inductor is more positive than the output side and the potential decreases in the direction of the current ($\Delta V_L < 0$). This was the situation in Figure 34.40b. If the current is decreasing ($dI/dt < 0$), the input side is more negative and the potential increases in the direction of the current ($\Delta V_L > 0$). This was the situation in Figure 34.41.

The potential difference across an inductor can be very large if the current changes very abruptly (large dI/dt). FIGURE 34.43 shows an inductor connected across a battery. There is a large current through the inductor, limited only by the internal resistance of the battery. Suppose the switch is suddenly opened. A very large induced voltage is created across the inductor as the current rapidly drops to zero. This potential difference (plus ΔV_{bat}) appears across the gap of the switch as it is opened. A large potential difference across a small gap often creates a spark.

FIGURE 34.43 Creating sparks.



Indeed, this is exactly how the spark plugs in your car work. The car's generator sends a large current through the *coil,* which is a big inductor. A switch in the *distributor* is suddenly opened, breaking the current. The induced voltage, typically a few thousand volts, appears across the terminals of the spark plug, creating the spark that ignites the gasoline. A similar phenomenon happens if you unplug appliances such as toaster ovens or hair dryers while they are running. The heating coils in these devices have quite a bit of inductance. Suddenly pulling the plug is like opening a switch. The large induced voltage often causes a spark between the plug and the electric outlet.

EXAMPLE 34.13 Large voltage across an inductor

A 1.0 A current passes through a 10 mH inductor coil. What potential difference is induced across the coil if the current drops to zero in 5.0 μs?

MODEL Assume this is an ideal inductor, with $R = 0\ \Omega$, and that the current decrease is linear with time.

SOLVE The rate of current decrease is

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{-1.0\text{ A}}{5.0 \times 10^{-6}\text{ s}} = -2.0 \times 10^5\text{ A/s}$$

The induced voltage is

$$\Delta V_L = -L\frac{dI}{dt} \approx -(0.010\text{ H})(-2.0 \times 10^5\text{ A/s}) = 2000\text{ V}$$

ASSESS Inductors may be physically small, but they can pack a punch if you try to change the current through them too quickly.

STOP TO THINK 34.6 The potential at a is higher than the potential at b. Which of the following statements about the inductor current I could be true?

a. I is from a to b and steady.
b. I is from a to b and increasing.
c. I is from a to b and decreasing.
d. I is from b to a and steady.
e. I is from b to a and increasing.
f. I is from b to a and decreasing.

Energy in Inductors and Magnetic Fields

An inductor, like a capacitor, stores energy that can later be released. It is energy released from the coil in your car that becomes the spark of the spark plug. You learned in Chapter 32 that electric power is $P_{elec} = I\,\Delta V$. As current passes through an inductor, for which $\Delta V_L = -L(dI/dt)$, the electric power is

$$P_{elec} = I\Delta V_L = -LI\frac{dI}{dt} \tag{34.37}$$

P_{elec} is negative because the current is *losing* electric energy. That energy is being transferred to the inductor, which is *storing* energy U_L at the rate

$$\frac{dU_L}{dt} = +LI\frac{dI}{dt} \tag{34.38}$$

where we've noted that power is the rate of change of energy.

We can find the total energy stored in an inductor by integrating Equation 34.38 from $I = 0$, where $U_L = 0$, to a final current I. Doing so gives

$$U_L = L\int_0^I I\,dI = \frac{1}{2}LI^2 \tag{34.39}$$

The potential energy stored in an inductor depends on the square of the current through it. Notice the analogy with the energy $U_C = \frac{1}{2}C(\Delta V)^2$ stored in a capacitor.

In working with circuits we say that the energy is "stored in the inductor." Strictly speaking, the energy is stored in the inductor's magnetic field, analogous to how a capacitor stores energy in the electric field. We can use the inductance of a solenoid, Equation 34.33, to relate the inductor's energy to the magnetic field strength:

$$U_L = \frac{1}{2}LI^2 = \frac{\mu_0 N^2 A}{2l}I^2 = \frac{1}{2\mu_0}Al\left(\frac{\mu_0 NI}{l}\right)^2 \tag{34.40}$$

We made the last rearrangement in Equation 34.40 because $\mu_0 NI/l$ is the magnetic field inside the solenoid. Thus

$$U_L = \frac{1}{2\mu_0}AlB^2 \tag{34.41}$$

But Al is the volume inside the solenoid. Dividing by Al, the magnetic field *energy density* inside the solenoid (energy per m^3) is

$$u_B = \frac{1}{2\mu_0}B^2 \tag{34.42}$$

We've derived this expression for energy density based on the properties of a solenoid, but it turns out to be the correct expression for the energy density anywhere there's a magnetic field. Compare this to the energy density of an electric field $u_E = \frac{1}{2}\epsilon_0 E^2$ that we found in Chapter 30.

Energy in electric and magnetic fields

Electric fields	Magnetic fields
A capacitor stores energy	An inductor stores energy
$U_C = \frac{1}{2}C(\Delta V)^2$	$U_L = \frac{1}{2}LI^2$
Energy density in the field is	Energy density in the field is
$u_E = \frac{\epsilon_0}{2}E^2$	$u_B = \frac{1}{2\mu_0}B^2$

EXAMPLE 34.14 Energy stored in an inductor

The 10 μH inductor of Example 34.12 was 5.7 cm long and 4.0 mm in diameter. Suppose it carries a 100 mA current. What are the energy stored in the inductor, the magnetic energy density, and the magnetic field strength?

SOLVE The stored energy is

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}(1.0 \times 10^{-5}\text{ H})(0.10\text{ A})^2 = 5.0 \times 10^{-8}\text{ J}$$

The solenoid volume is $(\pi r^2)l = 7.16 \times 10^{-7}\text{ m}^3$. Using this gives the energy density of the magnetic field:

$$u_B = \frac{5.0 \times 10^{-8}\text{ J}}{7.16 \times 10^{-7}\text{ m}^3} = 0.070\text{ J/m}^3$$

From Equation 34.42, the magnetic field with this energy density is

$$B = \sqrt{2\mu_0 u_B} = 4.2 \times 10^{-4}\text{ T}$$

34.9 *LC* Circuits

Telecommunication—radios, televisions, cell phones—is based on electromagnetic signals that *oscillate* at a well-defined frequency. These oscillations are generated and detected by a simple circuit consisting of an inductor and a capacitor in parallel. This is called an ***LC* circuit.** In this section we will learn why an *LC* circuit oscillates and determine the oscillation frequency.

FIGURE 34.44 shows a capacitor with initial charge Q_0, an inductor, and a switch. The switch has been open for a long time, so there is no current in the circuit. Then, at $t = 0$, the switch is closed. How does the circuit respond? Let's think it through qualitatively before getting into the mathematics.

As **FIGURE 34.45** shows, the inductor provides a conducting path for discharging the capacitor. However, the discharge current has to pass through the inductor, and, as we've seen, an inductor resists changes in current. Consequently, the current doesn't stop when the capacitor charge reaches zero.

FIGURE 34.44 An *LC* circuit.

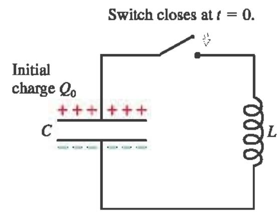
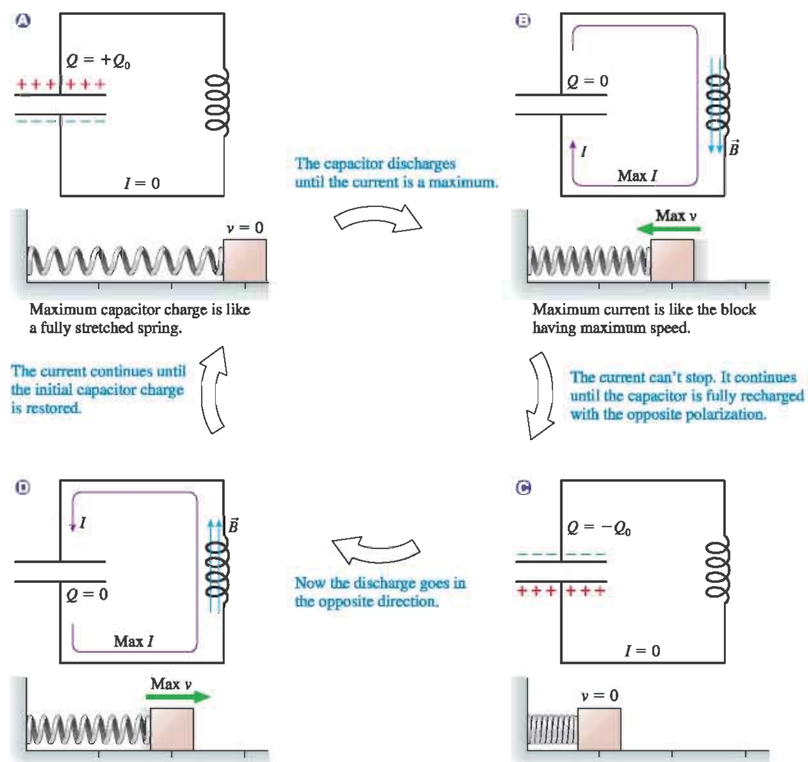


FIGURE 34.45 The capacitor charge oscillates much like a block attached to a spring.



A block attached to a stretched spring is a useful mechanical analogy. Closing the switch to discharge the capacitor is like releasing the block. The block doesn't stop when it reaches the origin; its momentum keeps it going until the spring is fully compressed. Likewise, the current continues until it has recharged the capacitor with the opposite polarization. This process repeats over and over, charging the capacitor first one way, then the other. That is, the charge and current *oscillate*.

The goal of our circuit analysis will be to find expressions showing how the capacitor charge Q and the inductor current I change with time. As always, our starting point for circuit analysis is Kirchhoff's voltage law, which says that all the potential differences around a closed loop must sum to zero. Choosing a cw direction for I, Kirchhoff's law is

$$\Delta V_C + \Delta V_L = 0 \tag{34.43}$$

You learned in Chapter 30 that the potential difference across a capacitor is $\Delta V_C = Q/C$, where Q is the charge on the top plate of the capacitor, and we found the potential difference across an inductor in Equation 34.36 above. Using these, Kirchhoff's law becomes

$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \tag{34.44}$$

A cell phone is actually a very sophisticated two-way radio that communicates with the nearest base station via high-frequency radio waves—roughly 1000 MHz. As in any radio or communications device, the transmission frequency is established by the oscillating current in an *LC* circuit.

Equation 34.44 has two unknowns, Q and I. We can eliminate one of the unknowns by finding another relation between Q and I. Current is the rate at which charge moves, $I = dq/dt$, but the charge flowing through the inductor is charge that was *removed* from the capacitor. That is, an infinitesimal charge dq flows through the inductor when the capacitor charge changes by $dQ = -dq$. Thus the current through the inductor is related to the charge on the capacitor by

$$I = -\frac{dQ}{dt} \tag{34.45}$$

Now I is positive when Q is decreasing, as we would expect. This is a subtle but important step in the reasoning, one worth thinking about because it appears in other contexts.

Equations 34.44 and 34.45 are two equations in two unknowns. To solve them, we'll first take the time derivative of Equation 34.45:

$$\frac{dI}{dt} = \frac{d}{dt}\left(-\frac{dQ}{dt}\right) = -\frac{d^2Q}{dt^2} \tag{34.46}$$

We can substitute this result into Equation 34.44:

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \tag{34.47}$$

Now we have an equation for the capacitor charge Q.

Equation 34.47 is a second-order differential equation for Q. Fortunately, it is an equation we've seen before and already know how to solve. To see this, rewrite Equation 34.47 as

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \tag{34.48}$$

Recall, from Chapter 14, that the equation of motion for an undamped mass on a spring is

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{34.49}$$

Equation 34.48 is *exactly the same equation,* with x replaced by Q and k/m replaced by $1/LC$. This should be no surprise because we've already seen that a mass on a spring is a mechanical analog of the *LC* circuit.

We know the solution to Equation 34.49. It is simple harmonic motion $x(t) = x_0 \cos\omega t$ with angular frequency $\omega = \sqrt{k/m}$. Thus the solution to Equation 34.48 must be

$$Q(t) = Q_0 \cos\omega t \qquad (34.50)$$

where Q_0 is the initial charge, at $t = 0$, and the angular frequency is

$$\omega = \sqrt{\frac{1}{LC}} \qquad (34.51)$$

The charge on the upper plate of the capacitor oscillates back and forth between $+Q_0$ and $-Q_0$ (the opposite polarization) with period $T = 2\pi/\omega$.

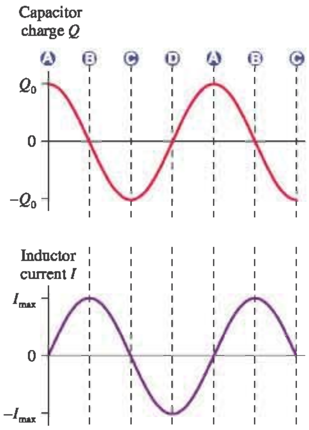
As the capacitor charge oscillates, so does the current through the inductor. Using Equation 34.45 gives the current through the inductor:

$$I = -\frac{dQ}{dt} = \omega Q_0 \sin\omega t = I_{max} \sin\omega t \qquad (34.52)$$

where $I_{max} = \omega Q_0$ is the maximum current.

An *LC* circuit is an *electric oscillator,* oscillating at frequency $f = \omega/2\pi$. **FIGURE 34.46** shows graphs of the capacitor charge Q and the inductor current I as functions of time. The letters over the graph match the labels in Figure 34.45, and you should compare the two. Notice that Q and I are 90° out of phase. The current is zero when the capacitor is fully charged, as expected, and the charge is zero when the current is maximum.

FIGURE 34.46 The oscillations of an *LC* circuit.



EXAMPLE 34.15 An AM radio oscillator

You have a 1.0 mH inductor. What capacitor should you choose to make an oscillator with a frequency of 920 kHz? (This frequency is near the center of the AM radio band.)

SOLVE The angular frequency is $\omega = 2\pi f = 5.78 \times 10^6$ rad/s. Using Equation 34.51 for ω gives the required capacitor:

$$C = \frac{1}{\omega^2 L} = \frac{1}{(5.78 \times 10^6 \text{ rad/s})^2(0.0010 \text{ H})}$$
$$= 3.0 \times 10^{-11} \text{ F} = 30 \text{ pF}$$

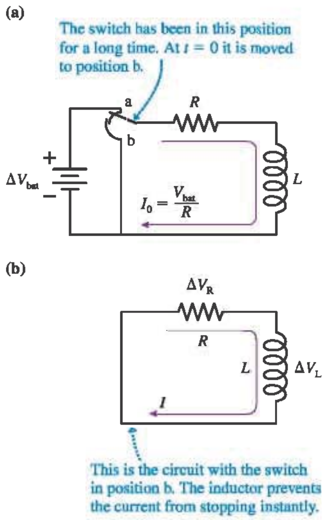
An *LC* circuit, like a mass on a spring, wants to respond only at its natural oscillation frequency $\omega = 1/\sqrt{LC}$. In Chapter 14 we defined a strong response at the natural frequency as a *resonance,* and resonance is the basis for all telecommunications. The input circuit in radios, televisions, and cell phones is an *LC* circuit driven by the signal picked up by the antenna. This signal is the superposition of hundreds of sinusoidal waves at different frequencies, one from each transmitter in the area, but the circuit responds only to the *one* signal that matches the circuit's natural frequency. That particular signal generates a large-amplitude current that can be further amplified and decoded to become the output that you hear.

Turning the dial on your radio or television changes a *variable capacitor,* thus changing the resonance frequency so that you pick up a different station. Cell phones are a bit more complicated. You don't change the capacitance yourself, but a "smart" circuit inside can change its capacitance in response to command signals it receives from the transmitter. The result is the same. Your cell phone responds to the one signal being broadcast to you and ignores the hundreds of other signals that are being broadcast simultaneously at different frequencies.

34.10 *LR* Circuits

14.1 Activ Physics ONLINE

FIGURE 34.47 An *LR* circuit.



A circuit consisting of an inductor, a resistor, and (perhaps) a battery is called an ***LR* circuit.** FIGURE 34.47a is an example of an *LR* circuit. We'll assume that the switch has been in position a for such a long time that the current is steady and unchanging. There's no potential difference across the inductor, because $dI/dt = 0$, so it simply acts like a piece of wire. The current flowing around the circuit is determined entirely by the battery and the resistor: $I_0 = \Delta V_{bat}/R$.

NOTE ▶ It's important not to open switches in inductor circuits because they'll spark, as Figure 34.43 showed. The unusual switch in Figure 34.47 is designed to make the new contact just before breaking the old one. Thus, there's never an open circuit across the switch. ◀

What happens if, at $t = 0$, the switch is suddenly moved to position b? With the battery no longer in the circuit, you might expect the current to stop immediately. But the inductor won't let that happen. The current will continue for some period of time as the inductor's magnetic field drops to zero. In essence, the energy stored in the inductor allows it to act like a battery for a short period of time. Our goal is to determine how the current decays after the switch is moved.

FIGURE 34.47b shows the circuit after the switch is changed. Our starting point, once again, is Kirchhoff's voltage law. The potential differences around a closed loop must sum to zero. For this circuit, Kirchhoff's law is

$$\Delta V_R + \Delta V_L = 0 \qquad (34.53)$$

The potential differences in the direction of the current are $\Delta V_R = -IR$ for the resistor and $\Delta V_L = -L(dI/dt)$ for the inductor. Substituting these into Equation 34.53 gives

$$-RI - L\frac{dI}{dt} = 0 \qquad (34.54)$$

We're going to need to integrate to find the current I as a function of time. Before doing so, rearrange Equation 34.54 to get all the current terms on one side of the equation and all the time terms on the other:

$$\frac{dI}{I} = -\frac{R}{L}dt = -\frac{dt}{(L/R)} \qquad (34.55)$$

We know that the current at $t = 0$, when the switch was moved, was I_0. We want to integrate from these starting conditions to current I at the unspecified time t. That is,

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{1}{(L/R)}\int_0^t dt \qquad (34.56)$$

Both are common integrals, giving

$$\ln I\Big|_{I_0}^{I} = \ln I - \ln I_0 = \ln\left(\frac{I}{I_0}\right) = -\frac{t}{(L/R)} \qquad (34.57)$$

We can solve for the current I by taking the exponential of both sides, then multiplying by I_0. Doing so gives I, the current as a function of time:

$$I = I_0 e^{-t/(L/R)} \qquad (34.58)$$

Notice that $I = I_0$ at $t = 0$, as expected.

The argument of the exponential function must be dimensionless, so L/R must have dimensions of time. If we define the **time constant** τ of the *LR* circuit to be

$$\tau = \frac{L}{R} \qquad (34.59)$$

then we can write Equation 34.58 as

$$I = I_0 e^{-t/\tau} \qquad (34.60)$$

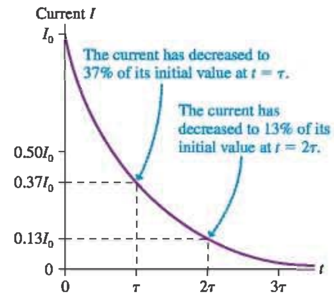
The time constant is the time at which the current has decreased to e^{-1} (about 37%) of its initial value. We can see this by computing the current at the time $t = \tau$.

$$I(\text{at } t = \tau) = I_0 e^{-\tau/\tau} = e^{-1} I_0 = 0.37 I_0 \qquad (34.61)$$

Thus the time constant for an *LR* circuit functions in exactly the same way as the time constant for the *RC* circuit we analyzed in Chapter 32. At time $t = 2\tau$, the current has decreased to $e^{-2} I_0$, or about 13% of its initial value.

The current is graphed in **FIGURE 34.48**. You can see that the current decays exponentially. The *shape* of the graph is always the same, regardless of the specific value of the time constant τ.

FIGURE 34.48 The current decay in an *LR* circuit.



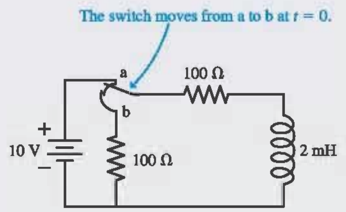
EXAMPLE 34.16 Exponential decay in an *LR* circuit

The switch in **FIGURE 34.49** has been in position a for a long time. It is changed to position b at $t = 0$ s.

a. What is the current in the circuit at $t = 5.0\ \mu s$?

b. At what time has the current decayed to 1% of its initial value?

FIGURE 34.49 The *LR* circuit of Example 34.16.



MODEL This is an *LR* circuit. We'll assume ideal wires and an ideal inductor.

VISUALIZE The two resistors will be in series after the switch is thrown.

SOLVE Before the switch is thrown, while $\Delta V_L = 0$, the current is $I_0 = (10\ \text{V})/(100\ \Omega) = 0.10\ \text{A} = 100\ \text{mA}$. This will be the initial current after the switch is thrown because the current through an inductor can't change instantaneously. The circuit resistance after the switch is thrown is $R = 200\ \Omega$, so the time constant is

$$\tau = \frac{L}{R} = \frac{2.0 \times 10^{-3}\ \text{H}}{200\ \Omega} = 1.0 \times 10^{-5}\ \text{s} = 10\ \mu\text{s}$$

a. The current at $t = 5.0\ \mu s$ is

$$I = I_0 e^{-t/\tau} = (100\ \text{mA}) e^{-(5.0\ \mu\text{s})/(10\ \mu\text{s})} = 61\ \text{mA}$$

b. To find the time at which a particular current is reached we need to go back to Equation 34.57 and solve for t:

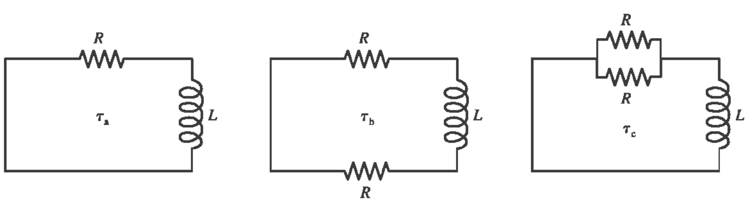
$$t = -\frac{L}{R}\ln\left(\frac{I}{I_0}\right) = -\tau\ln\left(\frac{I}{I_0}\right)$$

The time at which the current has decayed to 1 mA (1% of I_0) is

$$t = -(10\ \mu\text{s})\ln\left(\frac{1\ \text{mA}}{100\ \text{mA}}\right) = 46\ \mu\text{s}$$

ASSESS For all practical purposes, the current has decayed away in $\approx 50\ \mu s$. The inductance in this circuit is not large, so a short decay time is not surprising.

STOP TO THINK 34.7 Rank in order, from largest to smallest, the time constants τ_a, τ_b, and τ_c of these three circuits.



SUMMARY

The goal of Chapter 34 has been to understand and apply electromagnetic induction.

General Principles

Faraday's Law

MODEL Make simplifying assumptions.

VISUALIZE Use Lenz's law to determine the direction of the **induced current.**

SOLVE The **induced emf** is

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right|$$

Multiply by N for an N-turn coil.
The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Is the result reasonable?

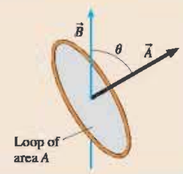
Lenz's Law

There is an induced current in a closed conducting loop if and only if the magnetic flux through the loop is changing.
The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

Magnetic flux

Magnetic flux measures the amount of magnetic field passing through a surface.

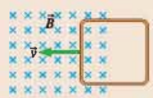
$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta$$



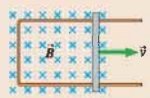
Important Concepts

Three ways to change the flux

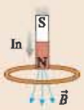
1. A loop moves into or out of a magnetic field.



2. The loop changes area or rotates.

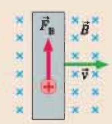


3. The magnetic field through the loop increases or decreases.



Two ways to create an induced current

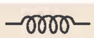
1. A **motional emf** due to magnetic forces on moving charge carriers.



2. An induced electric field due to a changing magnetic field.

Applications

Inductors



Solenoid inductance $L_{solenoid} = \dfrac{\mu_0 N^2 A}{l}$

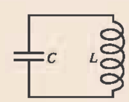
Potential difference $\Delta V_L = -L\dfrac{dI}{dt}$

Energy stored $U_L = \frac{1}{2}LI^2$

Magnetic energy density $u_B = \dfrac{1}{2\mu_0}B^2$

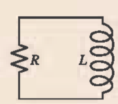
LC circuit

Oscillates at $\omega = \sqrt{\dfrac{1}{LC}}$



LR circuit

Exponential change with $\tau = \dfrac{L}{R}$



Terms and Notation

electromagnetic induction	area vector, $\vec{A}$	induced magnetic field	inductor
induced current	Lenz's law	electromagnetic wave	*LC* circuit
motional emf	induced emf, $\mathcal{E}$	primary coil	*LR* circuit
generator	Faraday's law	secondary coil	time constant, τ
eddy current	induced electric field	transformer	
magnetic flux, Φ_m	Coulomb electric field	inductance, L	
weber, Wb	non-Coulomb electric field	henry, H	

MP For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||| (challenging).

Problems labeled [ ] integrate significant material from earlier chapters.

CONCEPTUAL QUESTIONS

1. What is the direction of the induced current in FIGURE Q34.1?

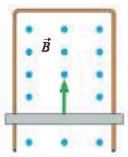


FIGURE Q34.1

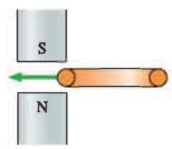


FIGURE Q34.2

2. You want to insert a loop of copper wire between the two permanent magnets in FIGURE Q34.2. Is there an attractive magnetic force that tends to *pull* the loop in, like a magnet pulls on a paper clip? Or do you need to *push* the loop in against a repulsive force? Explain.

3. A vertical, rectangular loop of copper wire is half in and half out of the horizontal magnetic field in FIGURE Q34.3. (The field is zero beneath the dotted line.) The loop is released and starts to fall. Is there a net magnetic force on the loop? If so, in which direction? Explain.

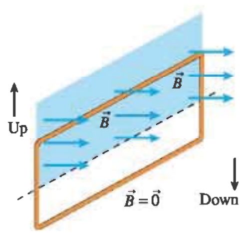


FIGURE Q34.3

4. FIGURE Q34.4 shows four different circular loops that are perpendicular to the page. The radius of loops c and d is twice that of loops a and b. The magnetic field is the same for each. Rank in order, from largest to smallest, the magnetic fluxes Φ_a to Φ_d. Some may be equal. Explain.

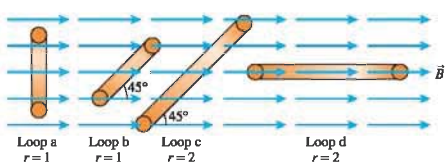


FIGURE Q34.4

5. Does the loop of wire in FIGURE Q34.5 have a clockwise current, a counterclockwise current, or no current under the following circumstances? Explain.
   a. The magnetic field points out of the page and is increasing.
   b. The magnetic field points out of the page and is constant.
   c. The magnetic field points out of the page and is decreasing.

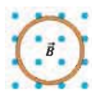


FIGURE Q34.5

FIGURE Q34.6

6. The two loops of wire in FIGURE Q34.6 are stacked one above the other. Does the upper loop have a clockwise current, a counterclockwise current, or no current at the following times? Explain.
   a. Before the switch is closed.
   b. Immediately after the switch is closed.
   c. Long after the switch is closed.
   d. Immediately after the switch is reopened.

7. FIGURE Q34.7 shows a bar magnet being pushed toward a conducting loop from below, along the axis of the loop.
   a. What is the current direction in the loop? Explain.
   b. Is there a magnetic force on the loop? If so, in which direction? Explain.

   Hint: A current loop is a magnetic dipole.

   c. Is there a force on the magnet? If so, in which direction?

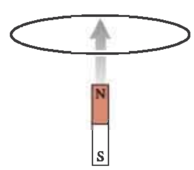


FIGURE Q34.7

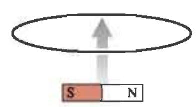


FIGURE Q34.8

8. A bar magnet is pushed toward a loop of wire as shown in FIGURE Q34.8. Is there a current in the loop? If so, in which direction? If not, why not?

9. FIGURE Q34.9 shows a bar magnet, a coil of wire, and a current meter. Is the current through the meter right to left, left to right, or zero for the following circumstances? Explain.
   a. The magnet is inserted into the coil.
   b. The magnet is held at rest inside the coil.
   c. The magnet is withdrawn from the coil.

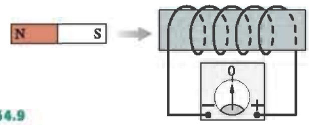


FIGURE Q34.9

10. FIGURE Q34.10 shows two coils of wire, a switch, and a current meter. Is the current through the meter right to left, left to right, or zero for the following circumstances? Explain.
    a. Just after the switch on the left coil is closed.
    b. Long after the switch on the left coil is closed.
    c. Just after the switch on the left coil is reopened.

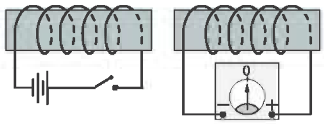


FIGURE Q34.10

11. Is the magnetic field strength in FIGURE Q34.11 increasing, decreasing, or steady? Explain.

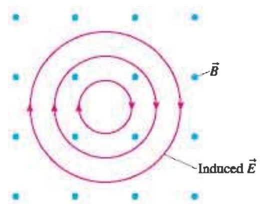


FIGURE Q34.11

12. a. Can you tell which of the inductors in FIGURE Q34.12 has the larger current through it? If so, which one? Explain.
    b. Can you tell through which inductor the current is changing more rapidly? If so, which one? Explain
    c. If the current enters the inductor from the bottom, can you tell if the current is increasing, decreasing, or staying the same? If so, which? Explain.

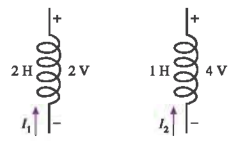


FIGURE Q34.12

13. An inductor with a 2.0 A current stores energy. At what current will the stored energy be twice as large?
14. An LC circuit oscillates at a frequency of 2000 Hz. What will the frequency be if the inductance is quadrupled?
15. Rank in order, from largest to smallest, the three time constants τ_a to τ_c for the three circuits in FIGURE Q34.15. Explain.

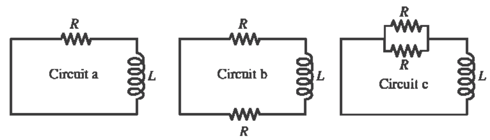


FIGURE Q34.15

16. For the circuit of FIGURE Q34.16:
    a. What is the battery current immediately after the switch closes? Explain.
    b. What is the battery current after the switch has been closed a long time? Explain.

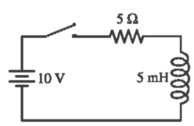


FIGURE Q34.16

EXERCISES AND PROBLEMS

Exercises

Section 34.2 Motional emf

1. | The earth's magnetic field strength is 5.0×10^{-5} T. How fast would you have to drive your car to create a 1.0 V motional emf along your 1.0-m-long radio antenna? Assume that the motion of the antenna is perpendicular to $\vec{B}$.
2. | A potential difference of 0.050 V is developed across a 10-cm-long wire as it moves through a magnetic field at 5.0 m/s. The magnetic field is perpendicular to the axis of the wire. What are the direction and strength of the magnetic field?

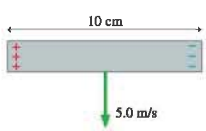


FIGURE EX34.2

3. | A 10-cm-long wire is pulled along a U-shaped conducting rail in a perpendicular magnetic field. The total resistance of the wire and rail is 0.20 Ω. Pulling the wire with a force of 1.0 N causes 4.0 W of power to be dissipated in the circuit.
   a. What is the speed of the wire when pulled with 1.0 N?
   b. What is the strength of the magnetic field?

Section 34.3 Magnetic Flux

4. | What is the magnetic flux through the loop shown in FIGURE EX34.4?

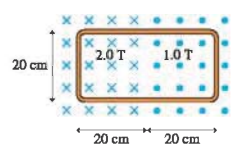


FIGURE EX34.4

5. || A 2.0-cm-diameter solenoid passes through the center of a 6.0-cm-diameter loop. The magnetic field inside the solenoid is 0.20 T. What is the magnetic flux through the loop when it is perpendicular to the solenoid and when it is tilted at a 60° angle?

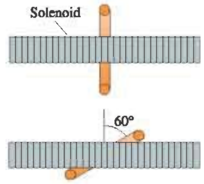


FIGURE EX34.5

6. | What is the magnetic flux through the loop shown in FIGURE EX34.6?

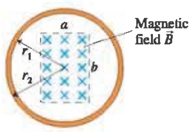


FIGURE EX34.6

Section 34.4 Lenz's Law

7. | There is a ccw induced current in the conducting loop shown in FIGURE EX34.7. Is the magnetic field inside the loop increasing in strength, decreasing in strength, or steady?

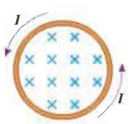


FIGURE EX34.7

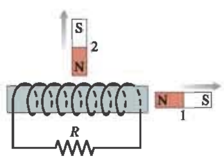


FIGURE EX34.8

8. | A solenoid is wound as shown in FIGURE EX34.8.
   a. Is there an induced current as magnet 1 is moved away from the solenoid? If so, what is the current direction through resistor R?
   b. Is there an induced current as magnet 2 is moved away from the solenoid? If so, what is the current direction through resistor R?

9. || The current in the solenoid of FIGURE EX34.9 is increasing. The solenoid is surrounded by a conducting loop. Is there a current in the loop? If so, is the loop current cw or ccw?

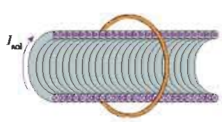


FIGURE EX34.9

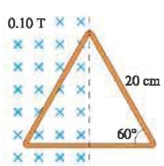


FIGURE EX34.10

10. | The metal equilateral triangle in FIGURE EX34.10, 20 cm on each side, is halfway into a 0.10 T magnetic field.
   a. What is the magnetic flux through the triangle?
   b. If the magnetic field strength decreases, what is the direction of the induced current in the triangle?

Section 34.5 Faraday's Law

11. | FIGURE EX34.11 shows a 10-cm-diameter loop in three different magnetic fields. The loop's resistance is 0.20 Ω. For each case, determine the induced emf, the induced current, and the direction of the current.

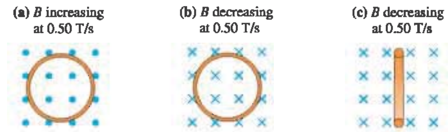


FIGURE EX34.11

12. | The loop in FIGURE EX34.12 is being pushed into the 0.20 T magnetic field at 50 m/s. The resistance of the loop is 0.10 Ω. What are the direction and the magnitude of the current in the loop?

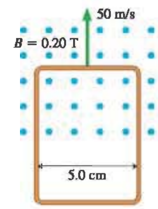


FIGURE EX34.12

13. || A 1000-turn coil of wire 2.0 cm in diameter is in a magnetic field that drops from 0.10 T to 0 T in 10 ms. The axis of the coil is parallel to the field. What is the emf of the coil?

14. | The resistance of the loop in FIGURE EX34.14 is 0.20 Ω. Is the magnetic field strength increasing or decreasing? At what rate (T/s)?

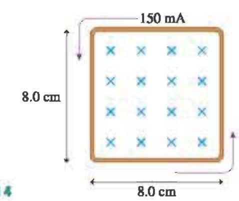


FIGURE EX34.14

Section 34.6 Induced Fields

15. || FIGURE EX34.15 shows the current as a function of time through a 20-cm-long, 4.0-cm-diameter solenoid with 400 turns. Draw a graph of the induced electric field strength as a function of time at a point 1.0 cm from the axis of the solenoid.

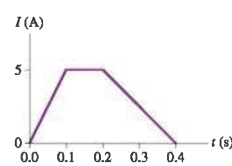


FIGURE EX34.15

16. || The magnetic field inside a 5.0-cm-diameter solenoid is 2.0 T and decreasing at 4.0 T/s. What is the electric field strength inside the solenoid at point (a) on the axis and (b) 2.0 cm from the axis?

17. ‖ The magnetic field in FIGURE EX34.17 is decreasing at the rate 0.10 T/s. What is the acceleration (magnitude and direction) of a proton at rest at points a to d?

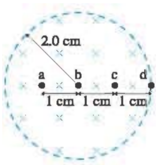


FIGURE EX34.17

Section 34.8 Inductors

18. | What is the potential difference across a 10 mH inductor if the current through the inductor drops from 150 mA to 50 mA in 10 μs? What is the direction of this potential difference? That is, does the potential increase or decrease along the direction of the current?

19. | The maximum allowable potential difference across a 200 mH inductor is 400 V. You need to raise the current through the inductor from 1.0 A to 3.0 A. What is the minimum time you should allow for changing the current?

20. | A 100 mH inductor whose windings have a resistance of 4.0 Ω is connected across a 12 V battery having an internal resistance of 2.0 Ω. How much energy is stored in the inductor?

21. ‖ How much energy is stored in a 3.0-cm-diameter, 12-cm-long solenoid that has 200 turns of wire and carries a current of 0.80 A?

Section 34.9 *LC* Circuits

22. ‖ An FM radio station broadcasts at a frequency of 100 MHz. What inductance should be paired with a 10 pF capacitor to build a receiver circuit for this station?

23. ‖ A 2.0 mH inductor is connected in parallel with a variable capacitor. The capacitor can be varied from 100 pF to 200 pF. What is the range of oscillation frequencies for this circuit?

24. ‖ An electric oscillator is made with a 0.10 μF capacitor and a 1.0 mH inductor. The capacitor is initially charged to 5.0 V. What is the maximum current through the inductor as the circuit oscillates?

Section 34.10 *LR* Circuits

25. | What value of resistor R gives the circuit in FIGURE EX34.25 a time constant of 10 μs?

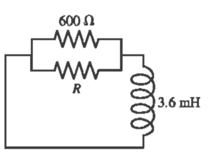


FIGURE EX34.25

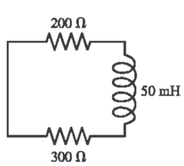


FIGURE EX34.26

26. | At $t = 0$ s, the current in the circuit in FIGURE EX34.26 is I_0. At what time is the current $\frac{1}{2}I_0$?

Problems

27. ‖ FIGURE P34.27 shows a 10 cm × 10 cm square bent at a 90° angle. A uniform 0.050 T magnetic field points downward at a 45° angle. What is the magnetic flux through the loop?

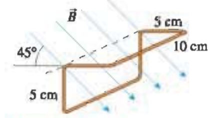


FIGURE P34.27

28. ‖ A 5.0-cm-diameter coil has 20 turns and a resistance of 0.50 Ω. A magnetic field perpendicular to the coil is $B = 0.020t + 0.010t^2$, where B is in tesla and t is in seconds.
    a. Draw a graph of B as a function of time from $t = 0$ s to $t = 10$ s.
    b. Find an expression for the induced current $I(t)$ as a function of time.
    c. Evaluate I at $t = 5$ s and $t = 10$ s.

29. ‖ A 20 cm × 20 cm square loop has a resistance of 0.10 Ω. A magnetic field perpendicular to the loop is $B = 4t - 2t^2$, where B is in tesla and t is in seconds.
    a. Determine B, $\mathcal{E}$, and I at half-second intervals from 0 s to 2 s.
    b. Use your results of part a to draw graphs of B and I versus time.

30. ‖ A 100-turn, 4.0-cm-diameter coil has a resistance of 1.0 Ω. A magnetic field perpendicular to the coil is $B = t - \frac{1}{4}t^2$, where B is in tesla and t is in seconds.
    a. Draw a graph of B as a function of time from $t = 0$ s to $t = 4$ s.
    b. Find an expression for the induced current $I(t)$ as a function of time.
    c. Evaluate I at $t = 1$, 2, and 3 s.

31. ‖ A 25-turn, 10-cm-diameter coil is oriented in a vertical plane with its axis aligned east-west. A magnetic field pointing to the northeast decreases from 0.80 T to 0.20 T in 2.0 s. What is the emf induced in the coil?

32. ‖ A 100-turn, 2.0-cm-diameter coil is at rest in a horizontal plane. A uniform magnetic field 60° away from vertical increases from 0.50 T to 1.50 T in 0.60 s. What is the induced emf in the coil?

33. ‖ A 100-turn, 8.0-cm-diameter coil is made of 0.50-mm-diameter copper wire. A magnetic field is perpendicular to the coil. At what rate must B increase to induce a 2.0 A current in the coil?

34. ‖ A circular loop made from a flexible, conducting wire is shrinking. Its radius as a function of time is $r = r_0 e^{-\beta t}$. The loop is perpendicular to a steady, uniform magnetic field B. Find an expression for the induced emf in the loop at time t.

35. ‖ A 10 cm × 10 cm square loop lies in the xy-plane. The magnetic field in this region of space is $B = (0.30t\hat{i} + 0.50t^2\hat{k})$ T, where t is in s. What is the emf induced in the loop at (a) $t = 0.5$ s and (b) $t = 1.0$ s?

36. ‖| A 20 cm × 20 cm square loop of wire lies in the xy-plane with its bottom edge on the x-axis. The resistance of the loop is 0.50 Ω. A magnetic field parallel to the z-axis is given by $B = 0.80y^2t$, where B is in tesla, y in meters, and t in seconds. What is the size of the induced current in the loop at $t = 0.50$ s?

37. ‖| A 2.0 cm × 2.0 cm square loop of wire with resistance 0.010 Ω has one edge parallel to a long straight wire. The near edge of the loop is 1.0 cm from the wire. The current in the wire is increasing at the rate of 100 A/s. What is the current in the loop?

38. ||| The rectangular loop in FIGURE P34.38 has 0.020 Ω resistance. What is the induced current in the loop at this instant?

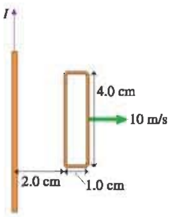


FIGURE P34.38

39. || A 4.0-cm-diameter loop with resistance 0.10 Ω surrounds a 2.0-cm-diameter solenoid. The solenoid is 10 cm long, has 100 turns, and carries the current shown in the graph. A positive current is cw when seen from the left. Determine the current in the loop at (a) $t = 0.5$ s, (b) $t = 1.5$ s, and (c) $t = 2.5$ s.

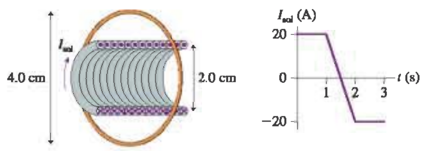


FIGURE P34.39

40. || FIGURE P34.40 shows a five-turn, 1.0-cm-diameter coil with $R = 0.10$ Ω inside a 2.0-cm-diameter solenoid. The solenoid is 8.0 cm long, has 120 turns, and carries the current shown in the graph. A positive current is cw when seen from the left. Determine the current in the coil at $t = 0.010$ s.

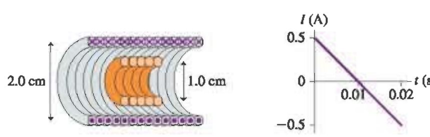


FIGURE P34.40

41. || FIGURE P34.41 shows two 20-turn coils tightly wrapped on the same 2.0-cm-diameter cylinder with 1.0-mm-diameter wire. The current through coil 1 is shown in the graph. Determine the current in coil 2 at (a) $t = 0.05$ s and (b) $t = 0.25$ s. A positive current is into the page at the top of a loop. Assume that the magnetic field of coil 1 passes entirely through coil 2.

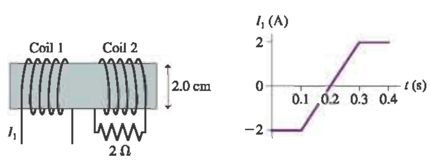


FIGURE P34.41

42. ||| A 50-turn, 4.0-cm-diameter coil with $R = 0.50$ Ω surrounds a 2.0-cm-diameter solenoid. The solenoid is 20 cm long and has 200 turns. The 60 Hz current through the solenoid is $I_{sol} = (0.50\text{ A})\sin(2\pi ft)$. Find an expression for I_{coil}, the induced current in the coil as a function of time.

43. || A loop antenna, such as is used on a television to pick up UHF broadcasts, is 25 cm in diameter. The plane of the loop is perpendicular to the oscillating magnetic field of a 150 MHz electromagnetic wave. The magnetic field through the loop is $B = (20\text{ nT})\sin\omega t$.

a. What is the maximum emf induced in the antenna?

b. What is the maximum emf if the loop is turned 90° to be perpendicular to the oscillating electric field?

44. || A 40-turn, 4.0-cm-diameter coil with $R = 0.40$ Ω surrounds a 3.0-cm-diameter solenoid. The solenoid is 20 cm long and has 200 turns. The 60 Hz current through the solenoid is $I = I_0\sin(2\pi ft)$. What is I_0 if the maximum induced current in the coil is 0.20 A?

45. || Electricity is distributed from electrical substations to neighborhoods at 15,000 V. This is a 60 Hz oscillating (AC) voltage. Neighborhood transformers, seen on utility poles, step this voltage down to the 120 V that is delivered to your house.

a. How many turns does the primary coil on the transformer have if the secondary coil has 100 turns?

b. No energy is lost in an ideal transformer, so the output power P_{out} from the secondary coil equals the input power P_{in} to the primary coil. Suppose a neighborhood transformer delivers 250 A at 120 V. What is the current in the 15,000 V line from the substation?

46. ||| A small, 2.0-mm-diameter circular loop with $R = 0.020$ Ω is at the center of a large 100-mm-diameter circular loop. Both loops lie in the same plane. The current in the outer loop changes from +1.0 A to −1.0 A in 0.10 s. What is the induced current in the inner loop?

47. ||| The square loop shown in FIGURE P34.47 moves into a 0.80 T magnetic field at a constant speed of 10 m/s. The loop has a resistance of 0.10 Ω, and it enters the field at $t = 0$ s.

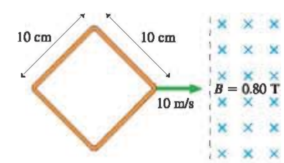


FIGURE P34.47

a. Find the induced current in the loop as a function of time. Give your answer as a graph of I versus t from $t = 0$ s to $t = 0.020$ s.

b. What is the maximum current? What is the position of the loop when the current is maximum?

48. || A 4.0-cm-long slide wire moves outward with a speed of 100 m/s in a 1.0 T magnetic field. (See Figure 34.27.) At the instant the circuit forms a 4.0 cm × 4.0 cm square, with $R = 0.010$ Ω on each side, what are

a. The induced emf?

b. The induced current?

c. The potential difference between the two ends of the moving wire?

49. || A 20-cm-long, zero-resistance slide wire moves outward, on zero-resistance rails, at a steady speed of 10 m/s in a 0.10 T magnetic field. (See Figure 34.27.) On the opposite side, a 1.0 Ω carbon resistor completes the circuit by connecting the two rails. The mass of the resistor is 50 mg.

a. What is the induced current in the circuit?

b. How much force is needed to pull the wire at this speed?

c. If the wire is pulled for 10 s, what is the temperature increase of the carbon? The specific heat of carbon is 710 J/kg C°.

50. | The 10-cm-wide, zero-resistance slide wire shown in FIGURE P34.50 is pushed toward the 2.0 Ω resistor at a steady speed of 0.50 m/s. The magnetic field strength is 0.50 T.

a. How big is the pushing force?
b. How much power does the pushing force supply to the wire?
c. What are the direction and magnitude of the induced current?
d. How much power is dissipated in the resistor?

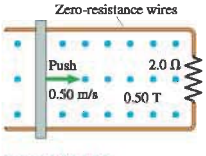


FIGURE P34.50

51. || Your camping buddy has an idea for a light to go inside your tent. He happens to have a powerful (and heavy!) horseshoe magnet that he bought at a surplus store. This magnet creates a 0.20 T field between two pole tips 10 cm apart. His idea is to build a hand-cranked generator with a rotating 5.0-cm-radius semicircle between the pole tips. He thinks you can make enough current to fully light a 1.0 Ω lightbulb rated at 4.0 W. That's not super bright, but it should be plenty of light for routine activities in the tent.

a. Find an expression for the induced current as a function of time if you turn the crank at frequency f. Assume that the semicircle is at its highest point at $t = 0$ s.
b. With what frequency will you have to turn the crank for the maximum current to fully light the bulb? Is this feasible?

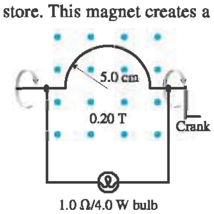


FIGURE P34.51

52. || You've decided to make a magnetic projectile launcher for your science project. An aluminum bar of length l slides along metal rails through a magnetic field B. The switch closes at $t = 0$ s, while the bar is at rest, and a battery of emf $\mathcal{E}_{bat}$ starts a current flowing around the loop. The battery has internal resistance r. The resistance of the rails and the bar are effectively zero.

a. Show that the bar reaches a terminal speed v_{term}, and find an expression for v_{term}.
b. Evaluate v_{term} for $\mathcal{E}_{bat} = 1.0$ V, $r = 0.10\ \Omega$, $l = 6.0$ cm, and $B = 0.50$ T.

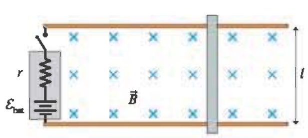


FIGURE P34.52

53. ||| A slide wire of length l, mass m, and resistance R slides down a U-shaped metal track that is tilted upward at angle θ. The track has zero resistance and no friction. A vertical magnetic field B fills the loop formed by the track and the slide wire.

a. Find an expression for the induced current I when the slide wire moves at speed v.
b. Show that the slide wire reaches a terminal speed v_{term}, and find an expression for v_{term}.

54. ||| FIGURE P34.54 shows a U-shaped conducting rail that is oriented vertically in a horizontal magnetic field. The rail has no electric resistance and does not move. A slide wire with mass m and resistance R can slide up and down without friction while maintaining electrical contact with the rail. The slide wire is released from rest.

a. Show that the slide wire reaches a terminal speed v_{term}, and find an expression for v_{term}.
b. Determine the value of v_{term} if $l = 20$ cm, $m = 10$ g, $R = 0.10\ \Omega$, and $B = 0.50$ T.

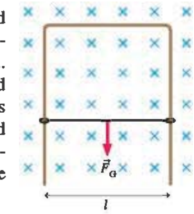


FIGURE P34.54

55. || Experiments to study vision often need to track the movements of a subject's eye. One way of doing so is to have the subject sit in a magnetic field while wearing special contact lenses with a coil of very fine wire circling the edge. A current is induced in the coil each time the subject rotates his eye. Consider an experiment in which a 20-turn, 6.0-mm-diameter coil of wire circles the subject's cornea while a 1.0 T magnetic field is directed as shown. The subject begins by looking straight ahead. What emf is induced in the coil if the subject shifts his gaze by 5° in 0.20 s?

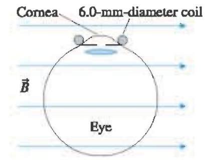


FIGURE P34.55

56. || A 10-turn coil of wire having a diameter of 1.0 cm and a resistance of 0.20 Ω is in a 1.0 mT magnetic field, with the coil oriented for maximum flux. The coil is connected to an uncharged 1.0 μF capacitor rather than to a current meter. The coil is quickly pulled out of the magnetic field. Afterward, what is the voltage across the capacitor?
Hint: Use $I = dq/dt$ to relate the *net* change of flux to the amount of charge that flows to the capacitor.

57. ||| The magnetic field at one place on the earth's surface is 55 μT in strength and tilted 60° down from horizontal. A 200-turn coil having a diameter of 4.0 cm and a resistance of 2.0 Ω is connected to a 1.0 μF capacitor rather than to a current meter. The coil is held in a horizontal plane and the capacitor is discharged. Then the coil is quickly rotated 180° so that the side that had been facing up is now facing down. Afterward, what is the voltage across the capacitor? See the Hint in Problem 56.

58. || The magnetic field inside a 4.0-cm-diameter superconducting solenoid varies sinusoidally between 8.0 T and 12.0 T at a frequency of 10 Hz.

a. What is the maximum electric field strength at a point 1.5 cm from the solenoid axis?
b. What is the value of B at the instant E reaches its maximum value?

59. || Equation 34.26 is an expression for the induced electric field inside a solenoid ($r < R$). Find an expression for the induced electric field outside a solenoid ($r > R$) in which the magnetic field is changing at the rate dB/dt.

60. || A 2.0-cm-diameter solenoid is wrapped with 1000 turns per meter. 0.50 cm from the axis, the strength of an induced electric field is 5.0×10^{-4} V/m. What is the rate dI/dt with which the current through the solenoid is changing?

61. II A solenoid inductor has an emf of 0.20 V when the current through it changes at the rate 10.0 A/s. A steady current of 0.10 A produces a flux of 5.0 μWb per turn. How many turns does the inductor have?

62. III A solenoid inductor carries a current of 200 mA. It has a magnetic flux of 20 μWb per turn and stores 1.0 mJ of energy. How many turns does the inductor have?

63. III You need to make a 100 μH inductor on a cylinder that is 5.0 cm long and 1.0 cm in diameter. You plan to wrap four layers of wire around the cylinder. What diameter wire should you use if the coils are tightly wound with no space between them? The wire diameter is small enough you don't need to consider the change in the coil's diameter for the outer layers.

64. a. What is the magnetic energy density at the center of a 4.0-cm-diameter loop carrying a current of 1.0 A?
 b. What current in a straight wire gives the magnetic energy density you found in part a at a point 2.0 cm from the wire?

65. II MRI (magnetic resonance imaging) is a medical technique that produces detailed "pictures" of the interior of the body. The patient is placed into a solenoid that is 40 cm in diameter and 1.0 m long. A 100 A current creates a 5.0 T magnetic field inside the solenoid. To carry such a large current, the solenoid wires are cooled with liquid helium until they become superconducting (no electric resistance).
 a. How much magnetic energy is stored in the solenoid? Assume that the magnetic field is uniform within the solenoid and quickly drops to zero outside the solenoid.
 b. How many turns of wire does the solenoid have?

66. I One possible concern with MRI (see Problem 65) is turning the magnetic field on or off too quickly. Bodily fluids are conductors, and a changing magnetic field could cause electric currents to flow through the patient. Suppose a typical patient has a maximum cross-section area of 0.060 m^2. What is the smallest time interval in which a 5.0 T magnetic field can be turned on or off if the induced emf around the patient's body must be kept to less than 0.10 V?

67. II FIGURE P34.67 shows the current through a 10 mH inductor. Draw a graph showing the potential difference ΔV_L across the inductor for these 6 ms.

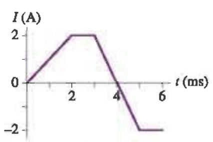


FIGURE P34.67

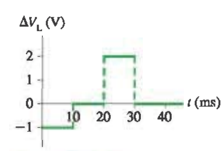


FIGURE P34.68

68. II FIGURE P34.68 shows the potential difference across a 50 mH inductor. The current through the inductor at $t = 0$ s is 0.20 A. Draw a graph showing the current through the inductor from $t = 0$ s to $t = 40$ ms.

69. II The current through inductance L is given by $I = I_0 \sin \omega t$.
 a. Find an expression for the potential difference ΔV_L across the inductor.
 b. The maximum voltage across the inductor is 0.20 V when $L = 50\ \mu$H and $f = 500$ kHz. What is I_0?

70. II The current through inductance L is given by $I = I_0 e^{-t/\tau}$.
 a. Find an expression for the potential difference ΔV_L across the inductor.
 b. Evaluate ΔV_L at $t = 0$ s, 1, 2, and 3 ms if $L = 20$ mH, $I_0 = 50$ mA, and $\tau = 1.0$ ms.
 c. Draw a graph of ΔV_L versus time from $t = 0$ s to $t = 3$ ms.

71. II An LC circuit is built with a 20 mH inductor and an 8.0 μF capacitor. The current has its maximum value of 0.50 A at $t = 0$ s.
 a. How long is it until the capacitor is fully charged?
 b. What is the voltage across the capacitor at that time?

72. II An LC circuit has a 10 mH inductor. The current has its maximum value of 0.60 A at $t = 0$ s. A short time later the capacitor reaches its maximum potential difference of 60 V. What is the value of the capacitance?

73. II The maximum charge on the capacitor in an oscillating LC circuit is Q_0. What is the capacitor charge, in terms of Q_0, when the energy in the capacitor's electric field equals the energy in the inductor's magnetic field?

74. II In recent years it has been possible to buy a 1.0 F capacitor. This is an enormously large amount of capacitance. Suppose you want to build a 1.0 Hz oscillator with a 1.0 F capacitor. You have a spool of 0.25-mm-diameter wire and a 4.0-cm-diameter plastic cylinder. How long must your inductor be if you wrap it with 2 layers of closely spaced turns?

75. II For your final exam in electronics, you're asked to build an LC circuit that oscillates at 10 kHz. In addition, the maximum current must be 0.10 A and the maximum energy stored in the capacitor must be 1.0×10^{-5} J. What values of inductance and capacitance must you use?

76. II The switch in FIGURE P34.76 has been in position 1 for a long time. It is changed to position 2 at $t = 0$ s.
 a. What is the maximum current through the inductor?
 b. What is the first time at which the current is maximum?

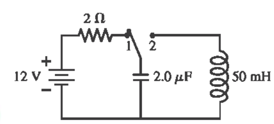


FIGURE P34.76

77. II The 300 μF capacitor in FIGURE P34.77 is initially charged to 100 V, the 1200 μF capacitor is uncharged, and the switches are both open.
 a. What is the maximum voltage to which you can charge the 1200 μF capacitor by the proper closing and opening of the two switches?
 b. How would you do it? Describe the sequence in which you would close and open switches and the times at which you would do so. The first switch is closed at $t = 0$ s.

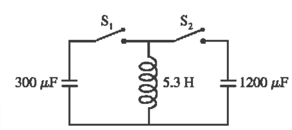


FIGURE P34.77

78. II The switch in FIGURE P34.78 has been open for a long time. It is closed at $t = 0$ s.
 a. What is the current through the battery immediately after the switch is closed?
 b. What is the current through the battery after the switch has been closed a long time?

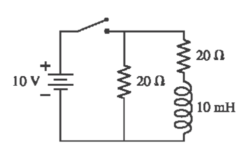


FIGURE P34.78

79. ‖ The switch in FIGURE P34.79 has been open for a long time. It is closed at $t = 0$ s. What is the current through the 20 Ω resistor
a. immediately after the switch is closed?
b. after the switch has been closed a long time?
c. immediately after the switch is reopened?

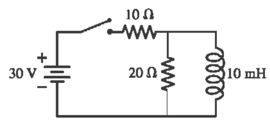


FIGURE P34.79

80. ‖‖ The switch in FIGURE P34.80 has been open for a long time. It is closed at $t = 0$ s.
a. After the switch has been closed for a long time, what is the current in the circuit? Call this current I_0.
b. Find an expression for the current I as a function of time. Write your expression in terms of I_0, R, and L.
c. Sketch a current-versus-time graph from $t = 0$ s until the current is no longer changing.

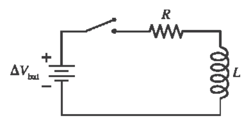


FIGURE P34.80

Challenge Problems

81. The metal wire in FIGURE CP34.81 moves with speed v parallel to a straight wire that is carrying current I. The distance between the two wires is d. Find an expression for the potential difference between the two ends of the moving wire.

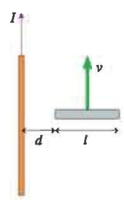


FIGURE CP34.81

82. A rectangular metal loop with 0.050 Ω resistance is placed next to one wire of the RC circuit shown in FIGURE CP34.82. The capacitor is charged to 20 V with the polarity shown, then the switch is closed at $t = 0$ s.
a. What is the direction of current in the loop for $t > 0$ s?
b. What is the current in the loop at $t = 5.0\ \mu$s? Assume that only the circuit wire next to the loop is close enough to produce a significant magnetic field.

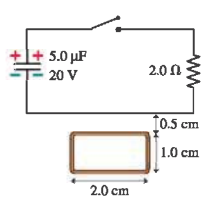


FIGURE CP34.82

83. The L-shaped conductor in FIGURE CP34.83 moves at 10 m/s across a stationary L-shaped conductor in a 0.10 T magnetic field. The two vertices overlap, so that the enclosed area is zero, at $t = 0$ s. The conductor has a resistance of 0.010 ohms *per meter*.
a. What is the direction of the induced current?
b. Find expressions for the induced emf and the induced current as functions of time.
c. Evaluate $\mathcal{E}$ and I at $t = 0.10$ s.

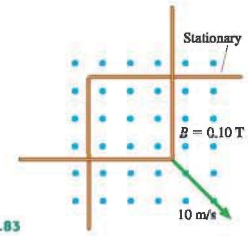


FIGURE CP34.83

84. A closed, square loop is formed with 40 cm of wire having $R = 0.10\ \Omega$, as shown in FIGURE CP34.84. A 0.50 T magnetic field is perpendicular to the loop. At $t = 0$ s, two diagonally opposite corners of the loop begin to move apart at 0.293 m/s.
a. How long does it take the loop to collapse to a straight line?
b. Find an expression for the induced current I as a function of time while the loop is collapsing. Assume that the sides remain straight lines during the collapse.
c. Evaluate I at four or five times during the collapse, then draw a graph of I versus t.

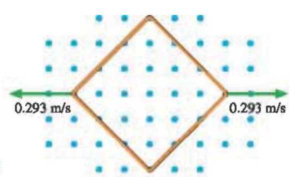


FIGURE CP34.84

85. Let's look at the details of eddy-current braking. A square loop, length l on each side, is shot with velocity v_0 into a uniform magnetic field B. The field is perpendicular to the plane of the loop. The loop has mass m and resistance R, and it enters the field at $t = 0$ s. Assume that the loop is moving to the right along the x-axis and that the field begins at $x = 0$ m.
a. Find an expression for the loop's velocity as a function of time as it enters the magnetic field. You can ignore gravity, and you can assume that the back edge of the loop has not entered the field.
b. Calculate and draw a graph of v over the interval $0\ \text{s} \le t \le 0.04$ s for the case that $v_0 = 10$ m/s, $l = 10$ cm, $m = 1.0$ g, $R = 0.0010\ \Omega$, and $B = 0.10$ T. The back edge of the loop does not reach the field during this time interval.

86. An 8.0 cm × 8.0 cm square loop is halfway into a magnetic field perpendicular to the plane of the loop. The loop's mass is 10 g and its resistance is 0.010 Ω. A switch is closed at $t = 0$ s, causing the magnetic field to increase from 0 to 1.0 T in 0.010 s.
   a. What is the induced current in the square loop?
   b. What is the force on the loop when the magnetic field is 0.50 T? Is the force directed into the magnetic field or away from the magnetic field?
   c. What is the loop's acceleration at $t = 0.005$ s, when the field strength is 0.50 T? If this acceleration stayed constant, how far would the loop move in 0.010 s?
   d. Because 0.50 T is the average field strength, your answer to c is an estimate of how far the loop moves during the 0.010 s in which the field increases to 1.0 T. If your answer is <8 cm, then it is reasonable to neglect the movement of the loop during the 0.010 s that the field ramps up. Is neglecting the movement reasonable?
   e. With what speed is the loop "kicked" away from the magnetic field?

   Hint: What is the impulse on the loop?

87. High-frequency signals are often transmitted along a *coaxial cable,* such as the one shown in FIGURE CP34.87. For example, the cable TV hookup coming into your home is a coaxial cable. The signal is carried on a wire of radius r_1 while the outer conductor of radius r_2 is grounded. A soft, flexible insulating material fills the space between them, and an insulating plastic coating goes around the outside.
   a. Find an expression for the inductance per meter of a coaxial cable. To do so, consider the flux through a rectangle of length l that spans the gap between the inner and outer conductor.
   b. Evaluate the inductance per meter of a cable having $r_1 = 0.50$ mm and $r_2 = 3.0$ mm.

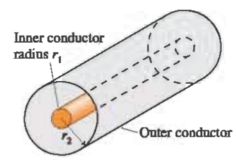


FIGURE CP34.87

STOP TO THINK ANSWERS

Stop to Think 34.1: d. According to the right-hand rule, the magnetic force on a positive charge carrier is to the right.

Stop to Think 34.2: No. The charge carriers in the wire move parallel to $\vec{B}$. There's no magnetic force on a charge moving parallel to a magnetic field.

Stop to Think 34.3: $F_b = F_d > F_a = F_c$. $\vec{F}_a$ is zero because there's no field. $\vec{F}_c$ is also zero because there's no current around the loop. The charge carriers in both the right and left edges are pushed to the bottom of the loop, creating a motional emf but no current. The currents at b and d are in opposite directions, but the forces on the segments in the field are both to the left and of equal magnitude.

Stop to Think 34.4: Clockwise. The wire's magnetic field as it passes through the loop is into the page. The flux through the loop decreases into the page as the wire moves away. To oppose this decrease, the induced magnetic field needs to point into the page.

Stop to Think 34.5: d. The flux is increasing into the loop. To oppose this increase, the induced magnetic field needs to point out of the page. This requires a ccw induced current. Using the right-hand rule, the magnetic force on the current in the left edge of the loop is to the right, away from the field. The magnetic forces on the top and bottom segments of the loop are in opposite directions and cancel each other.

Stop to Think 34.6: b or f. The potential decreases in the direction of increasing current and increases in the direction of decreasing current.

Stop to Think 34.7: $\tau_c > \tau_a > \tau_b$. $\tau = L/R$, so smaller total resistance gives a larger time constant. The parallel resistors have total resistance $R/2$. The series resistors have total resistance $2R$.