SCIENCE LABORATORY ANALYSIS MANUAL
(SLAM)

This Laboratory Reference Manual is your combined resource for General Chemistry (CH101/2 & CH151/2) and Introductory Physics (PH205/6 & PH255/6). Use this manual to assist you in completing laboratory requirements in both courses.

This manual will serve as a valuable reference for your future science and engineering courses at USMA.

Revised AY 18-1

The objectives of the Science laboratory program at USMA are to develop in each cadet:

- The ability to operate safely in a science laboratory.
- The skill of meticulous, careful, and accurate observation.
- The ability to ask questions about what he/she sees, hears, and reads.
- The ability to formulate questions for which data can be obtained.
- The skills necessary to obtain the quantitative data to answer questions.
- The ability to form conclusions from data.
- The ability to use evidence to back up conclusions.
- The ability to defend conclusions both orally and in writing.
- The ability to adapt a concept learned in one context to another situation.
- A basic knowledge of the principles of how to design a research effort.
- A realization of how scientific knowledge is obtained.
- A realization of the logical thinking process involved in scientific efforts.

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Chapter 1

Safety

A. General

Safety in the laboratory is the responsibility and concern of all present. Unsafe practices and accidents endanger cadets and instructors alike. The best safety precautions are a thinking mind and a concern for the work being accomplished. Always follow these general safety practices:

1. Always follow procedures exactly as outlined in this manual, unless told otherwise by your instructor. Deviation from stated procedures, even those that might seem minor, may cause serious accidents. For example, potassium cyanide will release the toxic gas hydrogen cyanide in an acidic solution, but is safe to use in a basic solution.

2. In the event of an accident **notify your instructor immediately** or have someone else notify him or her.

3. You are required by the chemical hygiene plan to **wash your hands** before leaving the laboratory. You will handle a variety of chemical reagents in the laboratory, many of which could harm you through contact with the skin or accidental ingestion. **Wash your hands** frequently when handling reagents; **wash your hands** anytime you suspect you may have come into contact with chemicals.

4. Protect your eyes and skin. You must **wear your goggles 100% of the time** when in lab to prevent eye injury (Figure 1-1). Skin is protected by proper wear of laboratory aprons/coats. Additionally, long pants are required at all times. Over-pants are available for cadets with temporary physical limitations that require the wear of short pants (e.g. bulky leg casts). Since contact lenses interfere with the removal of chemicals from your eyes, **contact lenses are prohibited in lab**. Never wipe your face or eyes with your hands while in the laboratory. If you feel the need to wipe your face, first wash your hands. If you feel you need to remove your goggles, wash your hands then step completely outside the lab before removing your goggles.

Figure 1-1. Lab Goggles

5. **Never eat or drink in the lab**. A water fountain is located in the hallway. No chewing gum or tobacco products are allowed in lab.

6. To work safely with chemicals you must know the potential hazards associated with their use. Safety Data Sheets (SDSs) are a valuable source of safety information about laboratory chemicals. Chemical manufacturers provide SDSs for each chemical purchased and used in the laboratory. Each lab has a yellow binder with a copy of the SDS for every chemical you will encounter in General Chemistry. Appendix D provides detailed information about SDSs.

B. Burns

1. Heat Burns

a) Cause. The most common laboratory accident is a burn resulting from touching a piece of hot equipment. Another cause of burns is accidental contact with a flame or hotplate.

b) Prevention. It is difficult to see a properly adjusted burner flame. Just as you should always assume a weapon is loaded, always assume equipment is turned ON and HOT. **Never reach across an open flame.** The metal barrel of the burner becomes very hot during operation; do not handle the burner by the barrel.

2. Chemical Burns

a) Cause. Another common burn is a *chemical* burn, which is usually caused by careless handling of chemicals such as acids and bases. A spill on skin or clothing must be thoroughly rinsed as soon as it occurs. **Inform your instructor immediately if you spill a chemical on your clothing or skin.**

b) Prevention

i) To prevent spills, carefully secure glassware with the proper clamp. If your set-up is knocked over, **get out of the way immediately**. Glassware can be replaced--you can't.

ii) Caution when transferring liquids can prevent chemical spills and burns. Carefully dispose of excess reagents in the designated waste container using the funnel provided.

iii) Spattering. **Never add water to an acid.** This is especially important in the dilution of a concentrated acid. To dilute an acid, pour a small amount of the acid down the side of the container of water and carefully mix before adding another portion. In this way, if a violent reaction causes a splatter of liquid, the majority of the liquid will be water rather than acid. Continue to add the acid in small portions. Dilution of an acid often liberates a large quantity of heat, therefore the container should never be held in your hand.

iv) "Bumping". Bumping occurs when a large vapor bubble forms rapidly and escapes from a heated test tube, taking a quantity of liquid with it (Figure 1-2). In some cases the heated liquid may suddenly escape from the test tube and injure someone in the path of the splash.

When heating liquids in a test tube, heat slowly and always point the test tube away from any person, either next to you or on the opposite lab bench, and away from sensitive equipment. The same precautions apply when heating liquids in beakers or flasks.

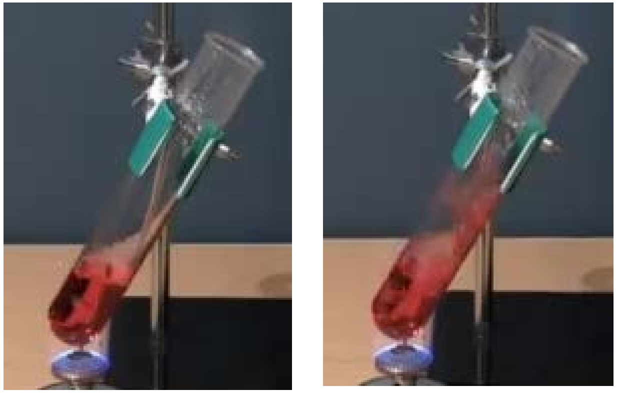


Figure 1-2. Bumping of a heated liquid in a test tube

Thenewboston (2013, June 17). Chemistry Lab -2- Test Tubes [video file]. Retrieved from http://www.youtube.com/watch?v=HQGIr89aGFQ

3. Corrective Measures

a) If any chemical come in contact with the eye, immediately proceed to the eye wash (Figure 1-3), depress the lever to the right of the basin, and flush the eye with water. Continue to flush until told to stop or for 15 minutes. Be prepared to assist other personnel. You may have to physically hold your eyes open with your hands. If you are assisting someone at the eyewash, you may have to force open their eyes to allow water to flush the eyes.

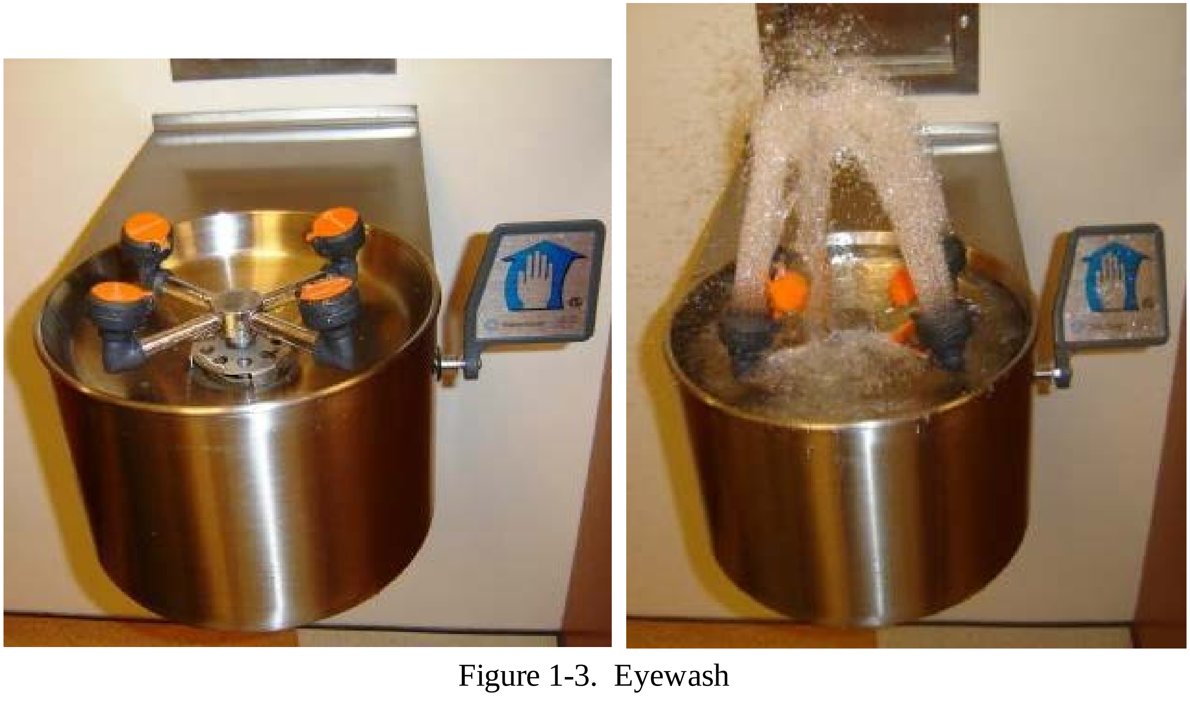


Figure 1-3. Eyewash

b) Spills on clothing are to be flushed with water. Use plenty of water as the clothing may retain the chemical and possibly cause injury to the skin underneath. In the event of a major spill on clothing, immediately move to the safety shower (Figure 1-4) and pull the chain to start the flow of water. Remove contaminated clothing.

Figure 1-4. Safety Shower

c) Spills on desktops and floors are to be wiped up with a wet sponge, rinsed, and wiped up again. Rinse the sponge to remove chemicals. If the spill is a concentrated acid, first pour sodium bicarbonate, $NaHCO_3$, over the area of the spill to neutralize the acid before wiping it up.

C. Fires

1. Prevention. A fire can be a dangerous situation, especially in a laboratory environment where chemicals may react and explode when heated. Every person in the lab should know how to respond in the event of a fire.

a) Know the location of fire-fighting equipment (Figure 1-5) and safety showers (Figure 1-4) in the laboratory.

Figure 1-5. Fire Extinguisher

b) Know the location of exits from the laboratory.

c) Wear lab aprons/coats and goggles at all times. When wearing long-sleeved shirts, keep the sleeves rolled down and buttoned.

d) Do not use flammable chemicals such as ether, acetone, carbon disulfide, or hydrogen gas in the vicinity of an open flame.

e) When you use a flame, be sure to remove all hazards from the area before ignition.

f) Never leave a burner or hot plate unattended.

2. Corrective Action. Should a fire occur, **immediately give a verbal alarm** and follow these guidelines:

a) Turn off all burners in the area and remove all flammable chemicals if you can do so without risk of injury.

b) Do not try to extinguish a fire if it is too large or you do not have the proper equipment. Leave the area immediately.

c) If clothing is on fire use the safety shower (Figure 1-4).

d) The size of the fire and the substance that is burning will determine the method to extinguish the fire. Small quantities of flammable liquid can be extinguished by covering with a glass plate, watch glass, or beaker. This will extinguish the fire by depriving it of oxygen. Allow the glass cover to remain in place for several minutes to ensure flammable material will not ignite again. The carbon dioxide fire extinguisher is used on larger chemical fires. The metal safety ring must be pulled to activate the extinguisher. Direct the spray at the base of the flames.

D. Vapor Hazards

1. Prevention. The vapors of some chemical reagents are hazardous. You will be informed of the specific dangers as they are encountered. Do not attempt to do an odor test unless specifically told to do so by the laboratory procedure or by the instructor. The proper procedure to test an odor is called **"wafting".** When attempting to smell the odor produced by a substance or chemical reaction, use the method shown in Figure 1-6. **Keep your head and nose away from the opening of the container**; use the motion of your hand to bring the vapor to your nose.

Figure 1-6. Wafting of vapors

2. Corrective Action. If, in the course of a reaction involving chemicals with potentially dangerous vapors, you feel unusual in any way, move quickly away from the area of the reaction or chemical and inform your instructor. If you notice anyone going faint, remove him/her from the immediate area and notify the closest instructor.

E. Cuts

1. Prevention. To prevent injury, follow the guidelines below.

a) Secure laboratory glassware properly to minimize accidental breakage. Always secure thermometers with a thermometer clamp.

b) Keep work areas neat and organized, with glassware away from the edges of the laboratory benches, or areas used for preparing laboratory reports.

c) Remove droppers and stirring rods from containers when not in use.

d) Inspect glassware carefully before use for cracks, nicks, or other defects.

e) Whenever you find glass tubing that is "frozen", such as stoppers stuck in bottles, do not use force to separate them. Take them to your instructor.

2. Corrective Action

a) If you should break glassware, notify your instructor as soon as possible. Ensure others are aware of the breakage and are kept clear of the area. Do not attempt to clean up the breakage yourself.

b) If directed to clean up breakage, use extreme caution when picking up fragments, particularly in a wet sink where glass may be difficult to see. Check areas carefully to ensure that all fragments have been picked up. Dispose of all glass fragments, regardless of size, in the cardboard bin clearly designated for broken glass.

c) Report any cuts to your instructor immediately. Damage to skin and surrounding tissues from glass fragments may be aggravated by the presence of toxic chemicals.

F. Lasers

The most common laser that you will come in contact with is a Class II Helium-Neon (HeNe) laser. In the event that you encounter a different type of laser in the laboratory, the primary operator or your instructor will brief you on the hazards of that particular type of laser. The HeNe laser produces a monochromatic, high intensity collimated beam of light that is hazardous to your retina. To avoid injury:

1. Treat the laser as a direct fire weapon.

2. Do not stare into the direct or specularly reflected laser beam.

3. Do not set up the laser at eye level. Direct the laser beam between nametag and belt buckle level. All cadets should stand during experiments involving lasers.

4. Do not walk through the beam; do not break the beam with your hand.

5. Take particular care when aligning the laser; predict and verify where the termination of each reflection will be. Control the laser beam to a safe, confined area.

6. Do not leave the laser unattended; turn the laser off when not in use.

7. Do not disassemble the laser.

G. Electricity

1. Chemistry. In Chemistry you may use a volt-ohm meters or low voltage power source. The D.C. power source, if not properly assembled, can deliver a damaging and painful shock to the user. Be sure your instructor approves your assembly prior to plugging in the power source or beginning the experiment. (See Table 1-1)

2. Physics. Each physics laboratory bench is equipped with an electrical transfer station that is capable of delivering enough current to kill you or your partners. Do not touch the bench power supply prior to your instructor verifying your equipment configuration and briefing you on its operation. **Remember that the current is what kills you, not the voltage.** The voltage is important only in that it determines how much current will flow through your body's resistance. (Refer to the Table 1-1 for the effects of electric current values you may encounter.)

Value (mA)	**Human Response**
8 – 15	Painful shock; individual can let go at will.
15 – 20	Painful shock; muscular control is lost; individual cannot let go.
20 – 75	Painful shock; severe muscular contractions with breathing extremely difficult. The person cannot let go.
100 – 200	Painful shock; ventricular fibrillation of the heart. (This is a fatal heart condition for which there is no known remedy or resuscitation – DEATH.)

Table 1-1. Current Values and the Human Response.

H. Radiation

In PH205/255 you will be using a radiation button source. The dosage of radiation that you will be exposed to during the lab is very small, but precautions should always be taken when dealing with radiation.

1. Principles of Radiation Safety. ALARA is a philosophy used when working with radiation. It is when one strives to keep exposure **A**s **L**ow **A**s **R**easonably **A**chievable. Considering ALARA, we ensure that **time** of exposure is always be minimized, and **distance** from the radiation source and **shielding** from the source are maximized.

2. Safety Precautions:

a) Do not handle radiation sources unnecessarily.

b) Do not put radiation sources in your pocket or mouth.

c) Wash hands after handling radiation sources.

d) Report any unsafe activities to the instructor.

Chapter 2

Characteristics and Use of Laboratory Equipment

A. Introduction

It is essential to develop good habits to avoid contamination and prevent accidents when working in a laboratory. Below are the good habits you are expected to develop and maintain when working in laboratories at West Point.

- Follow all prudent safety practices as directed by your instructor, the laboratory staff, and good common sense.
- Use caution when powering devices on and off. Always know if your device is turned on or if the device is connected to an energy source.
- Before use, wash all glassware with soapy water, rinse thoroughly, and perform a final rinse with distilled water three times. Distilled water is very expensive to produce and should not be used in great quantities, but only for final rinsing and making solutions.
- Replace caps on all reagent containers (both liquids and solids) when not in immediate use.
- Only place a clean utensil inside a reagent container or stock bottle.
- Never return used reagents to their original container, but dispose of extra reagent in an appropriate waste container.
- Dispose of all solutions/waste in an appropriate waste container. It's never a bad idea to ask if you aren't sure of proper waste disposal procedures.
- Clean up any spilled reagent immediately; maintain a neat working area and clean common use equipment, such as balances, after each use.
- Clean all equipment before returning it to its designated location.
- Wash your hands after each and every lab.

Understanding the capabilities and limitations of your laboratory equipment is an essential basic science skill. Selecting the appropriate device and knowing how to use it correctly will allow you to conduct your experiments quickly and efficiently. The precision of laboratory measuring devices discussed in this chapter can be found in Tables 2-1 through 2-4.

Each laboratory position is stocked with basic supplies. Learn the names of the pieces of laboratory equipment to make certain that you will always be using the correct equipment. It is your responsibility to see that all equipment at your assigned station is in good condition and ready for use by the next cadet assigned to that lab station. To keep the set of equipment complete for the next cadet, it is essential that any equipment shortage or breakage be reported to the instructor immediately so that replacement can be made. At the completion of each laboratory period thoroughly clean all equipment.

B. Chemistry Laboratory Equipment

1. Measuring Mass. Balances are precision instruments and proper care must be exercised in using them. Before using a balance, familiarize yourself with good practices common to all balances. Ensure the balance is level using the bubble level integrated into the instrument. Do not move or otherwise jar the balance. Gently place (do not drop) objects onto the balance pan. Never place chemicals directly on the balance pan as they will corrode and contaminate the pan in addition to causing incorrect results due to air currents and buoyancy errors. The balance pan should be kept perfectly clean by dusting with a brush after each use. Clean the workspace surrounding the balance to keep it free of dust and chemical residue.

a) Sartorius Balance. The mass of a sample between 0.01 grams and 320 grams will normally be determined on the Sartorius Balance Model ED323S, shown in Figure 2-1. To use the balance, follow the guidelines below.

i. Ensure the balance is level by checking to see if the bubble just above the yellow Tare button in the front left corner of the device is centered in the black ring. Remove the stainless steel lid. If necessary due to the large size of the substance to be measured, remove the glass shield. The glass shield and lid minimize balance fluctuations brought about by air flow and the instrument should be used with both in place when feasible. Press the blue button to turn the balance on or to bring it out of Standby mode.

ii. Before placing your sample on the pan, set the reading to 0.000 g by pressing either of the two yellow Tare buttons.

Figure 2-1. Sartorius Balance Model ED323S

iii. Place a container on the balance pan (e.g. weigh paper, plastic weigh boat, glass container). Record the digital results if you need to know the mass of container.

iv. Press one of the two yellow Tare buttons again to return the balance to 0.000 g. Taring deducts the mass of a container from the total mass of the container and its contents once the sample is added.

v. Add your sample to the container and record the digital weight after the reading stabilizes.

vi. Remove your sample and container from the balance. Use the brush to remove any spilled reagent from the balance area. Replace the stainless steel cover and turn the balance off if the lab is finished for the period. Clean the area around the balance as needed.

b) Sartorius Balance Model H110. This balance, shown in Figure 2-2, may be used to determine the mass of a sample between 0.1 mg (0.0001 g) and 110 grams. To use the balance, follow the guidelines below.

i. Ensure the balance is level by examining the bubble in the back left corner of the device. Press the yellow on/off button to turn the balance on.

ii. Before placing your sample on the pan, set the reading to 0.0000 g by pressing the red bar labeled T for tare.

iii. Open the sliding glass door on one or both sides of the balance. The glass minimizes balance fluctuations brought about by air flow and the mass should be read only with all glass doors closed. Place a container on the balance pan (e.g. weigh paper or plastic weigh boat). Record digital results if you need to know the mass of container.

iv. Press the red T bar again to return the balance to 0.000 g. Taring deducts the mass of a container from the total mass of the container and its contents once the sample is added.

Figure 2-2. Sartorius Balance Model H110

v. Add your sample to the container and record the digital weight after the reading stabilizes. You may notice dramatic fluctuations in the last digit, particularly if the balance is exposed to air currents or vibrations.

vi. Remove your sample and container from the balance. Use the brush to remove any spilled reagent from the balance area. After cleaning the balance, ensure all glass doors are closed. Turn the balance off if the lab is finished for the period. Clean the area around the balance as needed.

2. Measuring Volume of a Liquid and Handling Liquids in the Laboratory

a) Handling Liquids.

i. Contamination. Protect reagent bottles from contamination. Never put spatulas, eyedroppers, stirring rods, pipets, or anything else into a reagent bottle. Try to avoid taking a large excess of the reagent. However, if you should err and take more than you need, **do not return the excess to the original bottle**. Put the excess into a waste container. Use caution when handling container lids. Be mindful at all times of the location of the lid and handle lids in accordance with the guidelines shown in Figure 2-3 to prevent contamination. Always replace caps when not in use.

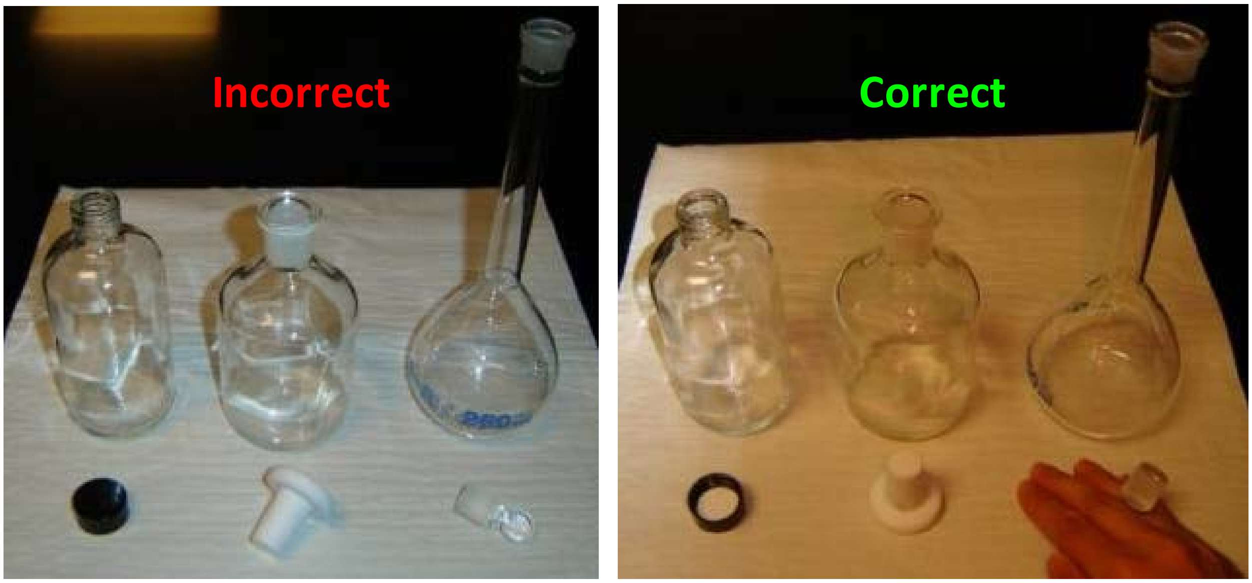
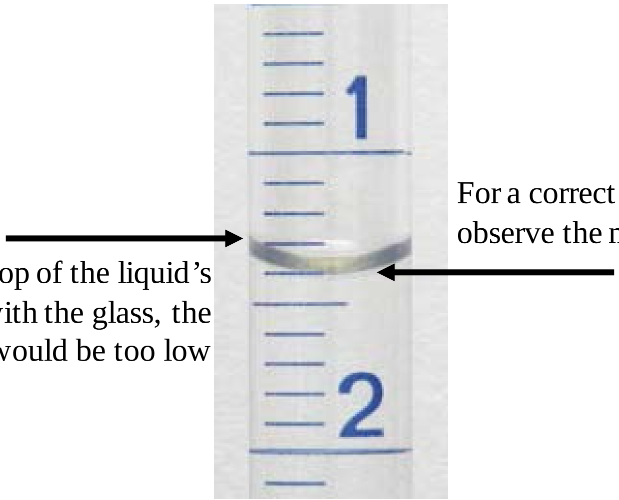


Figure 2-3. Handling Stoppers and Lids

ii. Labels. Always read the label on the reagent bottle before using it. When pouring a liquid reagent from the bottle, hold the bottle with the label up so that the reagent is poured from the side of the bottle without the label. This prevents the label from being soiled should the reagent run down the outside of the bottle. If any reagent is spilled on the outside of the bottle, wipe it off before returning the bottle to its original location.

iii. Pouring. Do not attempt to transfer reagents directly from the reagent bottle to a test tube or other small-mouthed container. Liquid reagents should be poured into a beaker first and then into the test tube.



For a correct reading of 1.40 mL, observe the meniscus at eye-level

If read at the top of the liquid's contact with the glass, the reading would be too low

Figure 2-4. Reading a meniscus

b) Reading a Meniscus. Several types of glassware are available for measuring liquids, depending on the precision desired. To use any of these devices properly, it is necessary to understand the nature of the meniscus. The surface of a liquid is called the meniscus. Water and aqueous solutions commonly used in introductory labs form a concave meniscus with the center of the surface at a lower level than the edges, because the liquid is attracted to the glass by similar intermolecular forces. Some liquids, such as mercury, form a convex meniscus with the center of the surface of the liquid at a higher level than the edges because the liquid has no appreciable attraction for glass. To maintain consistency, scientists arbitrarily decided to read the bottom of the concave meniscus and the top of the convex meniscus as shown in Figure 2-4. Always read the liquid volume with your eyes at the level of the meniscus. Otherwise, the calibration markings become parallactic and you will obtain inaccurate readings.

c) Approximate measurements of liquid volume**.** While accurate measurements of volume are important, time is wasted making extremely accurate measurements of volumes when approximate measurements would be sufficient. The approximate measuring devices described below may be used throughout the laboratory course unless more accurate measurements are required.

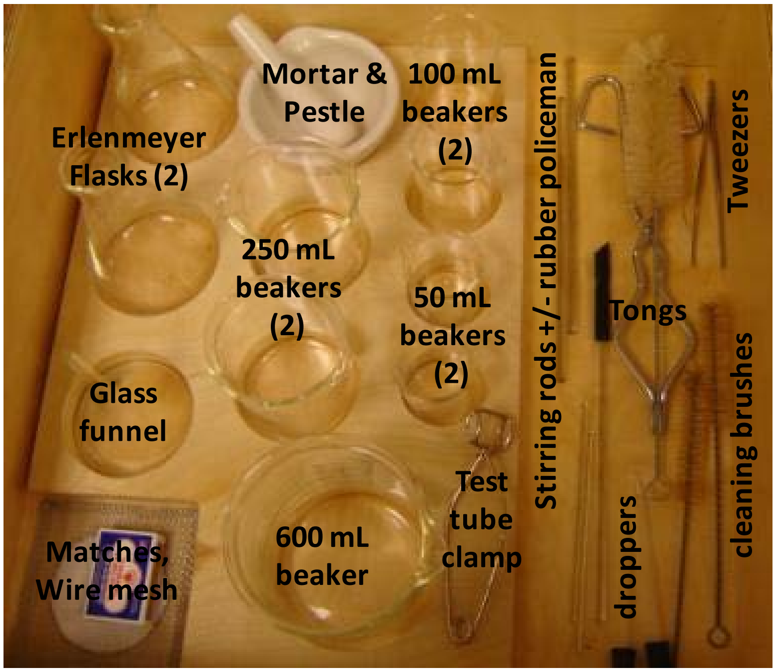


Figure 2-5. Top Drawer of Lab Station

i. Dropper. Each lab station is supplied with droppers, as shown in Figure 2-5. A dropper delivers drops with a volume of about 1/20th of a milliliter (20 drops ≅ 1 mL; 1 drop is roughly 50 µL). One full squeeze of a dropper will draw up a maximum of about 1 mL of solution. A dropper is normally used to measure approximate volumes of no more than 3-5 milliliters.

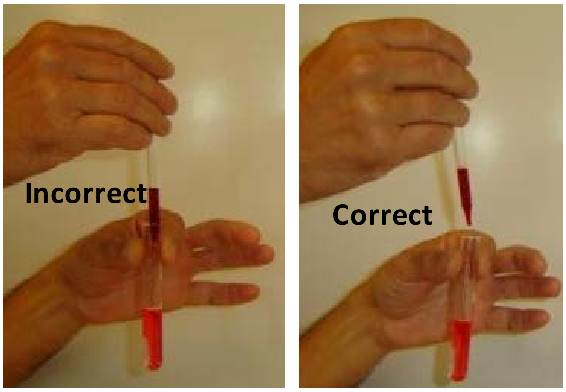


Figure 2-6. Use of Dropper

When using a dropper to transfer a reagent, use the technique illustrated in Figure 2-6. Add a drop or two and allow it to run down the side of the test tube into the liquid, and then shake to mix thoroughly. Do not touch the test tube with the tip of the dropper. Never use the same dropper for two different reagents. If you find a dropper missing, or if you are in doubt as to where a dropper belongs, request a replacement dropper from your instructor. Never return a chemical to its original container once it has been withdrawn. Expel the unused liquid into a waste beaker.

ii. Graduated Beakers and Flasks. Each lab station has a drawer containing multiple beakers and Erlenmeyer flasks (Figure 2-5). Remember these containers, while useful for transporting or transferring liquids, provide you only a rough guide to the amount of liquid in the container. They have a 5% error associated with their graduations. If you are asked to precisely measure a volume of liquid, you will need to use a device designed for precise measurements, described below.

d) Precise Measurements of liquid Volume. In experiments requiring more accurate measurements of volume, the following equipment is available:

i. Graduated Cylinder. Each lab station has a drawer containing two graduated cylinders, as shown in Figure 2-7. A graduated cylinder is used to measure volume. For volumes smaller than 10 mL, the volume that adheres to the walls of the graduated cylinder is appreciable. The 50 mL graduated cylinder located in your desk is calibrated in 1-mL increments. Read the volume to the nearest 0.1 mL but record the volume as your reading ± 0.5 mL, as that is the stated precision of the cylinder, as shown in Table 2-1.

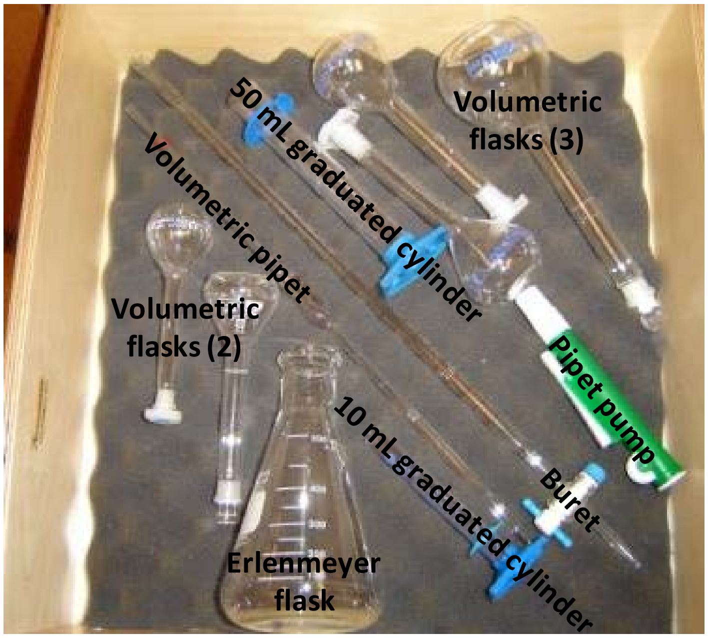


Figure 2-7. Second Drawer of Lab Station

ii. Volumetric Pipet. Each lab station has a drawer containing one 10 mL volumetric pipet and associated pump as shown in Figure 2-7. A volumetric pipet is used to measure one volume accurately. The volume of the solution to be delivered is made to the only calibration mark etched on the pipet. To measure liquid with a volumetric pipet, hold the glass pipet near the widest of the two ends and use your fingertips to gently but firmly insert the glass pipet into the rubber gasket end of the pipet pump. Ensure you gently grasp the glass pipet near the end close to the pump. Holding a pipet incorrectly while attaching it to the pump can result in a deep puncture wound if too much force is applied. Holding the pipet with the narrow tip in the desired liquid, rotate the thumbwheel downwards. This will cause the liquid to enter the pipet and simultaneously for the top of the pipet pump to rise. Continue to draw the liquid into the pipet until the meniscus is just above the calibration mark. Do NOT draw the solution into the pipet pump itself. While holding the pipet vertically over a clean waste beaker, rotate the thumbwheel upwards until the bottom of the meniscus reaches the calibration mark. The tip of the pipet is then touched to the side of the clean waste beaker to remove any excess solution. Without losing any solution from the pipet, transfer it vertically to the receiving vessel. Dispense the solution down the side of the receiving vessel by pressing the side lever and touch the tip to the side of the vessel to remove the last drop of liquid from the pipet. Any solution remaining in the tip is not to be blown out.

iii. Repipets. A repipet (Figure 2-8), is a calibrated dispenser used to transfer fixed volumes of liquids up to 20mL. The repipet is usually attached to the top of a large bottle of stock solution from which a measured amount of solution can then be drawn directly without pouring the solution into a secondary container, thus minimizing waste. The repipet will normally be adjusted by your instructor and primed to deliver the appropriate amount of reagent. To use the repipet, lift the cylinder straight up until it stops, then, making certain to hold the desired glassware under the outlet tip, depress the cylinder completely to release the measured amount of liquid.

iv. Burets. Each lab station has a 25 mL buret, as shown in Figure 2-7, a buret clamp, Figure 2-9, and a cabinet containing a buret stand, Figure 2-10. A buret is a graduated glass tube with a stopcock at the lower end for controlling the flow of solution.

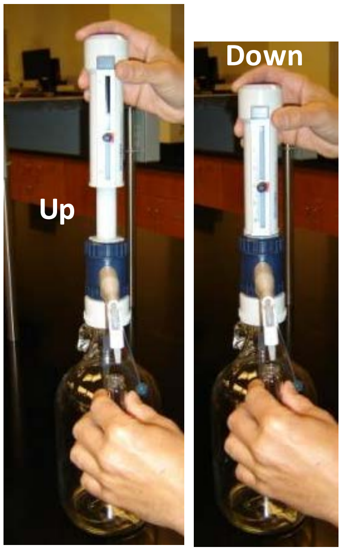


Figure 2-8. Re-pipet

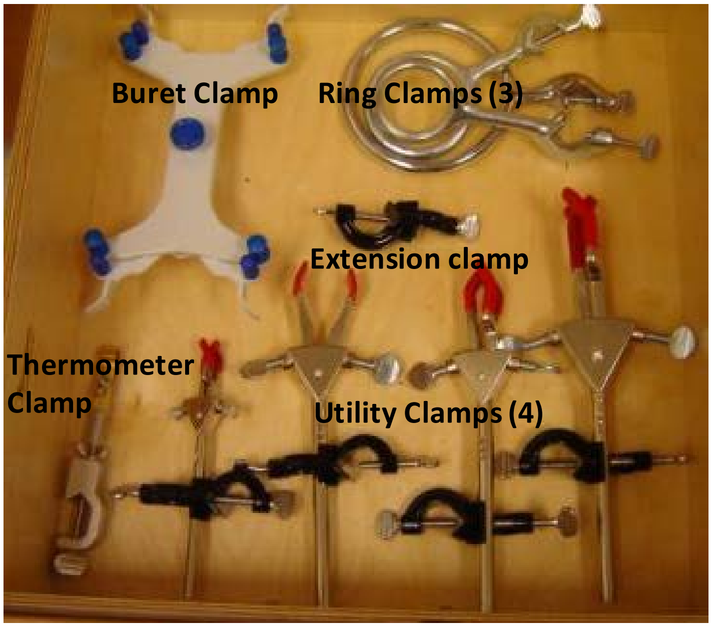


Figure 2-9. Third Drawer of Lab Station

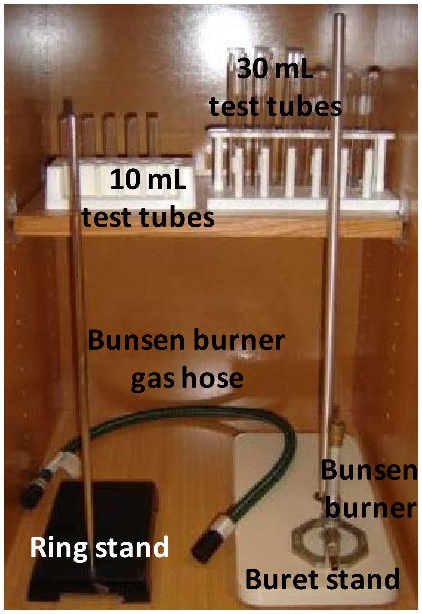


Figure 2-10. Cabinet at Lab Station

Always use a buret clamp to secure a buret on the support stand as shown in Figure 2-11. Before using a buret, first rinse it three times with roughly 5 mL portions of the solution with which it will be filled, making certain that all inner surfaces are wetted with each rinsing. This prevents contamination and dilution of the solution. These small portions should be drained through the tip of the buret into a waste beaker. Next fill the buret with solution using a glass funnel and drain a small amount until the bottom of the meniscus is on the graduated portion of the buret and the tip is free of air bubbles. Touch the tip against the side of a clean waste beaker to remove the drop clinging to the tip and then take the initial reading. It is not necessary to have the meniscus at zero, but it is important to record a careful reading of the meniscus at the start point before dispensing the solution.

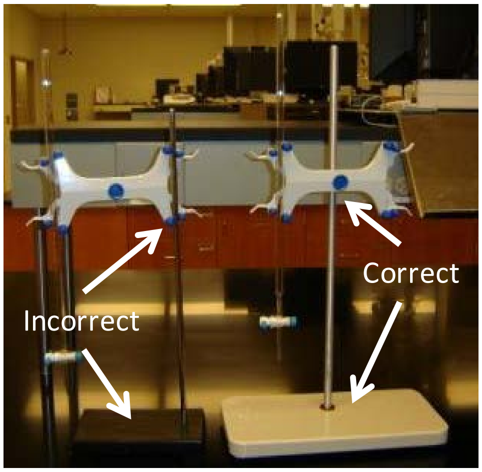


Figure 2-11. Proper Use of Buret

iv. Volumetric Flasks. Volumetric flasks are pear-shaped glass vessels that are calibrated to hold a precise volume of liquid when the bottom of the meniscus of the liquid is at the calibration mark. Volumetric flasks are used to prepare solutions with known concentrations. Their use is described in the next section.

Table 2-1. Device Uncertainty Data

	DEVICE	GRADUATION	PRECISION
Mass	Sartorius Balance ED323S	0.001 g	± 0.001 g for < 60 g ± 0.01 g for 60-320 g
	Sartorius Balance H110	0.0001g	± 0.001 g
Volume	Dropper	20 drops ≈1squirt≈1mL	± 0.025 mL
	10 mL Graduated Cylinder	0.2 mL	± 0.2 mL
	50 mL Graduated Cylinder	1 mL	± 0.5 mL
	50/100/250/600 mL beakers	10-50 mL	5%
	250 mL Erlenmeyer flask	10-50 mL	5%
	10 mL volumetric pipet	Single mark at 10 mL	± 0.01 mL
	repipet	0.1 mL	± 0.01 mL
	25 mL buret	0.1 mL	± 0.01 mL
	50 mL volumetric flask	Single mark at 50 mL	± 0.05 mL
	100 mL volumetric flask	Single mark at 100 mL	± 0.08 mL
	250 mL volumetric flask	Single mark at 250 mL	± 0.12 mL
Other	alcohol thermometer (0-100°C)	1°C	± 0.1°C

3. Preparing Solutions

a) Aqueous Solution from a Solid.

i. In preparing an aqueous solution by dissolving or dissociating a solid in water, place the solid in a volumetric flask using a clean, dry funnel. Flush the solid from the funnel into the flask with distilled water. When the volumetric flask is 3/4 full, dissolve the solid by swirling.

ii. Fill the volumetric flask with distilled water to within one centimeter of the calibration mark, remove the funnel, and fill the flask exactly to the mark with a clean dropper. Stopper the flask and invert the flask 10 times to thoroughly mix the contents, ensuring the stopper remains seated.

b) Aqueous Solution from a Liquid.

i. In preparing an aqueous solution from a liquid, pour the liquid into the volumetric flask using a clean funnel. The flask from which the liquid was obtained is washed two or three times with a few milliliters of distilled water. The washings are added to the volumetric flask. The funnel and the neck of the volumetric flask are rinsed with distilled water.

ii. Fill the volumetric flask with distilled water to within one centimeter of the calibration mark, remove the funnel, and fill the flask exactly to the mark with a clean dropper. Stopper the flask and invert the flask 10 times to thoroughly mix the contents, ensuring the stopper remains seated.

4. Physical Separation of Mixtures

a) Solid-Liquid

i. Decanting. Decanting is a method of separating the components of a liquid-solid (precipitate) solution. It is accomplished by allowing the solid to settle to the bottom of the vessel (the use of a centrifuge will hasten this) and then gently pouring off the liquid, leaving the solid at the bottom of the container. If the precipitate needs to be washed, a solvent in which the precipitate is insoluble may be added, mixed with the precipitate, and poured off in the same manner. Generally, several washings using small volumes of solvent are better than one washing using a large volume of solvent. When washing small amounts of precipitate in a small test tube, it is better to draw the washing solvent off with a dropper rather than pouring off the liquid.

Figure 2-12. Filter Paper

ii. Filtering. Filtering is a process used to separate a solid (precipitate) from a liquid by passing the liquid through filter paper. The filter paper will allow the liquid (filtrate) to pass through but will retain the precipitate (residue) on the filter paper. Correct filtration techniques prevent contamination of solutions and save time. A common type of filtration is using a glass funnel with filter paper, illustrated in Figure 2-12. Fold the filter paper in half and then half again. Spread out the folded paper in the glass funnel and wet it with distilled water. After the filtrate has drained from the precipitate, solvent may be added and allowed to drain through the filter paper. If the washing solvent is water, it can be added conveniently from the washing bottle, directing the stream at the filter paper above the precipitate.

5. pH Determinations and Titration

a) pH Test Strips. In many experiments the volume of liquid to be tested will be small and the use of the pH meter described below is not practical. In this case, it may be appropriate to use one of a variety of commercially available pH test strips. These strips are made using paper that has been treated with a substance that will change color within a specific pH range. These test strips are available in a wide variety of pH ranges. An acceptable procedure is to lay the pH strips on a clean, dry surface and extract a drop of the solution with a clean, dry stirring rod. Touch each colored square on the strip with the stirring rod and carefully compare the color to the provided chart to determine the pH of the solution.

b) Indicators. An indicator is a chemical that changes color depending on the pH of the solution to which it is added. A list of some common indicators, their color changes, and the pH range in which this color change occurs can be found in your textbook.

c) pH Meter. The pH meter can be used to directly measure the pH of a solution. Its operation varies depending on the model, but generally requires the meter to be suspended above a solution with a stir bar to ensure an accurate reading. Each lab station is equipped with a heater/stirrer as shown in Figure 2-13 to facilitate thorough mixing and an accurate reading.

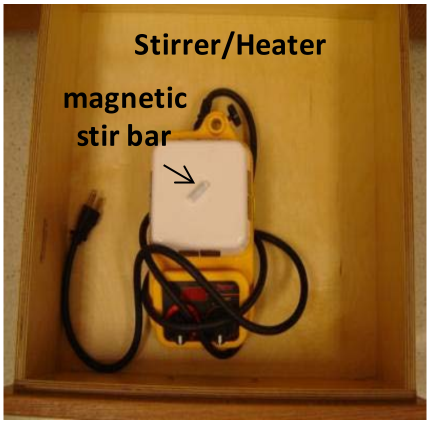


Figure 2-13. Bottom Drawer (right side) of Lab Station

d) Titration. Titration is a process used to determine a variety of information about a solution, including the precise concentration. A buret is used for measuring the added solution (titrant). An Erlenmeyer flask, wide-mouth bottle, or beaker may hold the solution being titrated. The titrant is added to the solution being titrated until the "equivalence point", that is the point where the number of particles required to react completely with the solution being titrated have been added, is reached. It is often impossible to determine the equivalence point directly. In this case, an indicator is added to the solution being titrated. The indicator is selected so that it will undergo a change in color or texture at or near the equivalence point. When the indicator changes color, the "end point" of the titration has been reached. Titrations are normally performed in two phases. In the first phase, a fairly rapid addition of the titrant is made to determine the approximate volume of the titrant necessary to reach the end point of the titration. Once this approximate volume has been determined, the subsequent runs are made fairly rapidly to within about ten milliliters of the end point. Then the rate of flow is decreased to a much slower rate so that the titration may be stopped when the first sign of a permanent color change in the indicator occurs. Subsequent runs are often made to obtain the average volume of titrant required to reach the end point.

6. Bunsen and Fisher Burners. The Bunsen burner is used to apply heat in the lab. The burner makes possible the nearly complete combustion of gaseous fuels by providing the correct ratio and mixing of fuel and air in the barrel of the burner. Since the Bunsen burner will be used in many of the experiments you perform, it is important for you to understand how it operates and how to adjust the air and gas to obtain maximum temperatures. Study Figure 2-14 and be sure that you understand the proper function of the components before attempting to use the Bunsen burner. Some lab stations are equipped with a Fisher Burner, which is the larger of the two burners at those stations and is chrome plated. The Fisher Burner produces a larger and hotter flame and should only be used when directed.

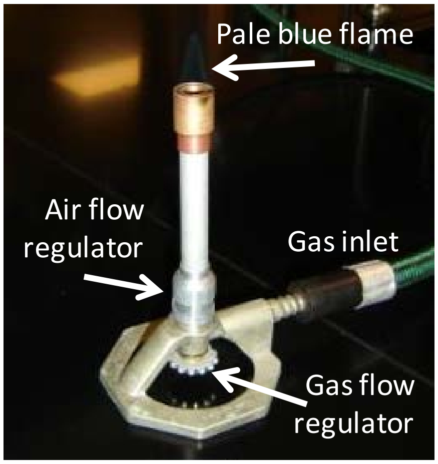


Figure 2-14. Bunser Burner

a) Procedures for Lighting a Bunsen Burner

i. While holding the burner upside down, completely **close the gas flow regulator** by rotating the knob to the right (clockwise as you look at the bottom of the burner).

ii. Slightly **re-open the gas flow regulator by rotating the knob one-half turn** to the left (counter-clockwise as you are looking at the bottom of the burner). Set the burner on the benchtop as shown in Figure 2-13 and attach the rubber hose to the gas inlet of the burner and the gas valve at your laboratory workstation.

iii. **Light a match** and hold it <u>away</u> from the burner.

iv. <u>After</u> the match is lit, **completely turn on the gas knob** at your bench by rotating the knob <u>all the way</u> to the left (counter-clockwise). Immediately bring the burning match to the side of the burner and then raise it straight up until the flame is level with the top of the burner. The burner should light.

v. **Adjust the flame** until it becomes almost colorless by manipulating the gas flow regulator and the air flow regulators <u>on the burner</u>. Do <u>not</u> attempt to adjust the burner flame by partially opening the gas valve at your workstation. The gas valve at your workstation should be fully open (when in use) or fully closed (when not in use). Control the flow of gas with the burner itself. When the proper amount of air and gas are entering the burner tube, the flame should have a pale blue inner cone with a surrounding pale violet cone. The hottest part of the flame is the area just on top of the inner blue cone. The coolest area is at the bottom of the cone just above the barrel. If the flame is yellow, you may need to reduce the amount of air by partially closing the air flow regulator on the burner. If the flame is blue, but too large, you may need to partially close the gas flow regulator on the burner.

b) Bunsen Burner Safety

i. If the burner does not light on the first attempt shut off the gas knob at your bench by turning it all the way to the right. Check that all rubber tubing connections are tight. Attempt to light the burner again. If it still doesn't work, consult your instructor. When you are finished using the burner, be sure to completely turn off the gas knob at your workstation.

ii. **Hot glassware looks exactly like cold glassware, and hot metal looks like cold metal**. Be patient and if in doubt assume the glass or metal is still hot. While hot glassware looks like cold glassware, extremely hot glass may turn red shortly before it begins to melt. If at any time you notice your glassware turning red, immediately remove the heat source.

iii. Never use a burner to directly heat volumetric glassware, bottles, or non-heat-resistant apparatus.

iv. Never use an open flame near flammables or combustibles.

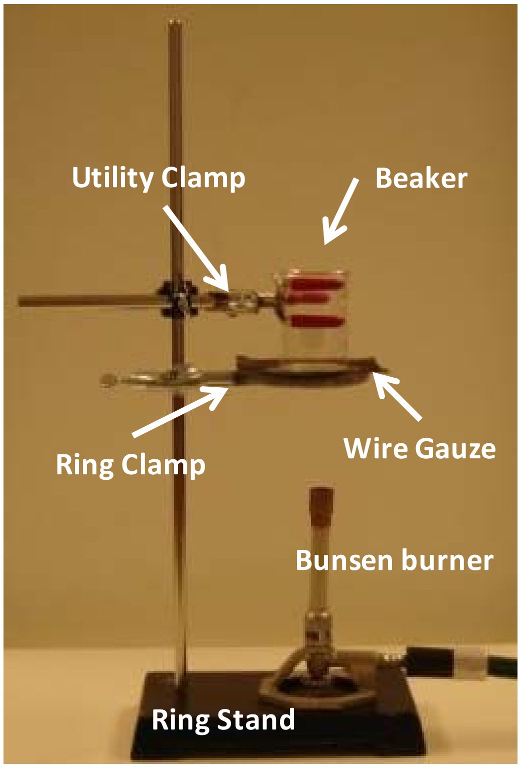


Figure 2-15. Common Heating Set-Up

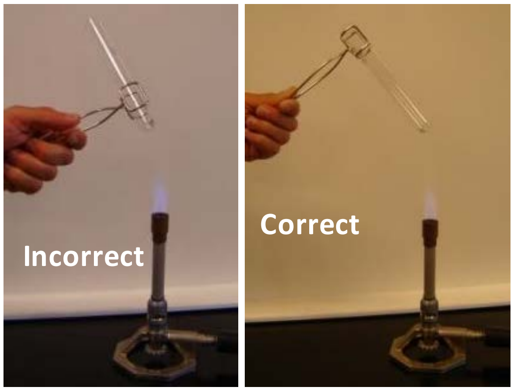


Figure 2-16. Test tube heating

v. Snorkel hoods remove fumes produced upon heating. The snorkels are located above the lab bench but should not be touched unless specifically directed. Ensure your heating set-up is not located close enough to the snorkel to melt the plastic cone. A common set-up for heating of a beaker and for heating a reagent in a test tube are shown in Figures 2-15 and 2-16.

7. MicroLab™. Each lab station is equipped with a MicroLab™ device and associated clear plastic equipment box (Figures 2-17 and 2-18). You will use MicroLab™ in many experiments. The power button for the MicroLab™ device is in the upper right corner of the white box covered by a green membrane. Once the power is turned on, the "o" in the word "Micr**o**Lab" will light up in green. When lab is over, always turn off the device to conserve energy.

Figure 2-17. Bottom Drawer (left side) of Lab Station

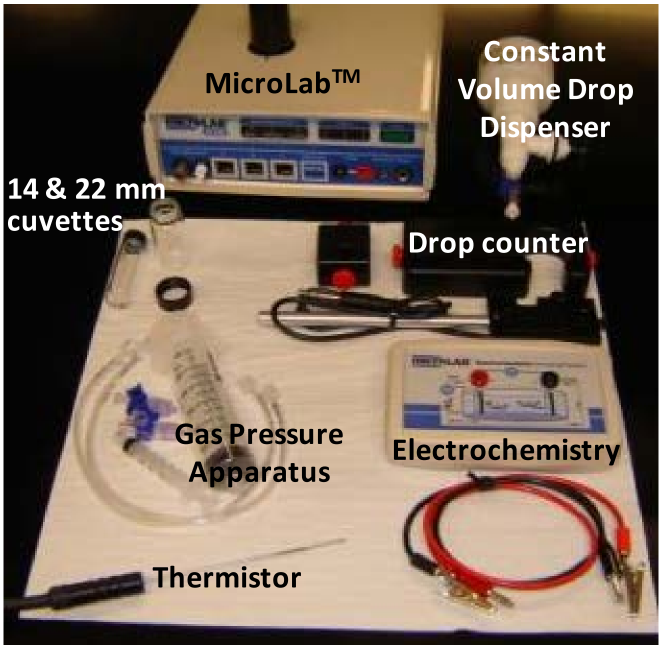


Figure 2-18. MicroLab™ and some Associated Equipment

C. Physics Laboratory Equipment

1. Triple-Beam Balance. The majority of triple beam balances in the Physics laboratories are O'Haus balances with a resolution of 0.1 grams. Cadets should verify the zero and ensure the cleanliness of the balance before use.

2. Digital Balance. The Adam HCB 1502 precision digital balance measures mass rounded to the nearest 0.05 g. This rounding creates uncertainty in mass measurements with this device, δm, of ±0.03g.

3. Digital Caliper. The digital calipers used in physics lab rooms have a resolution of .01 mm, but based on the manufacture’s specifications, are precise to ±0.03mm.

4. HeNe Laser. The Helium-neon (HeNe) gas lasers used as a collimated, coherent, monochromatic light source in physics laboratories are class II, 0.95 mW lasers made by Oriel Corporation or Uniphase. They are not eye safe (refer to section 1.F. Alignment of the laser and accompanying optical equipment is critical to collecting accurate data. Misalignment is a major source of systematic error. The HeNe laser produces a light beam of wavelength 632.8 ± 0.1 nm.

5. Multimeter

a) General Digital Multimeter Instructions. A multimeter is an instrument that can be used to measure current (amperes), resistance (ohms), and electric potential (volts). See Figure 2-19.

b) Using the Multimeter to Measure Resistance

i. A device that measures resistance is called an **ohmmeter**. The ohmmeter uses a self-contained battery to provide a known voltage to the material to be measured. When measuring resistance with a multimeter, the element being measured must be removed from its circuit and connected directly to the meter.

ii. Select the resistance mode **before** connecting the multimeter to the elements, or run the risk of damaging the multimeter. Select the resistance mode by rotating the dial (labeled ①in Figure 2-19) until it points to the position labeled "Ω." Place the leads in the resistance (②) and common (③) slots. To measure an individual resistor's resistance, R, connect the leads to opposite sides of the resistor after it has been removed from the circuit. To measure the equivalent resistance, R_{eq}, of a number of resistors in a circuit, connect the leads on opposite sides of the resistors in the circuit. **Do not turn on the power to your circuit.**

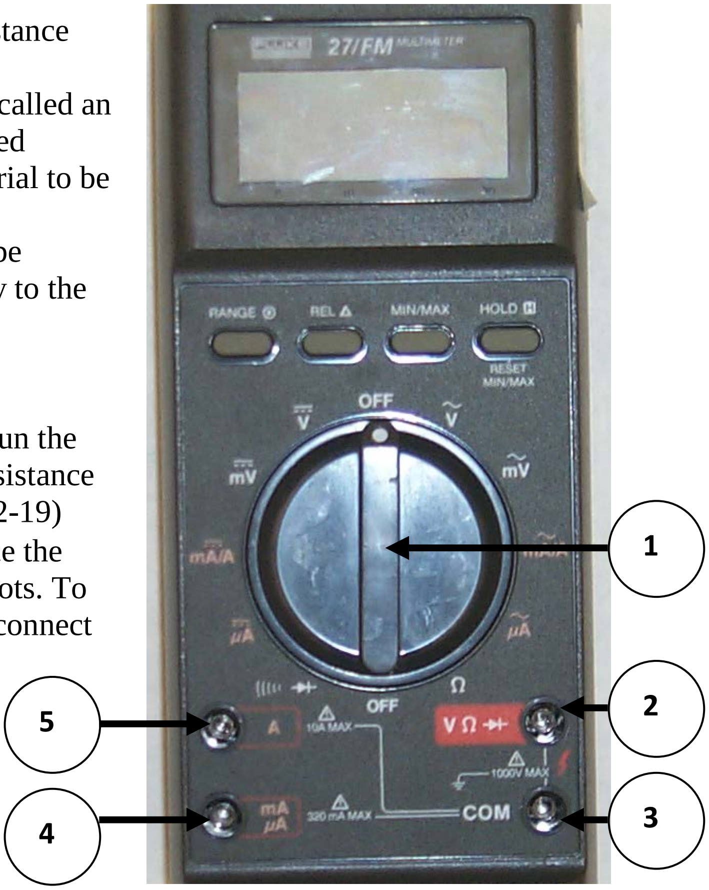


Figure 2-19. Digital Multimeter (front)

c) Using the Multimeter to Measure Electric Potential

i. A device that measures potential differences (voltage) is called a **voltmeter**. Potential difference is always measured *between* two points in a circuit, since only the *difference* in potential has physical consequence. A voltmeter should always be placed in **parallel** with the circuit segment across which potential difference is measured.

ii. To configure the multimeter to measure electric potential difference (voltage), select the voltage mode and scale by rotating the dial (①). To measure voltages greater than 0.3 V, set the dial to V. To measure voltages less than 0.3 V, set the dial to mV. To measure DC voltage, set the dial to one of the positions with the straight bars above the letters (⎓); to measure AC voltage, set the dial to one of the positions with the tilde (˜) over the letters. Select your mode and scale **before** connecting it to the circuit, or run the risk of damaging the multimeter. Place the leads in the volt (②) and common (③) slots.

d) Using the Multimeter to Measure Current, *i*

i. A device that measures the current in a circuit is called an **ammeter**. The current in a circuit is a measure of the number of electrons passing through a particular point per second. Unlike the voltmeter, the ammeter becomes part of the circuit. It must have current flowing through it to measure the current in that branch of the circuit.

ii. To configure the multimeter to measure current, select the ammeter mode and scale by rotating the dial (①). To measure current greater than 0.3 mA, set the dial to mA/A. To measure currents less than 0.3 mA, set the dial to μA. To measure DC current, set the dial to one of the positions with the straight bars above the letters (⎓); to measure AC current, set the dial to one of the positions with the tilde (˜) over the letters. Be sure the power to the circuit is turned off before connecting the ammeter.

iii. Break the circuit at the point you want to measure current. Connect one end of the break to the common slot of the ammeter (③) and the other to the amp slot (⑤) (if measuring a current greater than 320 mA) or milli/microamp slot (④) (if measuring a current less than 320 mA). Before activating the power, **be sure there is a resistive element in the same branch of the circuit as the ammeter**, or you will blow a fuse.

e) Uncertainties. The measurement of a quantity by a digital meter is suspect for two reasons: inaccuracy of the instrument and random uncertainties. The *accuracy* of a digital measuring device depends on three factors: calibration, component quality, and power source (battery). All of the digital meters in the department have been calibrated or have been checked against a calibrated meter. The quality of the devices is dependent on the cost of its components. You would expect a $200 voltmeter to be more precise and accurate than a $25 voltmeter. The tolerances of the components of the $200 voltmeter should be smaller than those of the $25 voltmeter. Based on these tolerances, the manufacturer guarantees that his device will give an accurate reading to a stated uncertainty.

The random uncertainties stem from our lack of knowledge on the round off criteria of the last digit, variations in the point of contact of the probes, and other physical fluctuations. This absolute uncertainty can be estimated by taking multiple measurements and using a standard deviation.

A better estimate uses the manufacturer's specifications (listed in Tables 2-2 through 2-4 for the laboratory multimeters). To use the table to find the contribution to the absolute uncertainty of a measurement due to the quality of the instrument, use the following formula: In Equation 3-1, 3-2, and 3-3, the measurement is the actual reading on the multimeter and all of the other quantities are taken from Tables 2-2 through 2-4. The row in the table that you use depends on the magnitude of the value shown on the multimeter display. Choose the row where the display value is **less** than the Range value but **more** than the Range value in the row directly above it.

Remember, the quality of the instrument is only one of the four factors used in determining the total absolute uncertainty of your measurement (see Chapter 3, Paragraph A-2).

Uncertainty in a Resistance Measurement

$$\delta R = \left[(R)\times(\text{relative uncertainty})\right] + \left[(\text{resolution})\times(\text{\# of least significant digits})\right] \quad (2\text{-}1)$$

Function	Upper Limit	Relative Uncertainty	Resolution	# of Least Significant Digits
Ω	320.0 Ω	.003	0.1 Ω	2
	3.200 kΩ	0.0025	0.001 kΩ	1
	32.00 kΩ	0.0025	0.01 kΩ	1
	320.0 kΩ	0.0025	0.1 kΩ	1
	3.200 MΩ	0.0025	0.001 MΩ	1
	32.00 MΩ	0.001	0.01 MΩ	1

Table 2-2. Specifications for Multimeter used as an Ohmeter

Uncertainty in a Voltage Measurement

$$\delta V = \left[(V)\times(\text{relative uncertainty})\right] + \left[(\text{resolution})\times(\text{\# of least significant digits})\right] \quad (2\text{-}2)$$

Function	Upper Limit	Relative Uncertainty	Resolution	# of Least Significant Digits
DC mV	320.0 mV	0.001	0.1 mV	1
DC V	3.200 V	0.001	0.001 V	1
	32.00 V	0.001	0.01 V	1
	320.0 V	0.001	0.1 V	1
	1000 V	0.001	1 V	1

Table 2-3. Specifications for Multimeter used as a Voltmeter

Uncertainty in a Current Measurement

$$\delta i = \left[(i)\times(\text{relative uncertainty})\right] + \left[(\text{resolution})\times(\text{\# of least significant digits})\right] \quad (2\text{-}3)$$

Function	Upper Limit	Relative Uncertainty	Resolution	# of Least Significant Digits
DC μA	320.0 μA	0.0075	0.1 μA	2
	3200 μA	0.0075	1 μA	2
DC mA	32.00 mA	0.0075	0.01 mA	2
	320.0 mA	0.0075	0.1 mA	2
DC A	10.00 A	0.0075	0.01 A	2

Table 2-4. Specifications for Multimeter used as an Ammeter

EXAMPLE. You measure the resistance of a resistor and the reading on the multimeter is 287.4 Ω. Based on Table 2-2, the *resolution* of this measurement is 0.1 Ω. Using Equation (2-1), the *absolute uncertainty* would be at least

$$(287.4\ \Omega)(0.003) + (0.1\ \Omega)(2) = 1.062\ \Omega.$$

If no other factors affect the measurement (see Chapter 3, Paragraph A, 2), the resistance reported as a confidence interval would be (287 ± 1) Ω.

Chapter 3

Data Collection and Reporting

A. Measurement

The values for many of the physical concepts that we study are based on the results of measurements. A large part of laboratory work consists of obtaining and recording measurements. Care in taking and reporting measurements is critical to arriving at valid conclusions.

1. Components of a Measurement. No measurement in an experiment is exact. There are specific elements to include when taking measurements in scientific experiments. For example, in an experiment to determine the acceleration due to gravity near the earth's surface, one might measure a value of $g = (9.807 \pm 0.001)\ \mathrm{m/s^2}$. As in this example, useful measurements must provide the following information:

a) The numerical value (magnitude) of the measured quantity. In our example, this value is $9.807\ \mathrm{m/s^2}$.

b) The Absolute Uncertainty (defined in Section A3 of this chapter) of the measured quantity is $0.001\ \mathrm{m/s^2}$. The number of digits used to report a measurement provides information about the uncertainty in that measurement. Ensure you follow the rules for significant figures as stated in Appendix E (Rules for Significant Figures) throughout your Chemistry and Physics courses.

c) The units of the measured quantity.

2. Factors Affecting Experimental Measurements.[1] **Every measurement has an uncertainty!** This uncertainty is not a mistake or blunder, but an estimate of how confident you are in your ability to make the measurement. There is no prescribed method for how to determine the absolute uncertainty for a single measured quantity, but it may be a function of one or more of the following factors:

a) The quality of the measuring instrument.

i. Mass can be measured with a triple beam balance or an electronic scale. Each instrument may measure mass to a different resolution. The resolution is the smallest graduation on a scale or the last decimal place in a digital readout. For example, if a cadet measures a mass using the triple beam balance as shown in Fig. 3-1, they might estimate the value of the mass to be between 273.35 and 273.40 grams. The resolution of the scale on the triple beam balance is 0.1 grams, but because of their ability to estimate they could reasonably say that the absolute

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[1] Taylor, John, R. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. 2nd ed. Sausalito, California: University Science Books, 1997, pp. 3-16.

uncertainty of the measurement is actually smaller, in the range of 0.05 grams. The absolute uncertainty of a measurement may be different than the resolution of the scale. As this example illustrates, it might be smaller. Too often we assume that the absolute uncertainty is based solely on the resolution of the measuring instrument. If any of the factors listed in paragraphs b-d that follow are important, the uncertainty might also be larger than the resolution. (Consider trying to use this balance inside a HMMWV that is driving down a bumpy road!)

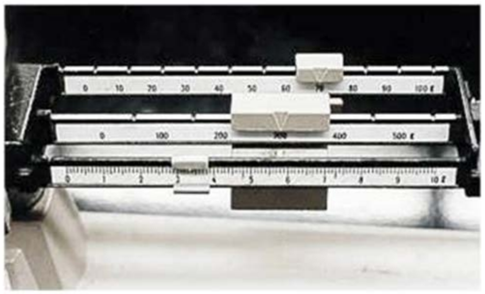
Figure 3-1. Triple Beam Balance

ii. Instruments used to take measurements may have a random variation in their response. An example of this is the wandering rest point when using a mass or a spring scale that may oscillate slightly.

iii. There may be uncertainty associated with the appropriateness of the measuring instrument in terms of the object being measured. For example, there would be greater uncertainty in measuring the length of a football field with a meter stick than there would be with a thirty-meter steel tape.

b) The nature of the quantity being measured. Sometimes measuring a quantity may be difficult because we are unable to determine exactly where or when to start or stop measuring. Also the sample may not be uniform or consistent. For example, the length of a rectangular sheet may be greater on one side than the other. Some quantities, such as a person’s blood pressure, naturally fluctuate on the time scale at which measurements are taken.

c) The limitations of humans. Human limitations (senses, reaction time, etc.) may limit the precision of a measurement. For example, if a time interval is measured with a stopwatch, the main source of uncertainty is not in the resolution of the stopwatch (many inexpensive stop-watches have a resolution of 0.01 seconds); rather, the largest contributors to the absolute uncertainty in the time measurement are usually the reaction times associated with starting and stopping the timer. A much better estimate of the absolute uncertainty for this type of time measurement would be 0.1 seconds.

d) The conditions under which the measurement is made. Most of our measurements are made in a laboratory and the environment will have little effect on our ability to make a measurement. But, if we had to read a scale with very little light, or operate a stopwatch in below freezing conditions, we may not be as confident in our measurement. Even in a laboratory, variations in temperature, air turbulence, or lighting can affect measurements in random ways. Included in this category are uncertainties in a measurement due to parallax. Parallax is the error that may occur when reading a scale because the observer’s eye and the pointer are not perpendicular. This error can be reduced by ensuring that the eye and the instrument’s pointer are in a line perpendicular to the plane of the scale.

3. Types of Uncertainty.

a) Absolute Uncertainty. The amount (often stated in the form $\pm\delta x$) that indicates the range in which the desired or true value most likely lies. Example: $(x \pm \delta x)$ m.

$$\text{Absolute Uncertainty} \equiv \delta x.$$

b) Relative (Fractional) Uncertainty. The ratio of the absolute uncertainty to the measured value. This is also known as the fractional uncertainty of the measured value and is often expressed as a percentage or fraction.

$$\text{Relative Uncertainty} \equiv \frac{\delta x}{|x|} \qquad (3\text{-}1)$$

Note: The absolute uncertainty of a measurement has units, but relative uncertainty does not.

c) Examples Using Absolute and Relative Uncertainties.

i. Determining the relative uncertainty from a confidence interval for a measured value: Given the confidence interval for a measurement of speed as $v = (66.3 \pm 0.5)$ m/s, calculate the relative uncertainty of the speed. In variable form, the confidence interval for the speed would be

$$(v \pm \delta v) \text{ m/s},$$

where v is the measured value and δv is the absolute uncertainty of the measured value. In order to calculate the relative uncertainty of the speed

$$\text{Relative Uncertainty} \equiv \frac{\delta v}{|v|} \qquad (3\text{-}2)$$

Substituting the appropriate values into Equation 3-2 results in

$$\text{Relative Uncertainty} = \frac{0.5 \text{ m/s}}{66.3 \text{ m/s}},$$

$$\underline{\underline{\text{Relative Uncertainty} = 0.0075 \text{ or } 0.75\%.}}$$

ii. Determining the absolute uncertainty from the relative uncertainty of a measured value. Given that the relative uncertainty of a measurement of resistance ($R = 243\ \Omega$) is 5.0%, calculate the absolute uncertainty and report the resistance as a confidence interval. Begin with the equation for relative uncertainty and solve for the absolute uncertainty (δR) as shown;

$$\text{Relative Uncertainty} = \frac{\delta R}{|R|}$$

$$\delta R = |R| * (\text{Relative Uncertainty})$$

$$\delta R = (243\ \Omega)(0.050) = 12.15\ \Omega$$

Using the rounding rules outlined below, we report this as a confidence interval

$$\boldsymbol{R = (240 \pm 10)}\ \Omega.$$

4. Reporting the Measurement.

a) Confidence Intervals. When we report the measured value, x, and the absolute uncertainty, δx, together, we call this a confidence interval.

b) Rounding Rule for Stating Uncertainties[2]. Since the absolute uncertainty is only an estimate, in most cases it is sufficient to round to one significant figure. This practice is based on the idea that if, for example, a quantity is uncertain to within one-tenth, then reporting uncertainty to the hundredths or thousandths place is meaningless. For relative uncertainty the number of significant figures depends on the precision of the measurement. In the core science courses the use of two significant figures will be used allowing for a comparison and analysis of data.

c) Rounding Rule for Writing Confidence Intervals[3]. The last significant figure in any stated answer will be of the same order of magnitude (in the same decimal position) as the last digit of the absolute uncertainty. For example, we would not report a number such as 2.1789345 meters if we were really only sure of the reading to within 0.01 meters (1 centimeter). Instead, we would report the value as 2.18 ± 0.01 meters, i.e., we estimate that the value lies somewhere between 2.17 and 2.19 meters.

d) Directly Measured Quantity table. Using a table is a good method of organizing your measurements and presenting them in a clear and concise manner. Use Table 3-1 below to report the details of your measurements.

<table>
<tr><th>Measured Quantity</th><th>Confidence Interval</th><th>Relative Uncertainty</th><th>Measurement Device</th></tr>
<tr><td rowspan="3">[Description of measured quantity, and the variable that represents the quantity]
Diameter of the plastic ball, d_{ball}</td><td>[Confidence interval of measurement]
(0.35 ± 0.01) m</td><td>[Relative Uncertainty]
2.9 %</td><td>[Accurate description of measurement device]
Meter stick</td></tr>
<tr><td colspan="3">Measurement Procedure: [describe how the measurement was taken (i.e. where/when you stopped/started your measurement, what settings were used)]
The ball was placed between two vertically oriented straight edges and marks were made at the base of the straight edges on a piece of paper. The distance between the two marks was measured with a meter stick.</td></tr>
<tr><td colspan="3">Justification of Uncertainty: [State the factor(s) of uncertainty that determined your estimate of the absolute uncertainty, the specific cause of the uncertainty and the absolute uncertainty associated with the specific cause]
The meter stick had tick marks every 1 cm, but could be estimated to a precision of ± 0.5 cm. The ball was not exactly spherical and measurements at different orientations resulted in differences of ± 1 cm.</td></tr>
</table>

Table 3-1. Measured Quantity Table Format

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[2] Ibid., p. 15.

[3] Ibid., p. 15.

e) Reporting Data in a Table. Often a table is required to report data when multiple trials are used. Use the following guidelines for presenting data in tables:

i. Each data column is labeled with the quantity, variable, and units.

ii. The **absolute uncertainty** of each directly measured quantity must be included.

(a) If the uncertainty is the same for each trial, then it can be recorded in the column heading (See Table 3-8 for an example).

(b) If the uncertainty is not the same for each trial, then the absolute uncertainty must be recorded in a separate column (See Table 3-9 for an example).

iii. The data is centered under the column heading and is displayed rounded to the decimal place of the corresponding absolute uncertainty.

B. Propagation of Uncertainty

The numerical values you obtain in measurements are frequently used to determine a final result for some quantity that is a function of those measured values, but cannot be measured directly itself. The uncertainty in your final numerical answer depends upon the uncertainties in the directly measured quantities. Error analysis is the assessment of the uncertainty in our measurements and how it affects the final results. To properly interpret the data you have collected, you must examine your results for probable experimental error. Conclusions based on a calculated value may be accepted or rejected on the basis of the error that is inherent in the findings. There are many techniques for propagating the uncertainty of calculated quantities; however, we will focus on error propagation by quadrature. In Introductory Physics, two methods will be utilized for evaluating the error of objective quantities. When you make a measurement, there is always some error present.

1. Introductory Method

The introductory method for error propagation is a simplified process that is derived utilizing partial derivatives and can be derived utilizing the method discussed in Appendix F. For the introductory method we have three general equations used for specific calculations. The first equation is when the propagation of uncertainty is calculated using an equation that is with sums and differences and the second general equation is used when your calculated value was determined from a function that was either a products and/or quotients relationship. The final equation is used when the function has variables raised to a power.

a) Uncertainty in Sums and Differences

Suppose that our function of interest is solely made up of sums and differences of other measured quantities that are **independent and random** from one another. The absolute uncertainties can be summed in quadrature to determine the function’s absolute uncertainty.

$$f(a, b, \ldots, c) = a - b + \ldots + c$$

$$\delta f = \sqrt{(\delta a)^2 + (\delta b)^2 + \ldots + (\delta c)^2} \qquad (3\text{-}3)$$

The derivation can be found in Appendix F.

For example, an infantry battalion's scout platoon leader's mission is to provide early warning on enemy movement in AO EINSTEIN. On day 2 of the operation the scout platoon leader identifies three enemy Antonov An-24 aircraft flying overhead and dropping enemy paratroopers into AO EINSTEIN. He observes and makes Table 3-2. In order to give an accurate SALUTE report to higher he wants to calculate the overall uncertainty in his calculation of enemy paratroopers.

P_1	(43±4) m
P_2	(44±6) m
P_3	(48±2) m

Table 3-2. Enemy Paratrooper Count from Antonov AN-24

The total number of enemy paratroopers, P, is the sum of the three An-24 enemy paratrooper drops, $(P_1 + P_2 + P_3)$, which equals 176 enemy paratroopers.

The absolute uncertainty in the total number of enemy paratroopers, δP, is found using Equation 3-3.

$$\delta P = \sqrt{(\delta P_1)^2 + (\delta P_2)^2 + (\delta P_3)^2}$$

$$\delta P = \sqrt{(4)^2 + (6)^2 + (2)^2}$$

$$\delta P = 7.483 \text{ enemy paratroopers}$$

We will round our absolute uncertainty to one significant figure and write our confidence interval as $P = (135 \pm 7)$ enemy paratroopers.

b) Uncertainty in Products and Quotients.

Suppose that our function of interest is solely made up of products and/or quotients of other measured quantities that are independent and random from one another. The relative uncertainties can be summed in quadrature to determine the functions uncertainty.

$$f(a, b, \ldots, c) = \frac{ab}{c}$$

$$\frac{\delta f}{|f|} = \sqrt{\left(\frac{\delta a}{|a|}\right)^2 + \left(\frac{\delta b}{|b|}\right)^2 + \cdots + \left(\frac{\delta c}{|c|}\right)^2} \qquad (3\text{-}4)$$

The derivation for this can be found in Appendix F.

For example, an infantry battalion's scout platoon leader's mission is to provide early warning on enemy movement in AO EINSTEIN. On day 2 of the operation the scout platoon leader identifies Antonov An-24 aircraft flying overhead and dropping enemy paratroopers into his AO. He observes and makes Table 3-3. In order to give an accurate SALUTE report to higher he wants to calculate the overall uncertainty in his calculation of enemy paratroopers.

n (# of Antonov An-24s)	P (# of enemy paratroopers jumped from one An-24)
(22±1) aircraft	(45±6) enemy paratroopers

Table 3-3. Measurement of Antonov An-24s and enemy paratroopers

The total number of enemy paratroopers is found using the function, $P_{Total} = nP$, and is equal to 990 enemy paratroopers. The relative uncertainty of the total number of enemy paratroopers is found using equation 3-4.

$$\frac{\delta P_{Total}}{|P_{Total}|} = \sqrt{\left(\frac{\delta n}{|n|}\right)^2 + \left(\frac{\delta P}{|P|}\right)^2}$$

Using algebra we can solve for the absolute uncertainty of the total number of enemy paratroopers.

$$\delta P_{Total} = |P_{Total}|\sqrt{\left(\frac{\delta n}{|n|}\right)^2 + \left(\frac{\delta P}{|P|}\right)^2}$$

$$\delta P_{Total} = |990|\sqrt{\left(\frac{1}{|22|}\right)^2 + \left(\frac{6}{|45|}\right)^2}$$

$$\delta P_{Total} = 139 \text{ enemy paratroopers}$$

Using appropriate rounding rules this becomes 100 enemy paratroopers and is expressed as a confidence interval, $P_{Total} = (1.0 \pm 0.1)\ x\ 10^3$enemy paratroopers.

c) Uncertainty in a Power

Suppose that our function of interest involves a variable that is raised to a power. The relative uncertainty can be found using the general equation below.

$$f(x) = x^n$$

$$\frac{\delta f}{|f|} = |n|\frac{\delta x}{|x|} \qquad (3\text{-}5)$$

d) Combining Uncertainty in a Complex Function

Suppose that we have a function that has powers, products and quotients, and sums and differences. We must break down the uncertainties and combine them appropriately to see what the overall uncertainty is in our function. Here is an example.

$$f(a, b, c, d) = \frac{a^2 b - c}{d^3}$$

In order to calculate the absolute uncertainty of our function, δf, we need to break down and look first just at the numerator and start combining uncertainties based off the math operation being used. Let us make the following expression

$$g(a,b) = a^2 b$$

We can find the absolute uncertainty of $g(a,b)$ combine Equations 3-4 and 3-5.

$$\delta g = |g|\sqrt{\left(2\frac{\delta a}{|a|}\right)^2 + \left(\frac{\delta b}{|b|}\right)^2}$$

We can rewrite our original function in terms of $g, c,$ and d.

$$f(g,c,d) = \frac{g-c}{d^3}$$

Now we can make a new function to express the numerator.

$$h(g,c) = g - c$$

We can find the absolute uncertainty in $h(g,c)$ by using the sums and differences rule from Equation 3-3.

$$\delta h = \sqrt{(\delta g)^2 + (\delta c)^2}$$

Finally, instead of writing our function in terms of $a, b, c,$ and d, we will write it in terms of h and d.

$$f(h,d) = \frac{h}{d^3}$$

We again can use the combination of Equation 3-4 and 3-5 to determine the absolute uncertainty in our function.

$$\delta f = |f|\sqrt{\left(\frac{\delta h}{|h|}\right)^2 + \left(3\frac{\delta d}{|d|}\right)^2}$$

As seen, this process can become long and cumbersome. There are more advance techniques that can be used to propagate error on calculations that reduce the complexity utilizing calculus and partial derivatives. This will be discussed in the following section.

2. Intermediate Method of Error Propagation

The introductory method discussed in the previous section was derived utilizing error propagation method discussed here and Appendix F. The error propagation begins by being evaluated in quadrature. For example to measure the density of a cube of wood, measuring its mass, m, and the length of one side of the cube, x, with calculations will determine the density.

$$\rho(m, x) = \frac{m}{x^3}$$

The uncertainty in your final numerical answer depends upon the uncertainties in the directly measured quantities. Error analysis is the assessment of how uncertainty our measurements and how it affects the final results. To properly interpret the data you have collected, you must examine your results for probable experimental error. Conclusions based on a calculated value may be accepted or rejected on the basis of the error that is inherent in the findings. There are many techniques for propagating the uncertainty of calculated quantities; however, in Physics we focus on error propagation by quadrature.

If the uncertainties in your measurements are *independent and random* in nature, a more realistic estimate of the absolute uncertainty is given by the following quadrature equation:

$$\delta f(x, y, \dots) = \sqrt{\left[\frac{\partial f}{\partial x}\delta x\right]^2 + \left[\frac{\partial f}{\partial y}\delta y\right]^2 + \cdots} \qquad (3\text{-}6)$$

Equation 3-6 looks complicated, but it is understandable if function $\boldsymbol{f}$ is broken down into each term. Each term of equation 3-6 consists of a partial derivative, $\boldsymbol{\frac{\partial f}{\partial x}}$, and a measurement absolute uncertainty, $\boldsymbol{\delta x}$. A partial derivative is the derivative of function, $\boldsymbol{f}$, with respect to one variable, $\boldsymbol{x}$, and treating the others as fixed. Equation 3-6 may be generalized to as many variables as desired, since each term is added in quadrature.

Applying Equation 3-6 to our density example yields the following:

$$\delta\rho(m, x) = \sqrt{\left[\frac{\partial\rho}{\partial m}\delta m\right]^2 + \left[\frac{\partial\rho}{\partial x}\delta x\right]^2}$$

Once the partial derivatives are taken the algebraic solution for propagating the error of the density equation, $\rho(m, x)$, is determined to be

$$\delta\rho(m, x) = \sqrt{\left[\frac{1}{x^3}\delta m\right]^2 + \left[\frac{-3m}{x^4}\delta x\right]^2}$$

For more examples of how to use the error propagation method with partial derivatives, see Appendix G.

C. Types of Experimental Error

When you make a measurement, there is always some error present. We can take steps to reduce errors due to sloppiness, poor preparation, and misinterpretation of data, but there are some errors we will not be able to avoid. Therefore, we must include an analysis of any error when considering the results and conclusions from our data. Errors can be divided into two categories: ***random*** and ***systematic***.

1. Random Error and Precision

a) Random errors are inherent in every measurement. Random errors affect the precision of a measurement. The precision of a measurement is the degree to which repeated measurements under unchanged conditions show the same results. Random errors are positive or negative fluctuations that cause similar measurements to be too high sometimes and too low at other times. When a single measurement of a quantity, x, is made the effect of these random errors should be estimated and reported as the absolute uncertainty, δx, in the measurement. This quantity is based on how precisely you think that you made the measurement. If many measurements are made, the effect of random errors decreases and normally averages out to nearly zero.

b) Relative uncertainty is a measure of the precision of the measurement. If a measurement has a high degree of precision, repeated measurements (using the same equipment and technique) should produce about the same results.

c) Random errors are due to uncertainties related to the experiment itself and are not mistakes. Some examples of sources of random errors will be addressed in later labs. ***Gross reading errors and miscalculations (otherwise known as "human" errors or mistakes [see 3. Below]) are not considered experimental (random or systematic) errors!***

2. Systematic Error and Accuracy

a) Systematic errors can cause measured values to be consistently either too high or too low. Systematic errors affect the accuracy of a measurement. The accuracy of a measurement indicates how close the measurement is to a true or commonly accepted value. We cannot reduce the effect of systematic errors by taking more measurements in the same manner. Examples of systematic error sources that can cause inaccuracies are instrument error, method of theory error, procedure error, and etc.

i. Instrument Error. The use of equipment that is incorrectly calibrated can lead to systematic errors. For example, not zeroing a balance before making a mass measurement.

ii. Method of Theory Errors. Method of theory errors are those that result from the procedure used or the failure to take into account all factors bearing on the measurement. For example, assuming that air drag is negligible when measuring the acceleration due to gravity by timing the fall of an object released from rest from the top of a tall building would cause systematic error in the result.

3. Mistakes. Mistakes include, but are not limited to, misreading a scale, copying data incorrectly or splashing and spillage of reagents during a reaction. These mistakes will largely affect the accuracy of your results. You can eliminate mistakes through careful laboratory

technique. In the event that a mistake occurs, that measurement should be discarded and a new measurement made. Mistakes *are neither random nor systematic!*

4. Determining the Relative Significance of Random and Systematic Errors

a) Number Line Analysis. Displaying an experimentally determined value as a confidence interval on a number line along with an accepted or reference value allows us to make a visual assessment of the relative importance of random and systematic errors in the experiment.

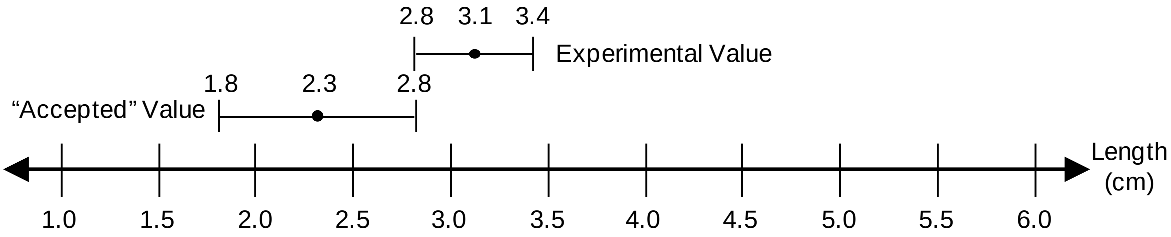

Figure 3-2. Number Line Formatting Example

i. Formatting Standards for Number lines. The following standards will be used when preparing number lines (see Figure 3-2):

(a) Scale is appropriately spaced and intervals are uniform; do not waste space unnecessarily.

(b) Number-line labeled with the title of the quantity and units.

(c) Lines representing the confidence interval for the values under comparison.

(d) Lines are labeled correctly (i.e. experimental, accepted, manufacturer, reference).

(e) The lines are marked at the midpoint (dot) and endpoints (vertical line).

(f) The midpoint and endpoints values are present and displayed with the correct significant figures.

ii. Determining the Dominant Form of Error. Through analysis of the number line, one can determine which type of error, random or systematic, is the dominant form or error in an experiment. Once the dominant form of error has been identified, then the process of determining meaningful improvements to the experiment can begin. By utilizing the flowchart in Fig. 3-3, the dominant form of error of any number line can be determined.

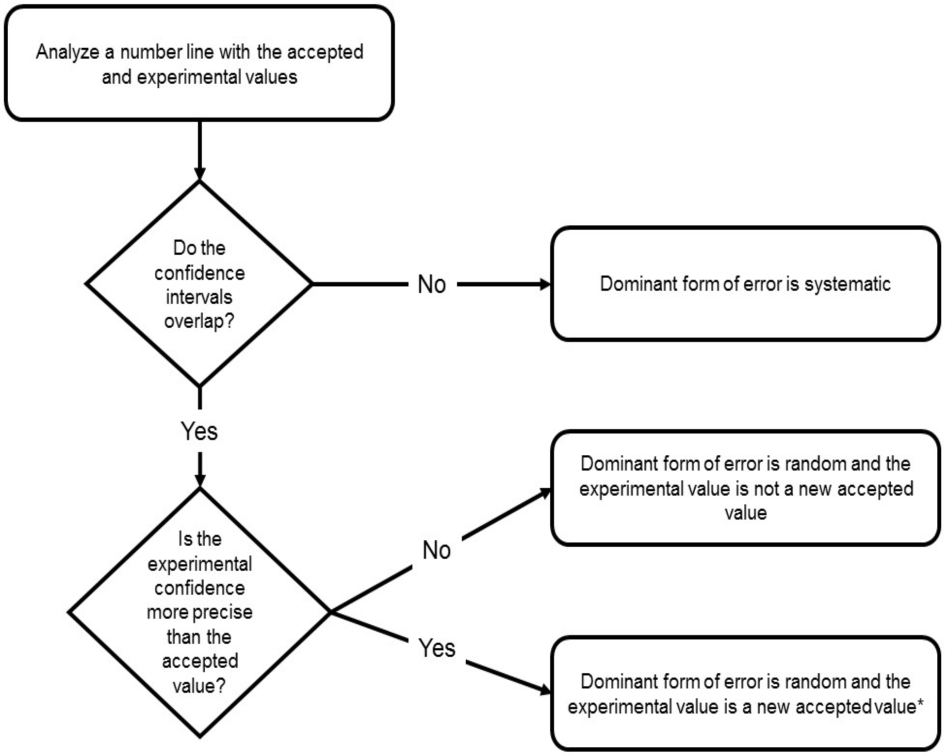

Figure 3-3. Number line analysis flowchart.

*If the experimental confidence interval is more precise than the accepted value, it may be the new accepted value. This is highly unlikely for any laboratory conducted in USMA undergraduate courses.

The following two examples are analyzed with the flowchart from Fig. 3-3.

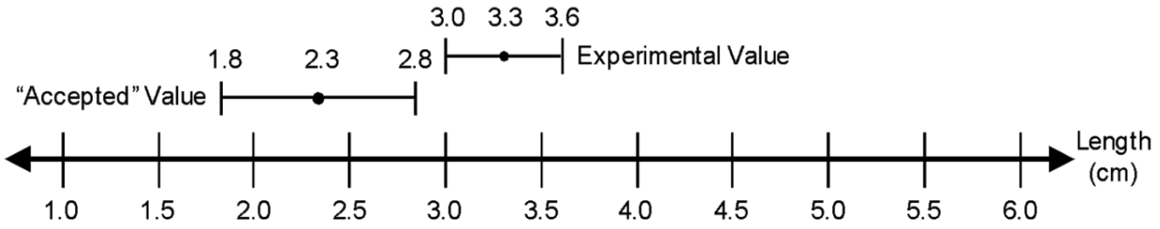

Figure 3-4. Systematic Error Example

Looking at the experimental value and accepted value we ask if the confidence intervals overlap. Since they do not, the dominant form of error is systematic error.

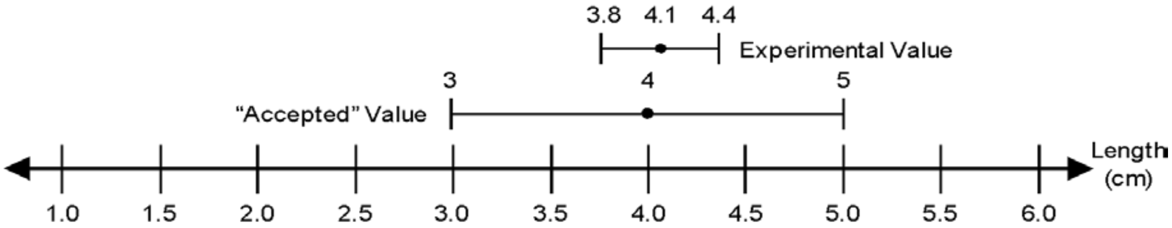

Figure 3-4. Random Error Example

Now consider Figure 3-4. Using the Fig. 3-3 flowchart, we determine whether or not the confidence intervals overlap. Since they do overlap we follow up with asking if the experimental relative uncertainty is smaller than relative uncertainty of the accepted value. After calculating the relative uncertainties, the experimental relative uncertainty, 0.0098, is smaller than the accepted value's relative uncertainty, 0.25, therefore the dominant form of error is random and the experimental value may be considered as the new accepted value.

Note: When the accepted value is an exact number (as is the case for the permeability of free space, μ_0), plot the accepted value as a point without a confidence interval. If the exact value lies inside the confidence interval, an experimenter should conclude that random error was dominant in the experiment. If the exact value lies outside the confidence interval, then an experimenter should conclude that the experiment was inaccurate and systematic error was dominant.

iii. Error Source Analysis. Once you have identified the form of error, systematic or random, it is important to analyze the sources that may have contributed to your dominant form of error. In order to analyze error or identify the main sources of error in an experiment, a thorough review of conditions, procedures, theory, and relative uncertainties must be completed. Ascertaining the sources contributing to the error in an experiment can be either an exhaustive process of elimination or an abbreviated process using judgement and intuition.

(a) **Identifying sources of error** in an experiment where **random error dominates**, we can start by conducting a relative uncertainty analysis. With random error, the relative uncertainty in our objective quantity is most likely greater than that of the accepted quantity's relative uncertainty (Reference Figure 3-3). In order to reduce the objective quantity's relative uncertainty we must look at the independent quantity, dependent quantity, and control variables' relative uncertainties to identify those that are the greatest and contributing the most to the objective quantity's relative uncertainty. Once the greatest contributors are identified, you must seek ways to reduce the measurements absolute uncertainty and as a result reduce the objective quantity's relative uncertainty. This process will reduce the objective's relative uncertainty and make your experimental value more precise.

An example of this can be seen when conducting an experiment to determine the drag coefficient of an object. In Table 3-4 are listed the measurements of one such experiment. In Figure 3-5 is a number line showing the accepted value and experimental value. Since the relative uncertainty of the accepted value (0.17) is smaller than the relative uncertainty of the experimental value (0.31), we must determine sources of error contributing to this high relative uncertainty. Looking at Table 3-4, the diameter of the object and the slope (the slope comes from conducting a linear regression analysis of the data – see Appendix H) have a high relative uncertainty compared to the relative uncertainties of the acceleration due to gravity and the density of air. We must try and determine how to reduce the relative uncertainty in the diameter and in our slope, which will make our calculated drag coefficient more precise. Possible solutions may be changing the instrument used to measure the diameter or possibly changing the measured value and the method used to collect data.

Variable	Measured Value	Absolute Uncertainty	Relative Uncertainty
Diameter of object, d	0.21 m	.03 m	0.14
Slope	360 m2kg s2	30m2kg s2	0.083
Density of Air,	1.20 kg/m^3	0.02 kg/m^3	0.017
Acceleration due to gravity, g	9.807 m/s^2	0.001 m/s^2	0.00010

Table 3-4. Drag Coefficient Data Table Example.

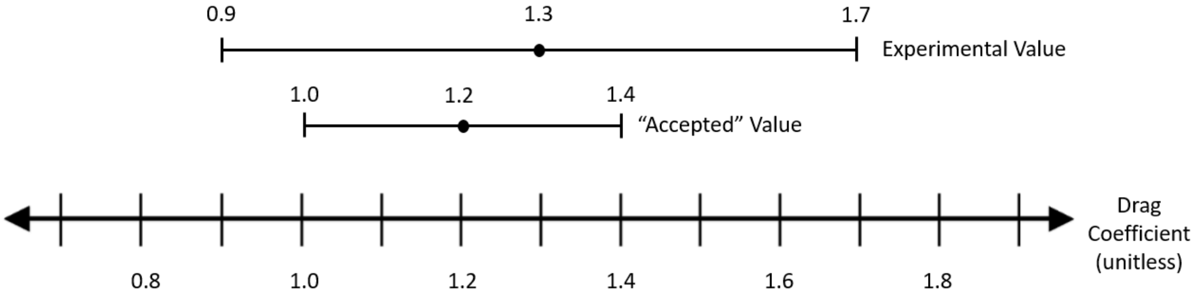

Figure 3-5. Drag Coefficient - Number Line Example

(b) In order to **identify sources of error** in an experiment where **systematic error dominates,** a more exhaustive approach must be taken due to the numerous sources of systematic error. Table 3-5 is a checklist for assisting with systematic error identification. Pay special attention to understanding how each variable, or "letter," in the model equation could skew an experimental value, because this step directs a detailed analysis of the measurements, procedures, and conditions.

Theory
1. Ensure model equation and theory are correct 2. Determine how each variable/input in the model equation could contribute to the systematic error (only after model equation review, and **MOST** important step)* *Review the variables and inputs for a linear regression (x-axis and y-axis)
Execution
1. Assess procedure execution 2. Review measurement procedures 3. Check conversions
Procedures
1. Review appropriateness of procedures 2. Assess procedures against assumptions
Conditions
1. Evaluate lab environment's impact on experiment 2. Check conditions of accepted and experimental values 3. Assess limitations of lab equipment and their impact

Table 3-5. Systematic Error Source Checklist

If a lab group has collected the data in Figure 3-6 through the model equation 3-7, then they would be close to the accepted value of their object's drag coefficient, but systematic error would still be dominating their experiment.

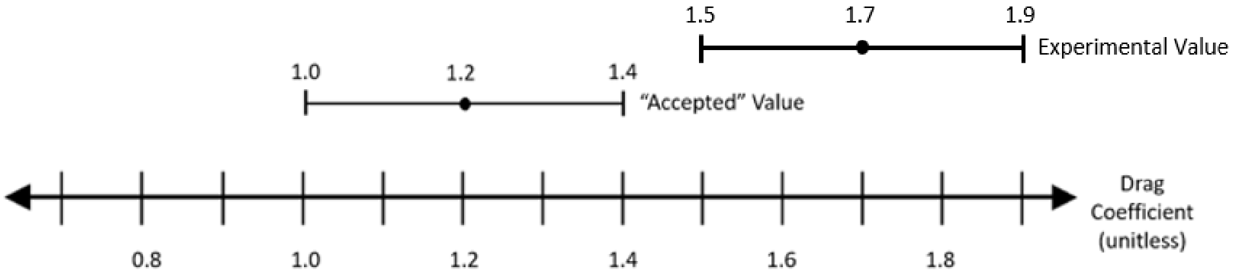

Figure 3-6. Drag Coefficient –Systematic Error Analysis Example

$$C = \frac{8g}{(slope)\rho\pi d^2} \tag{3-7}$$

By applying the checklist in Table 3-5, the group should develop findings similar to Table 3-6 in order to explain the systematic error with a consistently incorrect variable.

	g	slope	ρ, density of air	*d*, diameter
How	• Greater than actual	• Less than actual • y-axis: terminal velocity squared (too little) • x-axis: mass (too much)	• Less than actual	• Less than actual
Execution	• Given, N/A	• Improperly dropping object • mass scale not zeroed • Not dropping object high enough to reach terminal velocity • Sensor is measuring in ft/s but wrote down m/s	• Given, N/A	• Measured radius instead of diameter • Incorrectly measured the object's diameter • Dropped the wrong face of the object for exposed diameter
Procedures	• Given, N/A	• Inadequate motion sensor chosen for experiment • Low sampling rate of sensor limited velocity readings • Dropping at a height of 4 inches prevented object from reaching terminal velocity	• Given, N/A	• Instructions said to measure the wrong diameter • Instructions said to use a tape measure when calipers were needed • Need multiple copies of the object to prevent error from deformation
Conditions	• Value of g in lab is greater than average g (highly unlikely)	• A/C system reduced object's terminal velocity (highly unlikely)	• Given air density was at standard pressure at sea level • Laboratory is hot, humid, and several hundred feet above sea level, thus reducing density	• Object warmed during experiment and expanded the diameter (unlikely) • Object significantly deformed after falling

Table 3-6. Systematic Error Analysis Example

Note: The following table contains examples of c**ommon sources of systematic error.**

Common Sources of Systematic Error	**Examples of Common Source of Systematic Error**
Incorrect Theory	Wrong assumptions Incorrect model equation
Failure to follow Procedures	Throwing an object, instead of releasing (incorrect execution) Incorrect measurement Measured in grams, but reported kilograms (failure to convert)
Flawed Procedures	Measured at inappropriate locations/angles Procedures and assumptions do not match
Imperfect conditions	Environment adversely affected experiment Lab environment does not match reference Limitations of lab equipment

Table 3-7. Common Sources of Systematic Error

b) Percent Difference. To determine the agreement of an experimentally determined value with the accepted or theoretical value, we can also calculate percent difference. Percent difference shows us the accuracy of our results and is a measure of the systematic error in the same way that relative uncertainty is a measure of our random error. Using Equation 3-8 we can calculate the percent difference and round to two significant figures.

$$\%\, difference = \frac{|experimental\ value - accepted\ value|\ x\ 100}{|accepted\ value|} \qquad (3\text{-}8)$$

For example, if you conducted an experiment to measure the acceleration due to gravity, g, and obtained an experimental value of (10.0 ± 0.5) m/s^2, the percent difference from the accepted value of (9.807 ± 0.001) m/s^2 would be

$$\%\, difference = \frac{|10.0 - 9.807|\ x\ 100}{|9.807|}$$

$$\%\, difference = 2.0\%$$

D. Reduction of Errors

1. General. Although errors can never be eliminated entirely, care, patience, and awareness can reduce them. Mistakes caused by gross misreading of instruments are unjustifiable, and you may be asked to repeat the experiment(s). Detection of faulty equipment and its replacement can avoid some error. Analysis of the error in the final result of an experiment should suggest methods that can improve the accuracy and precision of the experiment. The experimentalist should determine the dominant source of error and first seek to minimize or eliminate it. Repeating the experiment can produce better results if the dominant form of error is addressed. It

is this iterative process that results in the high degree of precision and accuracy to which many physical constants have been measured.

2. Example of Analysis and Reduction of Error.

An experiment to determine the spring constant of a spring was done by attaching a hanging mass and allowing the mass to oscillate. The model equation used to describe the physical situation was $k = \frac{4\pi^2 m}{T^2}$, where m is the mass of the hanging mass (not the spring) and T is the period of oscillation. The mass and the period were measured in the lab, and the result plotted on a number line as shown in Figure 3-7.

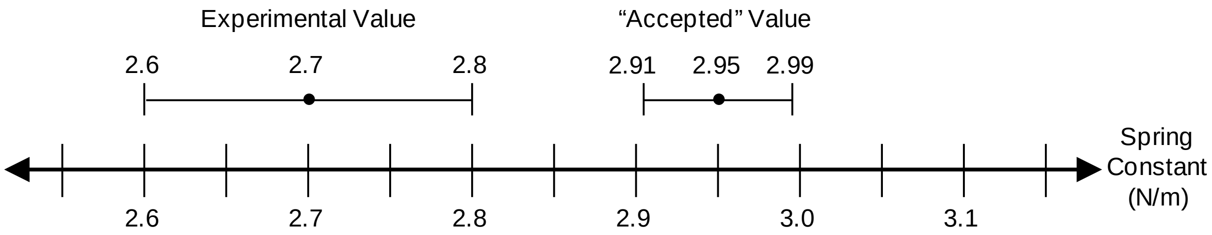

Figure 3-7. Spring Constant Example 1

An analysis of Figure 3-7 shows that the experimental value is not precise compared to the accepted value since the confidence interval of the experimental value is larger than that of the accepted value. The experimental value is also not accurate since there is no overlap of the confidence intervals. Utilizing the flowchart in Figure 3-3, the dominate form of error is systematic since the experimental value does not overlap therefore it is not accurate. The analysis of this systematic error begins with analyzing the assumptions from the model and the model equation $k = \frac{4\pi^2 m}{T^2}$. Since our experimental value is smaller than the accepted value we consider the model equation and what measured values may be reducing our experimental value. We observe that the mass, m, could be too small or that our measurement of the period, T, was too large. Both of these make the overall spring constant, k, smaller. A possible source of systematic error in this experiment could be the fact that the mass of the spring was ignored in the model; this could account for this discrepancy. The effect of this possible systematic error should be analyzed to determine if it is consistent with the results. Since the spring constant, k, is directly proportional to the mass, a value of m that is too small (ignoring the mass of the spring) would yield a calculated value of k that is too small. Since the experimental value is smaller than the accepted value, this systematic error is consistent with the results. Improvements to reduce or eliminate this systematic error should be explored. The model equation can be modified to include the mass of the spring, or the effect of the mass of the spring can be minimized with a larger hanging mass. The experiment was repeated with larger hanging masses to further evaluate the effect of neglecting the mass of the spring. The improved results with the larger hanging mass are shown in Figure 3-8.

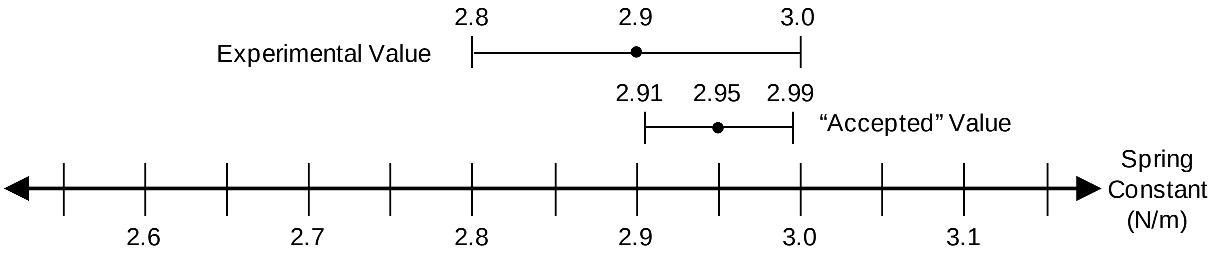

Figure 3-8. Spring Constant Example 2

An analysis of Figure 3-8 for the improved experiment shows that while the experimental value is now accurate due to overlap of the confidence intervals, it is still not precise compared to the accepted value. The dominant form of error is now random error. The relative uncertainties of each measured quantity should be compared to determine which is the least precise, contributing the most random error to the experiment. After comparing the relative uncertainties of the hanging mass and the square of the period, it was determined that the square of the period was the least precise. In order to improve the experiment further, 15 trials using the same procedure were conducted in order to reduce the effect of random errors in the measurement of the period. The results of the experiment after the second improvement are shown in Figure 3-9.

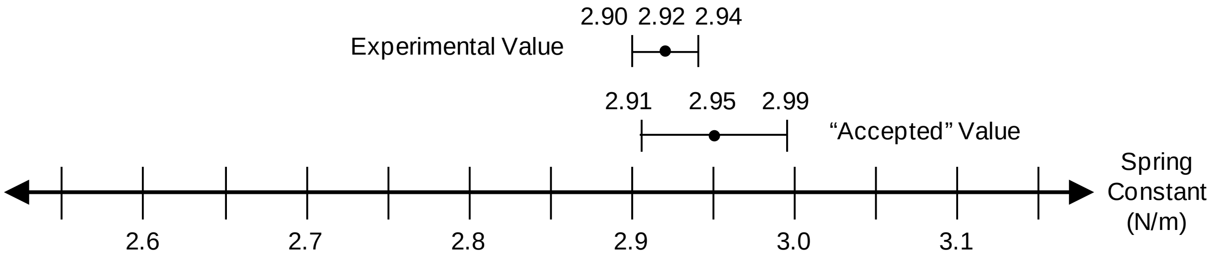

Figure 3-9. Spring Constant Example 3

An analysis of the number line after the second improvement shows that the experiment is both accurate and precise compared to the reference value. Because the experimental value has exceeded the precision of and is consistent with the reference value, it is not obvious if our efforts to further improve this experiment should be directed at reducing random or systematic errors.

E. Statistical Evaluation of Results

One method of acquiring a confidence interval from a set of experimental data is to use the **mean** and the **standard deviation**. We can use this method when we make several similar measurements and are confident that the greatest impact on the uncertainty of our measurements is due to random errors.

1. Mean. In many experiments, we make repeated measurements of the same quantity, say *x*, and obtain a set of measurements, $\{x_1, x_2, \ldots, x_N\}$, where N is the total number of measurements

made. Our best estimate for the value of the quantity x will be the average or mean[4], $\bar{x}$, which is defined as

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} \tag{3-9}$$

This can be written more compactly as

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{3-10}$$

2. Standard Deviation. The standard deviation[5] of the measurements, $x_1, \ldots, x_N$, is an estimate of the average uncertainty of the measurements. The deviation is the difference between an individual measurement x_i and the mean, $\bar{x}$. In order to make a single estimate of the uncertainty for our set of measurements, $x_1, \ldots, x_N$, we need to calculate the mean square deviation of all the measurements from the mean. Since some deviations may be positive and some may be negative we square all of the deviations and then take the square root of the average. Thus the standard deviation, σ_x, of the set $x_1, \ldots, x_N$, is

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} \tag{3-11}$$

Equation 3-11 is commonly referred to as the standard deviation of an entire population. If the number of measurements we make is relatively small, this definition of the standard deviation may slightly underestimate the uncertainty. Therefore, we will use the **sample standard deviation**[6] which is defined as

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \tag{3-12}$$

The function STDEV or STDEV.S in Microsoft Office Excel will calculate the sample standard deviation for a set of numbers.

3. Standard Deviation of the Mean[7]. A more precise method of acquiring a confidence interval for an experimental result is to use the mean and the standard deviation of the mean (SDOM or $\boldsymbol{\sigma_{\bar{x}}}$). The SDOM is defined as

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \tag{3-13}$$

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[4] Ibid., pp. 97-98.
[5] Ibid., pp. 98-99.
[6] Ibid., p. 100.
[7] Ibid., p. 149.

where σ_x is the sample standard deviation, as given in Eq. 3-12, of a set of values, $x_1,\ldots,x_N$, and N is the total number of values. The standard deviation is the average uncertainty in each of the values whereas the SDOM is the average uncertainty in all the values considered as a group. This method can be used when several similar measurements are made and the experimenter is confident that the greatest impact on the uncertainty in the measurements is due to random errors and therefore that the data set is **normally distributed**.

By using the standard deviation of the mean as the uncertainty in *x*, we can write x as a confidence interval:

$$x = \bar{x} \pm \sigma_{\bar{x}} \qquad (3\text{-}14)$$

Here, we see that the absolute uncertainty δx is equal to $\sigma_{\bar{x}}$. With this we are stating that we expect 68% of any measurement of *x* to fall within this range. If we had an application where we needed to know the value with more certainty, say 95% for example, we would write the confidence interval as

$$x = \bar{x} \pm 2\sigma_{\bar{x}} \qquad (3\text{-}15)$$

For most experimental purposes, the 68% confidence is sufficient, but the experimenter needs to consider the specific application when determining how much confidence is sufficient.

4. Example. Suppose that we wish to determine the muzzle speed of a projectile launcher by setting the launcher to a particular angle and measuring the distance the projectile travels. We can get a good estimate of the random variations by conducting the experiment 10 times. The data from such an experiment is shown in Table 3-8. You can use Eq. 3-10 to calculate the mean and Eq. 3-13 to calculate the standard deviation of the mean. You can also use Excel to do the calculations by using the AVERAGE function to determine the mean and the STDEV function to determine the standard deviation. Either way, the results are:

mean horizontal distance, $\bar{x}$ = 122.49 cm
standard deviation, σ_x = 2.1299 cm
SDOM = 0.6735 cm.

The confidence interval for the horizontal distance using the mean and the standard deviation is

x = (122 ± 2) cm.

Use the standard deviation as the uncertainty when you are interested in the scatter of individual measurements. For instance, you might want to conduct the experiment at different launch angles, but only have time to conduct one trial at each additional angle. You could use the standard deviation from the angle you did ten trials at as an estimate

Trial Number	Horizontal Distance, *x* (±0.1cm)
1	118.5
2	123.7
3	120.4
4	123.1
5	123.6
6	121.3
7	123.3
8	126.3
9	123.0
10	121.7

Table 3-8. Measurements for Projectile Launch Experiment.

of the uncertainty for the other trials; i.e., take the uncertainty in the horizontal distance as δx = 2 cm.

The confidence interval for the horizontal distance using the mean and the SDOM is

$$x = (122.5 \pm 0.7) \text{ cm.}$$

Use the SDOM as the uncertainty when you are interested in how uncertain you are in the mean of the data set. For instance, if you were only going to conduct the experiment ten times at one launch angle, you would use the SDOM as the uncertainty in your horizontal distance. i.e., δx = 0.7 cm. Notice that the reported value for the horizontal distance is more precise when the SDOM is used as the absolute uncertainty of the horizontal distance, δx, than when the standard deviation is used as δx.

5. Counting Experiments. Some experiments will count the number of times a random event occurs in a given amount of time, such as the decay of a radioactive source.

a) Single counting measurement. You may measure the activity of one or more radioactive samples by counting the number of radioactive decays within a certain time period. Each nucleus in the radioactive source decays at a random moment, so that the number of decays within a certain time period is not completely predictable. We would not, in fact, expect the number of decays in two different time periods of the same length to be exactly the same. Because of this randomness of the process, we use the best estimate of the number of decays per unit time as the measure of activity. The correct way to report this measurement, including its random variability, is to give a best estimate (an actual measurement) and the range that we are confident that the quantity lies within. The measurement of the number of radioactive decays, C, in a given time period, reported as a confidence interval, is

$$C = C_{actual} \pm \delta C \qquad (3\text{-}16)$$

We estimate the absolute uncertainty, δC using the **Square-Root Rule for Counting Experiments**[8] which states that the uncertainty in any counted number of random events, as an estimate of the true average number, is the square root of the counted number. Thus, the number of counts, C, within some fixed time period is

$$C = C_{actual} \pm \sqrt{C_{actual}} \qquad (3\text{-}17)$$

Equation 3-17 is based on a single measurement of the number of counts.

b) Repeated counting measurements. If more than one measurement is taken for the same radioactive sample, then it is reasonable to expect the reliability of our measurement to increase. Stated in another way, we would expect the absolute uncertainty to decrease. Applying the mean, and standard deviation of the mean to this application, we arrive at Equation 3-18 as a more

---

[8] Ibid., p. 48.

general expression for the number of counts in which N represents the number of identical measurements,

$$C = \overline{C}_{actual} \pm \frac{\sqrt{\overline{C}_{actual}}}{\sqrt{N}} \qquad (3\text{-}18)$$

For example, if an experiment for the number of counts in a 1 minute interval was conducted five times with results of 325, 337, 329, 339, and 371, then

$$\overline{C}_{actual} = \frac{325+337+329+339+371}{5} = \frac{1701}{5} = 340.2\,\text{counts}$$

$$\delta C = \frac{\sqrt{340.2}}{\sqrt{5}} = 8.249\,\text{counts}$$

$$C = (340 \pm 8)\text{counts}$$ [9]

As with the standard deviation of the mean, this confidence interval means that we are 68% confident that the "true" number of counts in one minute from this source lies between 332 and 348. Note that this procedure is exactly equivalent to measuring the count rate by taking a longer count; if rather than averaging five 1-minute counts, we took one 5-minute count and recorded 1701 counts and divided by 5 minutes, we would get the same number of counts in 5 minutes with the same uncertainty.

F. Graphing

Graphs are often a necessary and useful means of presenting experimental data. When properly developed and carefully prepared, a graph can make data easier to analyze, evaluate and compare.

1. General Requirements. Graphs prepared for laboratory requirements will contain the information listed here and illustrated in Figure 3-10:

- A meaningful title describing what the graph portrays, more descriptive than y vs. x.
- Correct variables used on the correct axes (independent quantity on the horizontal-axis and dependent quantity on the vertical-axis).
- Labeled axes that show the property being graphed (time, volume, length, etc.) and the units (s, mL, cm, etc.).
- Equation of trendline.

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[9] Ibid., problem 4.19, p. 115.

- Square of Pearson correlation coefficient (R^2).
- Slope of the trendline with uncertainty (from regression statistics) and correct units.
- Legend when two or more data sets are present on the same graph
- Error bars on data points.

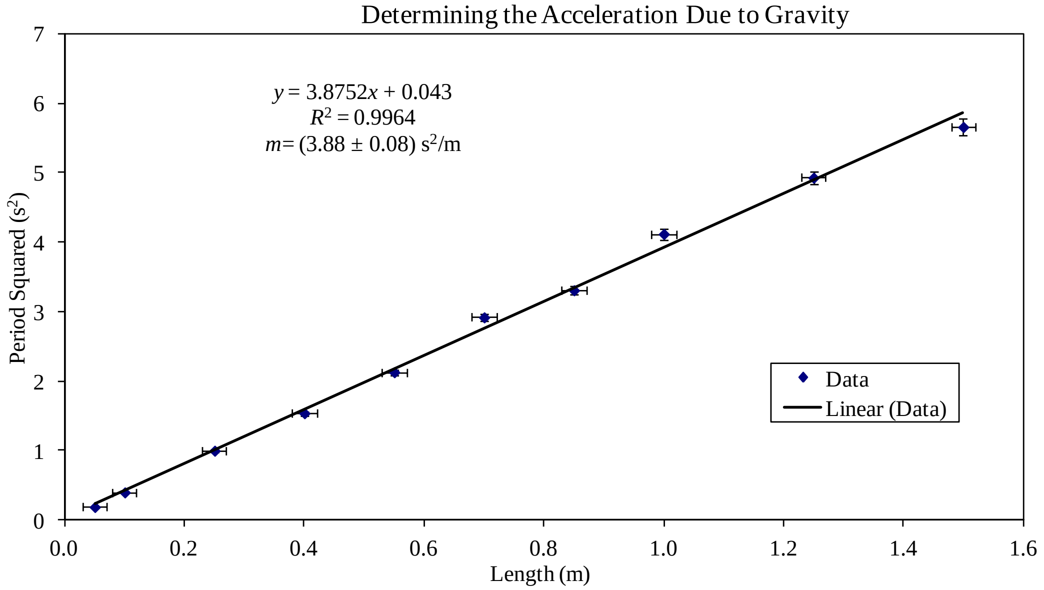


Figure 3-10. Sample Graph

2. Linear vs. Non-Linear Relationships. The relationships between variables may have many forms. The R^2 value represents how well the trendline fits the data. An R^2 value of 1 means the trendline goes through every single data point and an R^2 of 0 means the there is no correlation between the independent and dependent quantities. A trendline with negative slope will lead to an R^2 value between -1 and 0. While an R^2 value of one (1.00) is optimal, this will rarely be the result when dealing with experimental data because of random errors. Although the R^2 parameter can tell us if there is a relationship between two variables, it doesn't always tell us whether the relationship is a linear or a non-linear relationship. See Figure 3-13 for an example of a data set with a R^2 correlation > 0.95, yet the underlying relationship is not linear, the period, T, is a function of $\sqrt{l}$, not l. Oftentimes, a more detailed statistical analysis technique would be required to determine whether a relationship is a linear or a non-linear relationship. This is a discussion best covered in MA206, Probability and Statistics. For the purposes of PH205 and PH206 "test the model" labs, graphing of data will only be performed after a model equation has been derived. From derived model equations, lab groups will be able to determine the relationship between the dependent and independent variables before graphing the collected data.

3. Non-Linear Relationships. When the points plotted represent experimental data, the points will probably not lie on a simple curve. Most theoretical relationships between variables have simple lines of best fit. However, experimental errors can cause plotted experimental points to deviate from simple curves. In spite of this deviation, a simple curve is usually drawn. The curve does not have to go through all the points, but should generally follow them. Figure 3-11 shows

three different curves drawn through the same set of points to illustrate how a correctly drawn curve should look. It is important in experimental work to obtain a sufficiently large number of data points to be sure of the shape of the curve.

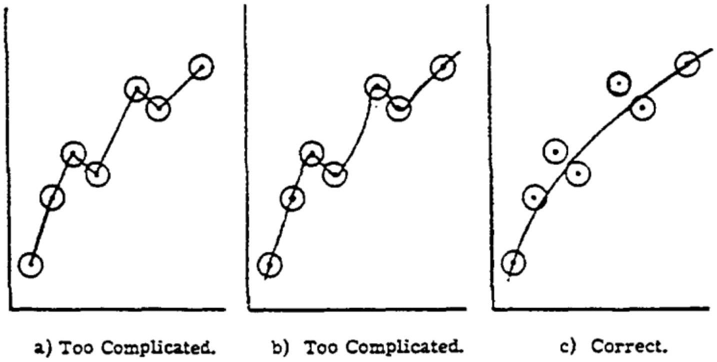


Figure 3-11. Drawing Curves Through Experimental Points.

4. Linear Relationships. It is usually desirable to plot experimental data so that one obtains a straight line. When a graph is a straight line, it is easy to obtain additional information from the graph and to express the relationship between the dependent and independent quantities as an equation.

In order to obtain a straight line graph, we are often required to manipulate our model equation and graph something other than the independent vs. the dependent variables. The independent variable is the physical parameter, such as length, mass, etc., that you control in the experiment and whose value determines the dependent variable. The dependent variable is the physical parameter that you observe or measure based on changes to the independent variable. The independent/dependent quantities are the correct form of the variable from the linearized model equation (in the $y=mx+b$ form).

An example of this process is determining the acceleration due to gravity using a simple pendulum. The acceleration due to gravity, g, can be calculated by measuring the period and length of a simple pendulum. The mathematical relationship between period, T, and length, l, is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{3-19}$$

In order to write this equation in the familiar straight line format ($y = mx + b$), we can leave the square root terms $\sqrt{l}$ and $\sqrt{g}$ or to make it more clear we square both sides of the equation. It then becomes

$$T^2 = \left(\frac{4\pi^2}{g}\right)l + 0 \tag{3-20}$$

We call Equation 3-20 our model equation for the experiment. Here we can see that the independent variable is the length, l and the independent quantity ("x") is also l. Whereas, the dependent variable is the period, T, and the dependent quantity ("y") is the period squared, T^2 (See Appendix I on Linearization). If we use the dependent and independent quantities, the slope ("m") of the line is $\frac{4\pi^2}{g}$ and the y-intercept ("b") is zero. After collecting data in the laboratory, reported in Table 3-9, we plot the length vs the square of the period, and the graph should resemble Figure 3-10.

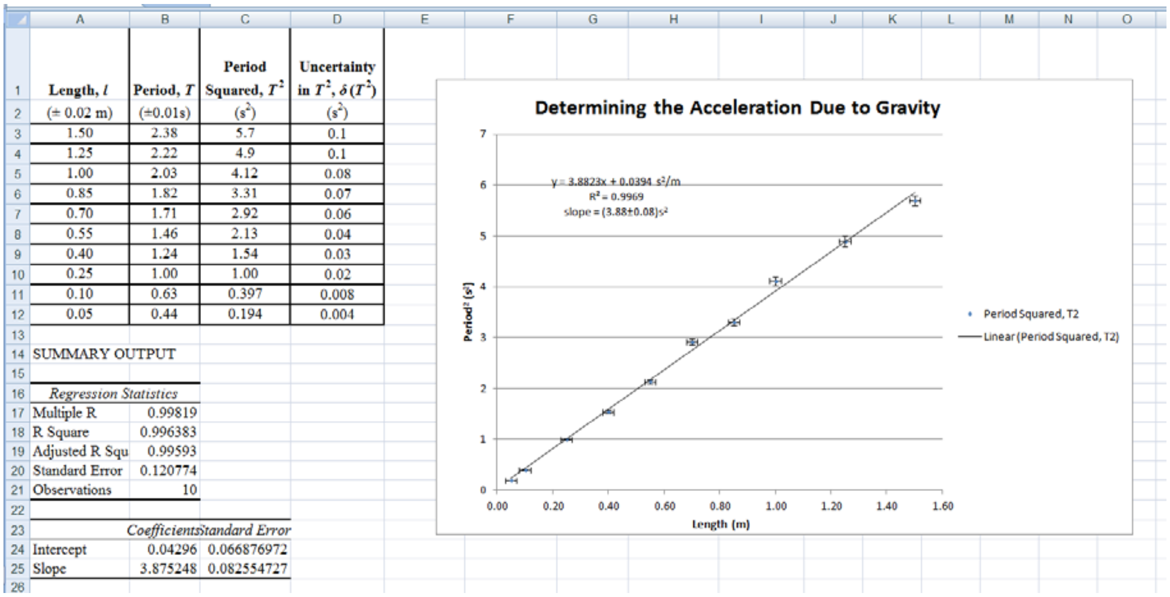
Length, l (± 0.02 m)	**Period, T** (±0.01s)	**Period Squared, T^2** (s^2)	**Uncertainty in T^2, $\delta(T^2)$** (s^2)
1.50	2.38	5.7	0.1
1.25	2.22	4.9	0.1
1.00	2.03	4.12	0.08
0.85	1.82	3.31	0.07
0.70	1.71	2.92	0.06
0.55	1.46	2.13	0.04
0.40	1.24	1.54	0.03
0.25	1.00	1.00	0.02
0.10	0.63	0.397	0.008
0.05	0.44	0.194	0.004

Table 3-9. Data Collected for a Simple Pendulum.

Note: For directly measured quantities where the absolute uncertainty of the measured quantity for each measurement of the parameter does not change, we report the absolute uncertainty in the column heading. For the calculated value of the period squared, we apply the method of partial derivatives to find the absolute uncertainty of the period squared. Thus, we make a new column reporting each unique absolute uncertainty of the calculated parameter.

A powerful tool of linearization is that we can determine the uncertainty in the slope by performing linear regression on the data using Microsoft Excel. See Appendix H (Graphing, Linear Regression, and Curve Fitting) for instructions on how to perform linear regression. After completing regression, the uncertainty is displayed as part of the slope's confidence interval on the graph.

All of the information from tabular, graphical and numerical analysis of data can be combined together in a clear and concise format as shown in Figure 3-12 below. This information now becomes an effective tool for you to present your data and results and support your conclusions.



Length, l (± 0.02 m)	Period, T (±0.01s)	Period Squared, T^2 (s^2)	Uncertainty in T^2, $\delta(T^2)$ (s^2)
1.50	2.38	5.7	0.1
1.25	2.22	4.9	0.1
1.00	2.03	4.12	0.08
0.85	1.82	3.31	0.07
0.70	1.71	2.92	0.06
0.55	1.46	2.13	0.04
0.40	1.24	1.54	0.03
0.25	1.00	1.00	0.02
0.10	0.63	0.397	0.008
0.05	0.44	0.194	0.004

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.99819
R Square	0.996383
Adjusted R Squ	0.99593
Standard Error	0.120774
Observations	10

	Coefficients	*Standard Error*
Intercept	0.04296	0.066876972
Slope	3.875248	0.082554727

Figure 3-12. Final Linearized Graph with Slope and Uncertainty

Once the value for the slope is determined, the acceleration due to gravity can be calculated by re-arranging the terms of the slope in Equation 3-20,

$$slope = \frac{4\pi^2}{g}$$

$$g = \frac{4\pi^2}{slope}$$

$$g = \frac{4\pi^2}{3.856686\,\mathrm{s^2/m}}$$

$$g = 10.2363577\,\mathrm{m/s^2}$$

To determine the absolute uncertainty of the acceleration due to gravity, we must use the either the Introductory or Intermediate Method for propagation of error as described in Section B. Here we will use the intermediate method for this example.

$$\delta g = \left|\frac{dg}{d\left(slope\right)}\right|\delta\left(slope\right)$$

$$\delta g = \left|\frac{d}{d\left(slope\right)}\left[\frac{4\pi^2}{slope}\right]\right|\delta\left(slope\right),$$

$$\delta g = \left|-\frac{4\pi^2}{\left(slope\right)^2}\right|\delta\left(slope\right),$$

$$\delta g = \frac{4\pi^2}{\left(3.857\ \text{s}^2/\text{m}\right)^2}\left(.097\ \text{s}^2/\text{m}\right),$$

$$\delta g = .257\ \text{m/s}^2.$$

The acceleration due to gravity expressed as a confidence interval is

$$\boldsymbol{g = (10.2 \pm 0.3)\ \text{m/s}^2}$$

**For more detailed information on how and why to linearize a model equation as well as another example of the process, see Appendix J (Linearization).

5. Common Mistake. A common mistake is a failure to understand the difference between the dependent/independent variables and the independent/dependent quantities. In the example above, this mistake would lead to plotting the period, T, vs. the length l (not T^2 versus l). Doing this would lead to a graph that looks like Figure 3-13.

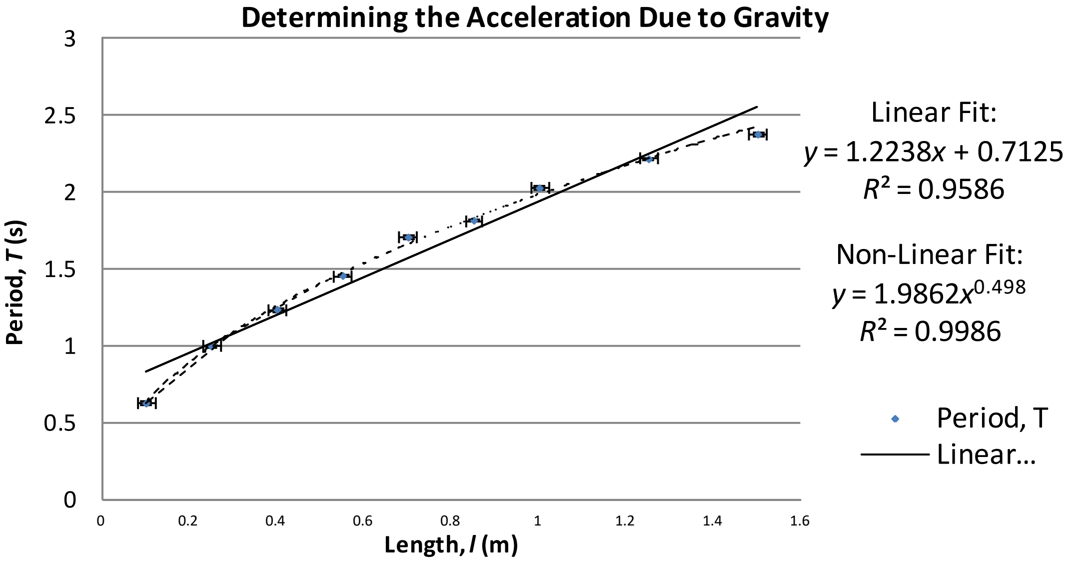


Figure 3-13. Example Graph of Incorrect Linearization

The solid trend line above represents the linear fit to our collected data. Although this trend line has a correlation of $R^2 > 0.95$ (quite close to 1!), the data points plotted do not represent a linear relationship. The data points appear to form a curve. As we saw above in Equation 3-20 and in Section 4, the linear relationship is not between period and length, but rather between the square of the period and the length.

Chapter 4

Laboratory Reports

A. Introduction

1. Lab Reports. Written laboratory reports are a means of communicating to the reader the results of an experiment, the method used to obtain those results, and the conclusions of the experimenter as to the reliability and meaning of the results.

2. Audience. The writer must remember when preparing the report that it is for the reader and, as such, should be tailored to the intended audience. A student working on a subset of a larger work may write a report for a lead experimenter. This audience would obviously require almost no background into the need for or theory of the experiment. Similarly, an article for publication in a technical journal may assume a certain level of knowledge by the reader. In contrast, a report written for publication in a general journal of undergraduate research would require a much more thorough introduction to satisfy the broad range of backgrounds of different readers.

3. Format. Because of the different possible audiences for reports, there is no standard format or procedure. However, all reports begin with some form of introduction which states the purpose of the experiment and the scientific model or hypothesis to be tested. This is followed by a description of the experiment itself and the results obtained. The report ends with a discussion of the important conclusions drawn from the results and suggestions for future work. The written report also includes acknowledgments and references as appropriate.

4. Primary Literature. Each scientific discipline and journal has its own requirements as to the specific format for written reports of scientific information. Few undergraduate programs include what is known as primary literature in introductory courses. If you continue to study in a science discipline you will find that reading and understanding primary literature is one of the most essential skills a scientist must have.

B. Chemistry Written Laboratory Report

Cadets in CH151 and CH152 will have the opportunity to read primary literature and are required to write laboratory reports, or sections of laboratory reports, using the format described below.

1. **Cover Sheet**. Ensure that the cover sheet is formatted as specified in the Documentation of Academic Work. Include all lab group members listed in alphabetical order.

2. **Abstract.** The abstract may be the most important 200 words a scientist ever writes. This short paragraph completely summarizes from start to finish the purpose, result, and significance of your experiment. The abstract should start by providing a basic framework, then move into the specific background of the study. The results of the study should be clearly stated, followed

by a conclusion that puts the results in perspective relative to currently known information in the field of study, and in the greater scientific community. A well-written abstract stimulates the reader's interest in the study.

3. **Introduction**. Clearly state the scientific purpose of the experiment. Provide background that explains why this experiment is important. Give sufficient detail so the reader understands how your experiment fits into the big picture, what information the experiment will provide, and why it is important. Present the fundamental concepts, reactions or equations and explain how these concepts, reactions or equations apply to your experiment. State any assumptions that were made and explain the impact on the experiment.

4. **Materials and Methods**. This technical section describes the equipment, supplies, and procedures used in the experiment. Include sufficient detail so that a reasonably competent individual could independently replicate your experiment and obtain the same result. If needed, include a picture or diagram and refer to it when describing the procedures used to perform the experiment. State what quantities were actually measured during the laboratory and the method used to obtain these quantities (for example, we measured the optical absorbance of the solution at 470 nm with the MicroLab^{TM} spectrophotometer). Present the uncertainty of each measured quantity and discuss how these uncertainties were determined. Describe statistical methods used in determining the measured quantities.

5. **Results**. This section consists of data and observations from the experiment, but does not include any assessment regarding the meaning of the data. Present all of your directly measured data, physical constants, or other experimental parameters (with uncertainties and proper units). The data should be easy to understand and are often presented in a table format with supporting graphs when applicable. Include text, captions, or legends as needed to explain how the data is organized or presented.

6. **Discussion**. This section interprets the facts presented in the results section and is often the longest section in the written report. As needed, refer back to the Introduction section and address the significance of the results relative to the scientific objective of the experiment. Specifically discuss any evidence of systematic and/or random error in the data. If systematic error is the dominant form of error, identify at least one source that is consistent with the direction of your discrepancy and analyze its effects on the measurement of the objective quantity. If random error is the dominant form of error, identify at least one likely source and analyze its effects on the measurement of the objective quantity. Focus on the relative uncertainties of the measured quantities and identify which quantity made the largest random error contribution.

7. **Conclusion**. A good conclusion begins with a clear and concise summary of the research findings. Make an assessment of how well your results satisfied your experimental objective. Suggest changes to the experiment that would reduce the dominant form of error and propose future research steps to be taken to discover more information relevant to the scientific principles explored in the report.

8. **References**. List all of the sources you used in preparing your report in accordance with the Documentation of Academic Work.

C. Physics Written Laboratory Report

In PH205/6 and PH255/6, you will be required to write a laboratory report. Use the format below and address each of the items listed in each section.

1. **Cover Sheet**. Ensure that the cover sheet is formatted as specified in the Documentation of Academic Work. Include all lab group members listed in alphabetical order.

2. **Introduction**. State the scientific objective(s) of the experiment, for example, validate a theory or measure a particular quantity, and how you accomplished that objective. Give sufficient detail so the reader understands not only what you are doing, but why.

3. **Theory**. Present the fundamental physics theories involved. Start with common equations that your readers will accept without question. Ensure that you define all symbols and parameters you use. State any assumptions you must make in order to use these equations must also be stated in this section. While the equations demonstrate the appropriate physics in the experiment, you need to show how these equations apply specifically to each experiment by rearranging these equations into a model equation and identifying the quantities to be measured from the model equation. If linear regression is used to analyze the data, explicitly show the relationship between the slope of the regression line and the objective quantity.

4. **Procedure**. Explain the “mechanics” of the experiment itself. The first step is to briefly explain the experimental apparatus. Include a picture or diagram and refer to it when describing the procedures used to perform the experiment. State what physical quantities are actually measured during the laboratory and the method used to obtain these quantities (for example, we measured the voltage across resistor 1 with a digital multimeter). Present the uncertainty of each measured quantity and discuss how these uncertainties were determined. Describe statistical methods used in determining the measured quantities.

5. **Results**. Present all of your directly measured data, physical constants, or other experimental parameters (with uncertainties and proper units). You will also present any statistical analysis and/or regression results. Present any plots of your data on a full-page graph. Present the calculation of the objective quantity using the derived equations developed in the Theory and Procedure sections. Present the calculation of the absolute uncertainty of the objective quantity. Present the rationale behind selecting the method used to determine the absolute uncertainty. The details of the calculations may be placed in an appendix, but in any case, complete calculations must be present. Report the experimental value as a confidence interval.

6. **Discussion**. Present a number line with the objective quantity and reference value confidence intervals. Discuss the significance of the number line (precision, accuracy, dominant form of error). Discuss any evidence of systematic and/or random error in the data. If systematic error is the dominant form of error, identify at least one source that is consistent with

the direction of your discrepancy and analyze its effects on the measurement of the objective quantity. Revisit the Theory section and address the significance of the systematic error introduced by your assumptions. If random error is the dominant form of error, identify at least one likely source and analyze its effects on the measurement of the objective quantity. Focus on the relative uncertainties of the measured quantities and identify which quantity made the largest random error contribution.

7. **Conclusion**. Make an assessment of how well you met your experimental objective. One of the goals of experimental procedure is to learn from the tasks performed and pave the way to the next iteration of the experiment, building on what you have learned, not repeating the same procedure. Suggest a change(s) to the experiment that would reduce the dominant form of error (refer to the number line and propagation of error calculation as appropriate for establishing priority).

8. **End Notes**. List all of the sources you used in preparing your report in accordance with the Documentation of Academic Work.

Appendix A

Naming Binary Compounds

A. Introduction

A thorough knowledge of nomenclature will greatly aid your development in chemistry. You must learn the basic rules and use the rules frequently as the course progresses. A binary compound is one that is formed from two elements. For most binary compounds, the ending -ide is substituted for the usual ending of the element that appears last in the name. Many elements form cations or anions whose charges can be predicted by their positions on the periodic table. The more you understand the periodic table, the less you need to memorize.

B. Metal-Nonmetal Binary Compounds.

1. Name the metal first followed by the nonmetal with -ide substituted for the usual ending. If the metal can have more than one oxidation state, more than one compound may be formed with a given nonmetal, such as $FeCl_2$ and $FeCl_3$. Systematic naming uses Roman numerals to specify the oxidation state of the metal. It reduces the amount of memory work required. Examples:

$FeCl_2$: Fe^{2+}, iron(II) chloride
$FeCl_3$: Fe^{3+}, iron(III) chloride
Cu_2O: Cu^{+}, copper(I) oxide
CuO: Cu^{2+}, copper(II) oxide
$SbCl_5$: Sb^{5+}, antimony(V) chloride

2. Metals which do not have more than one oxidation state are exceptions to the Roman numeral system. These exceptions include Group 1A metals, Group 2A metals, and the AZCA metals (Aluminum, Zinc, Cadmium, and Silver). Examples:

$NaCl$	sodium chloride
Mg_3N_2	magnesium nitride
Ag_2O	silver oxide

3. The algebraic sum of the oxidation numbers of the metal and the nonmetal must be zero since the compound has no net charge. Examples:

$NaCl$: Na^{+} (Group 1A) +1 oxidation state x 1 sodium ion = +1
Cl^{-} (Group 7A) -1 oxidation state x 1 chloride ion $\underline{= \ -1}$
0

Mg_3N_2: Mg^{2+} (Group 2A) +2 oxidation state x 3 magnesium ions = +6
N^{3-} (Group 5A) −3 oxidation state x 2 nitride ions $\underline{= -6}$
0

Ag_2O: Ag^+ (AZCA metal) +1 oxidation state x 2 silver ions = +2
O^{2-} (Group 6A) -2 oxidation state x 1 oxide ion $\underline{= -2}$
0

C. Nonmetal - Nonmetal Binary Compounds.

This system uses the Greek prefixes mono- (1), di- (2), tri- (3), tetra- (4), penta- (5), etc., to indicate the number of atoms of each nonmetal that is present. The nonmetal that is lowest in electronegativity (the electronegativity values for each element as shown on most Periodic Tables) is named first, followed by the other nonmetal with <u>-ide</u> substituted for the usual ending. Variable oxidation states are common among many of the nonmetals; therefore, the naming system used must allow for this. Examples:

NO	nitrogen monoxide
NO_2	nitrogen dioxide
N_2O	dinitrogen monoxide
PCl_3	phosphorous trichloride
PCl_5	phosphorous pentachloride

D. Common Names and Other Exceptions to the Rules

1. Certain binary compounds have acquired nonsystematic names by which they are known exclusively. These include water (H_2O) and ammonia (NH_3). Notice that the formula of the last compound is inverted with respect to the customary position of the hydrogen and nonmetal.

2. Compounds containing polyatomic ions are not binary compounds but are named by similar rules. Treat the atoms that make up the polyatomic ion as a single entity. The only change to the name of the polyatomic ion is to drop the word "ion". The ammonium ion, NH_4^+, is treated as if it were a Group IA metal. Examples:

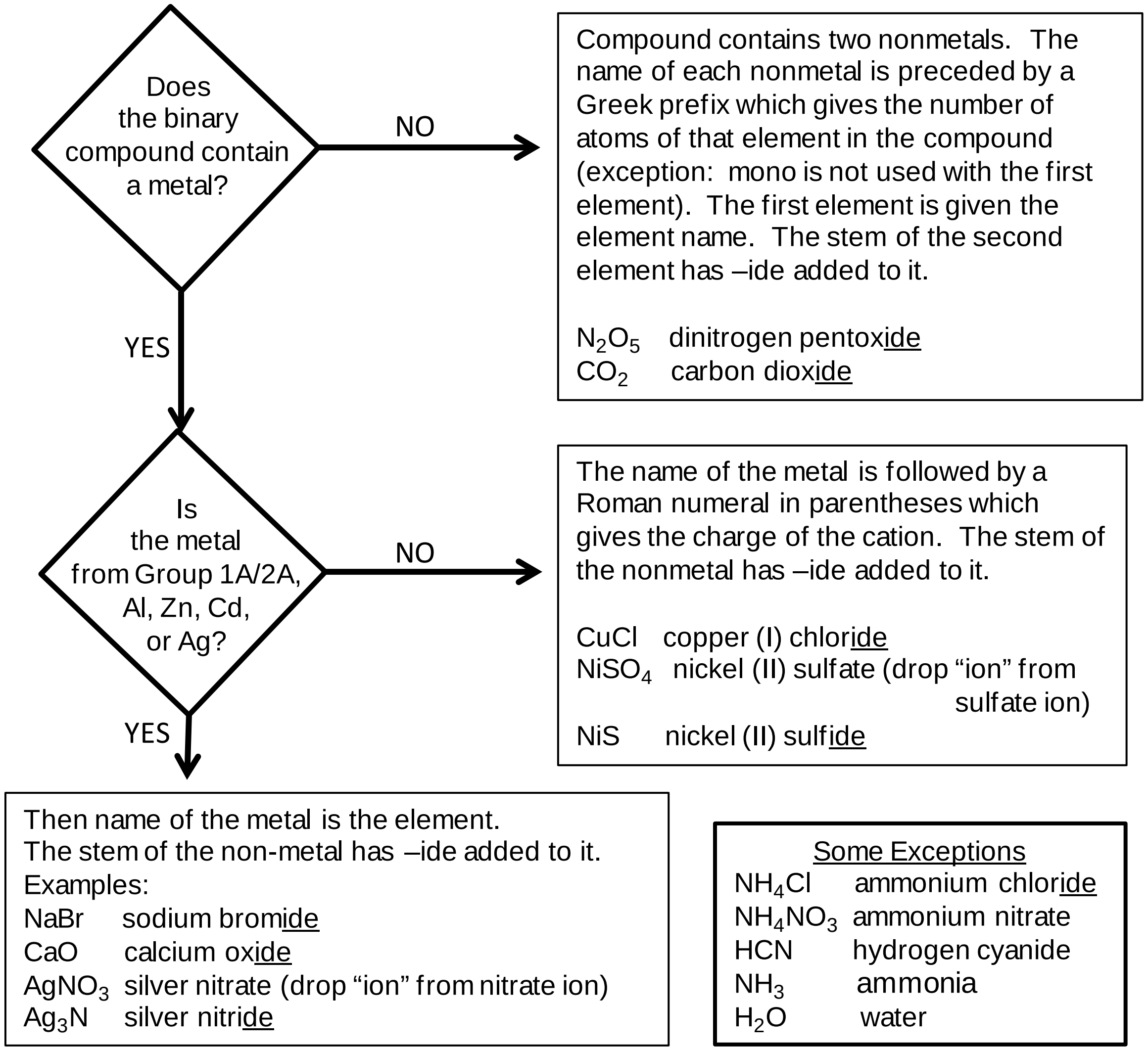
KCN	potassium cyanide
$Cu(OH)_2$	copper(II) hydroxide
$CoSO_4$	cobalt(II) sulfate
NH_4CO_3	ammonium carbonate

NOMENCLATURE FLOWCHART FOR BINARY COMPOUNDS

This flow chart will help you to name binary compounds. It is based upon the premise that you are able to:

1. classify elements as metals or nonmetals.
2. determine whether an element will form a cation or an anion.
3. identify the charge of that cation or anion.

Once you have mastered the above concepts, follow the directions in the diagram below to correctly name binary compounds.



Compound contains two nonmetals. The name of each nonmetal is preceded by a Greek prefix which gives the number of atoms of that element in the compound (exception: mono is not used with the first element). The first element is given the element name. The stem of the second element has –ide added to it.

N_2O_5	dinitrogen pentoxide
CO_2	carbon dioxide

The name of the metal is followed by a Roman numeral in parentheses which gives the charge of the cation. The stem of the nonmetal has –ide added to it.

CuCl	copper (I) chloride
$NiSO_4$	nickel (II) sulfate (drop “ion” from sulfate ion)
NiS	nickel (II) sulfide

Then name of the metal is the element.
The stem of the non-metal has –ide added to it.
Examples:

NaBr	sodium bromide
CaO	calcium oxide
$AgNO_3$	silver nitrate (drop “ion” from nitrate ion)
Ag_3N	silver nitride

Some Exceptions

NH_4Cl	ammonium chloride
NH_4NO_3	ammonium nitrate
HCN	hydrogen cyanide
NH_3	ammonia
H_2O	water

Appendix B

Visual Descriptions of Solutions

A. Introduction

Although modern technology now makes photographs of solutions practical, it is still necessary to accurately describe the physical properties of solutions. Including the elements explained here will allow you to provide a more thorough description of the observations you make about solutions. Additionally, paying close attention to the subtle changes in a solution's color, clarity, or presence of solid particles may provide you valuable insights into the chemical reactions that may be reflected in some of these changes.

B. Clarity

Clear - no visible particles, light passes through solution without scattering

Translucent - no visible particles, some light passes through solution, some light is scattered

Cloudy - visible particles, light is scattered, but some light passes through

Opaque - large or very dense particles completely block light

C. Solid Particles

If particles are present, the sample is a heterogeneous mixture normally classified as one of several types of suspension. Consider the particle size, color, density (whether they tend to sink to the bottom, float at the surface, or adhere to the walls of the container). Assess if the particles clump together, form fibrous connections, or disperse evenly throughout the sample.

D. Three Factors used in Describing Colors:

The Inter-Society Color Council of the National Bureau of Standards (ISCC-NBS) defines specific terms to be used when describing color. The information provided here is a simplified version of the ISCC-NBS color system.

1. Lightness: dark, medium, light

2. Saturation: grayish, moderate, strong, vivid

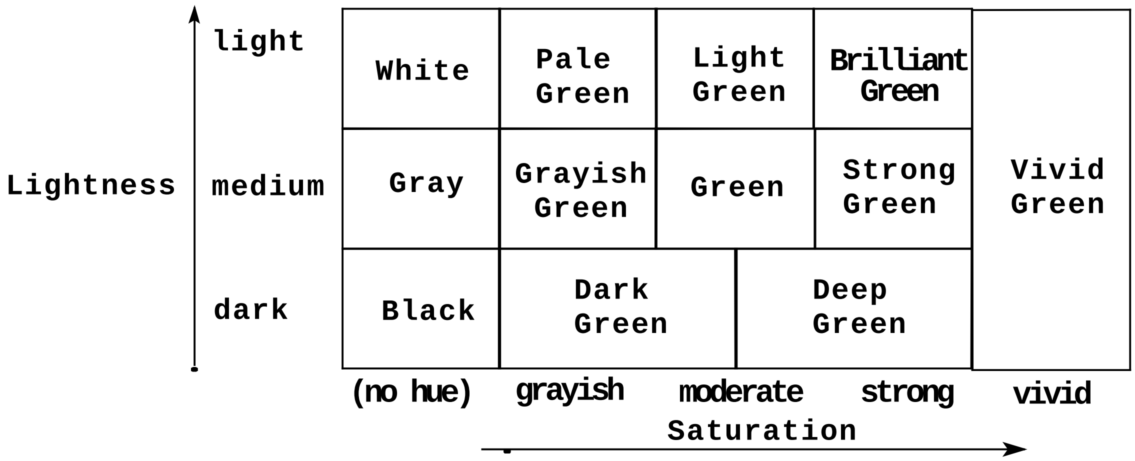
deep = dark and strong
pale = light and grayish
brilliant = light and strong

3. Hue: commonly used descriptions

red
reddish orange
orange
orange yellow
greenish yellow
yellowish green
green
bluish green
greenish blue
blue
purplish blue
violet
purple
reddish purple
purplish red
purplish pink
pink
brownish pink
brownish orange
reddish brown
brown
yellowish brown
olive
olive green

Example

Hue = Green



Lightness	(no hue)	grayish	moderate	strong	vivid
light	White	Pale Green	Light Green	Brilliant Green	Vivid Green
medium	Gray	Grayish Green	Green	Strong Green	
dark	Black	Dark Green		Deep Green	

Saturation

Appendix C

Naming Organic Compounds

A. Introduction

The ability to interpret structures of organic compounds is important in understanding basic Chemistry, Biochemistry and more advanced science information. Learning how to name and draw organic structures opens up an entire language of scientific knowledge that greatly increases your ability to interpret and discuss fields that affect human health, the environment, and industrial processes. The naming rules for organic compounds, or carbon containing compounds, can be found in your textbook and several examples are on your Reference Data Cards.

B. General Rules for Naming an Organic Molecule

1. Identify the longest chain of carbon atoms. This is referred to as the “backbone” of the organic molecule. The longest chain must include multiple bonds.

2. Identify any multiple bonds, substituent groups, and/or functional groups present in the molecule.

a) Bonds within the backbone:

i. If there are only single bonds in a hydrocarbon chain, then the molecule belongs to the general group “alkane” and will have an “–ane” suffix.
ii. If there are double bonds (one or more in a hydrocarbon backbone), then the molecule belongs to the general group “alkene” and will have the “–ene” suffix.
iii. If there are triple bonds (one or more in a hydrocarbon backbone), then the molecule belongs to the general group “alkyne” and will have the “–yne” suffix.

b) Substituent groups. Any of the groups identified in Table C-1: Common Substituent Groups for Organic Molecules.

Table C-1: Common Substituent Groups for Organic Molecules

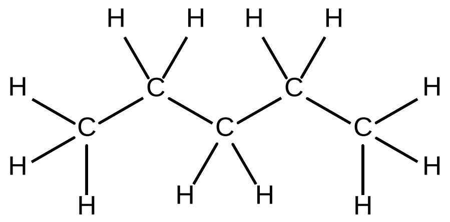
Formula	Name	Formula	Name
$-CH_3$	methyl	-Br	Bromo
$-CH_2CH_3$ or $–C_2H_5$	ethyl	-Cl	Chloro
$-CH_2CH_2CH_3$	propyl	-F	Fluoro
$-CH(CH_3)_2$	isopropyl	-CN	Cyano
$-CH=CH_2$	ethenyl (vinyl)	$-NO_2$	Nitro
$–C_6H_5$	phenyl	$-NH_2$	amino
$– CH_2C_6H_5$	benzyl	-D	deuterio

c) Functional groups. Any of the functional groups drawn in the Functional Groups Table on your Chemistry Reference Data Card (RDC). The presence of one of these groups will change the ending on the name of the molecule.

3. Sequentially number the carbon atoms in the backbone. A carbon atom with a functional group takes precedence over a multiple bond. A multiple bond takes precedence over a substituent group. The carbon chain will be numbered so that the functional group carbon has the lowest number. If there is not a functional group, then the carbon chain will be numbered so that the multiple bond carbon has the lowest number. If there is not a functional group or a multiple bond, then the carbon chain will be numbered so that the substituent group carbon has the lowest number.

4. Name the molecule. Numbers are used to identify the carbons with substituent groups and multiple bonds. When different substituent groups are present, list them in alphabetical order with the number in front identifying the specific carbon to which they are attached. Endings on the name should be appropriate for the functional group present or for bonding within the backbone.

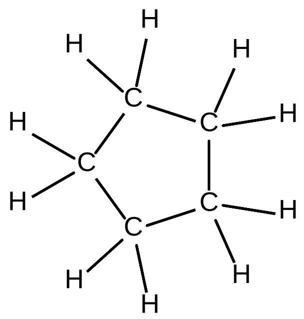
EXAMPLE 1, an alkane: $CH_3CH_2CH_2CH_2CH_3$ (which may also be written C_5H_{12}) The name of the molecule is “pentane”.



Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and end the name with the suffix “–ane” which lets the reader know all the carbon to carbon bonds in the hydrocarbon are single.

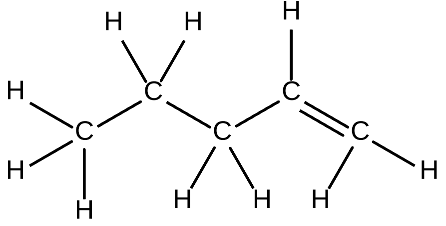
Cyclic alkanes all have the general formula C_nH_{2n} and the carbons are arranged to form a ring. They are named based on the number of carbons with the prefix “cyclo-”.

EXAMPLE 2, a cycloalkane. C_5H_{10} The name of this molecule is “cyclopentane”.



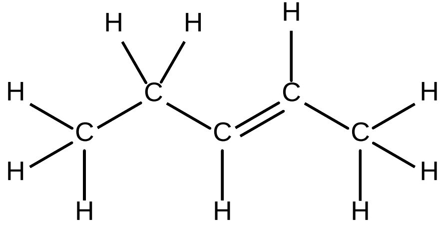
Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and use the suffix “–ane” which lets the reader know all of the carbon to carbon bonds are single and the prefix cyclo- tells the reader that it is a cycloalkane.

EXAMPLE 3, an alkene. $CH_3CH_2CH_2CHCH_2$ (which may also be written C_5H_{10}), is “1-pentene”.



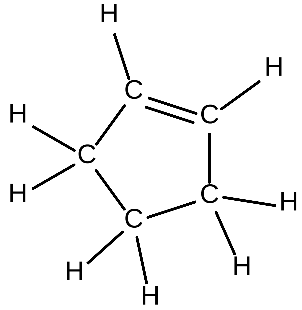
Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and use the suffix “–ene” which lets the reader know at least one of the carbon to carbon bonds is a double bond and specifically there is <u>one</u> double bond and it is located between the first and the second carbon in the chain.

EXAMPLE 4, an alkene: $CH_3CH_2CHCHCH_3$ (which may also be written C_5H_{10}) is “2-pentene”.



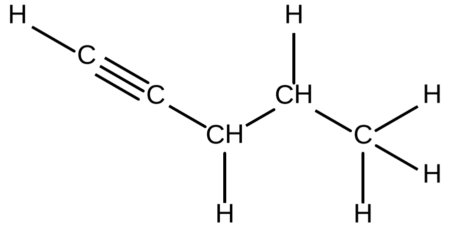
Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and use the suffix “–ene” which lets the reader know at least one of the carbon to carbon bonds is a double bond and specifically there is <u>one</u> double bond and it is located between the second and third carbon in the chain.

EXAMPLE 5, an alkene: C_5H_8, the name is “1-cyclopentene.”



Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and use the suffix –ene which lets the reader know at least one of the carbon to carbon bonds is a double bond and specifically there is one double bond located between the first and the second carbon in the chain (so that it has the lowest number possible) and the prefix cyclo- tells the reader that it is a cycloalkene.

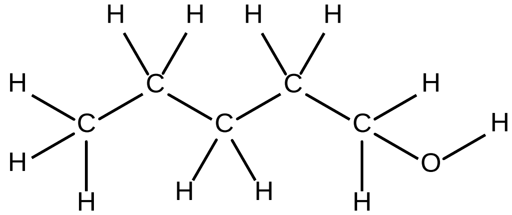
EXAMPLE 6, an alkyne $CH_3CH_2CH_2CCH$ is “1-Pentyne”.



Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and end the name with the suffix “–yne” which lets the reader know at least one of the carbon to carbon bonds is a triple bond and specifically there is only one triple bond and it is located between the first and the second carbon in the chain.

What if the compound contains a functional group?

EXAMPLE 7, a functional group containing molecule:



Follow the Steps to name this molecule:
Step 1. Identify the number of carbons in the backbone: five, so we will use the penta-prefix.
Step 2. Identify any multiple bonds, substituent groups, and/or functional groups:

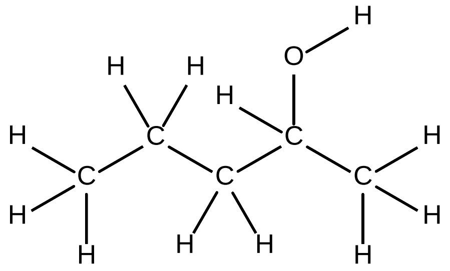
   a. no multiple bonds so normally would follow alkane rules
   b. no substituent groups
   c. there is an alcohol functional group so it will take precedence for naming the molecule and the name will end in –ol, to identify it as an alcohol.

Step 3. Number the carbons so that the carbon with the attached functional group has the lowest number.
Step 4. Name the molecule. <u>The name for this molecule is “1-pentanol”.</u>

Explanation: There are five carbons, thus we utilize the prefix pent- for “five” and end the name with the suffix –ol which lets the reader know it is an alcohol. Further, the 1- in the name tells the reader that the alcohol functional group is attached to the end carbon, making it a primary alcohol.

BUT what if the alcohol was on a carbon inside the carbon chain?

EXAMPLE 8, a molecule that has a functional group not on a terminal carbon:

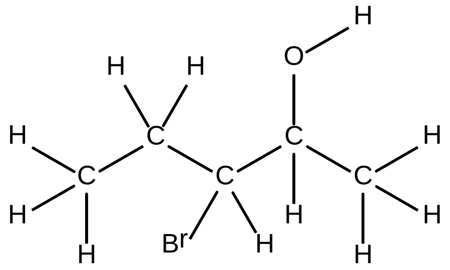


<u>The name is “2-pentanol”.</u>

Explanation: There are five carbons which demands the prefix pent- for “five” and the name will end in the suffix –ol which lets the reader know it is an alcohol. Further, the 2- in the name tells the reader that the alcohol functional group is attached to the second carbon in the chain, making it a secondary alcohol.

What happens when there is a substituent group and a functional group?

EXAMPLE 9, a molecule with both a functional group and a substituent group:



Follow the Steps to name this molecule:
Step 1. Identify the number of carbons in the backbone: five, so we will use the penta-prefix.
Step 2. Identify any multiple bonds, substituent groups, and/or functional groups:

   d. no multiple bonds so normally would follow alkane rules
   e. there is a bromine substituent group
   f. there is an alcohol functional group so it will take precedence for naming the molecule and the name will end in –ol, to identify it as an alcohol.

Step 3. Number the carbons so that the carbon with the attached functional group has the lowest number.
Step 4. Name the molecule. <u>The name is “3-bromo, 2-pentanol”.</u>

Explanation: There are five carbons which utilizes the prefix pent- for “five” and the name will end in the suffix –ol which lets the reader know it is an alcohol. The 2-bromo tells the reader that the second carbon is bonded to a bromine and the 2-pentanol tells the reader that the alcohol functional group is attached to the second carbon in the chain, making it a secondary alcohol. Note that the groups are listed in alphabetical, not numeric, order.

Appendix D

Safety Data Sheets

A. Introduction

Many chemicals have dangers associated with their use. You can avoid these dangers by reading the **Safety Data Sheets (SDS)** (previously called Material Safety Data Sheets, or MSDS) for the chemicals you use. All chemical manufacturers are required by law to provide a SDS for each chemical the laboratory purchases for your use. The information given relates to the risks involved when using a specific chemical. These sheets are found in a yellow binder located in each chemistry laboratory. The SDS provides a wealth of information about chemicals and all laboratory workers rely on them to provide the information they need to work safely in the lab.

B. Interpreting an SDS

A typical SDS for sodium chloride, NaCl, is provided in this section as an example. The sheet is divided into several sections: Identification, Toxicity Hazards, Health Hazard Data, Physical Data, Fire and Explosion Data, Reactivity Data, Spill or Leak Procedures, and Additional Precautions and Comments. The identification section provides additional names by which the compound is known (table salt, for example), the CAS (Chemical Abstract Service) number, and the Fisher catalog product numbers (Fischer Scientific is a major chemical supplier). The CAS number is especially useful because you can use it to conduct searches in several databases to obtain more sources of information concerning the chemical.

1. The Toxicity Hazards section contains results of studies detailing the toxicity of the compound in various animal and inhalation tests. Sodium chloride is a well-studied compound, so many such tests have been performed. There are several common abbreviations used:

HMN	human
IVN	Intravenous
LD50	Lethal Dose 50. The single dose of a substance that causes the death of 50% of an animal population from exposure to the substance by any route other than inhalation.
LDLo	Lethal Dose low. The lowest dose of a substance introduced by any route, other than inhalation, reported to have caused death in humans or animals.
ORL	Oral Dose
SKN	Skin

Thus, the listing ORL-RAT LD50: 3000 mg/kg indicates that when NaCl was given orally to a test sample of rats, the dose that killed 50% of the rats was 3000 mg NaCl per kg body weight (BW). Extrapolating directly from rats to humans (assuming average BW = 80 kg), it would take 3000 mg NaCl / kg BW x 80 kg BW/person = 2.40 x 10^5 mg/person or 240 g / person to kill half the individuals in this group of exposed humans. Obviously, the LD50 for NaCl is well above the amount one would expect to accidentally ingest in the lab, so NaCl does not pose a significant risk to you in this regard.

2. The Health Hazard Data section indicates that inhalation, ingestion, or skin absorption may be harmful, and that NaCl irritates mucous membranes and the upper respiratory tract. This may surprise you since table salt is a common household item and a substance we consume every day. However, it is well know that NaCl(s) will sting the eyes and that prolonged exposure of the skin to salt water can be harmful. While spilling a small amount of NaCl on the skin will not hurt you, this warning illustrates the principle of trying to minimize contact with any chemical. The section also gives the treatment for getting salt in the eyes: flush with water for at least 15 minutes.

3. The Fire and Explosion Hazard Data and Reactivity Data sections provide information about chemical incompatibilities and other chemical reaction dangers. The SDS indicates that NaCl does not combust but that it may react with strong oxidizing agents or strong acids. The steps to be taken if material is released or spilled generally refers to large, industrial amounts of the chemical. Specific information will be provided in the experimental procedures for materials with unusual handling characteristics.

4. Waste disposal methods are also given. Again, they generally refer to quantities much larger than those used in normal laboratory experiments. The Handling and Storage section gives some practical advice on how to deal with the compound, as well as recommended safety equipment that should be on hand.

5. The Additional Precautions and Comments section details specific dangers associated with this compound. Sodium chloride is known to react violently with lithium and bromine trifluoride under certain conditions. Extreme caution should be used if these substances and NaCl are used in the same reaction.

C. Other Useful Sources of Safety Information

1. The Merck Index. Similar information in a more compact form can be found in the Merck Index. This reference gives the “bottom line” on the toxicity of chemicals and their incompatibilities. In the case of NaCl, the index lists under Human Toxicity: “Not generally considered poisonous. Accidental substitution of NaCl for lactose in baby formulas has caused fatal poisoning.” While the information in the Merck Index is not as complete as in the MSDS, it is generally sufficient for our purposes. It also provides information about the common usages of the chemicals listed, with special emphasis on medical usages. References to the chemical literature are also provided.

2. CRC Handbook of Chemistry and Physics (CRC). This reference contains a wide range of data in the area of health, safety, and environmental protection. It also contains directions for the handling and disposal of laboratory chemicals. Each lab bench has a copy of the CRC.

Example Safety Data Sheet

Sodium Chloride ACC# 21105

Section 1 - Chemical Product and Company Identification

SDS Name: Sodium Chloride
Catalog Numbers: AC424290030, S641212, S641350LB, S641500, S71988, S71989, S71989-1, S78446,S78449, S78449-1, BP358 1, BP358 10, BP358 212, BP358-1, BP358-212, BP3581, BP35810, BP3581000,BP358212, XXB64017.5KG
Synonyms: Common salt, halite, rock salt, saline, salt, sea salt, table salt
Company Identification:

Fisher Scientific
1 Reagent Lane
Fairlawn, NJ 07410

For information, call: 201-796-7100
Emergency Number: 201-796-7100
For CHEMTREC assistance, call: 800-424-9300
For International CHEMTREC assistance, call: 703-527-3887S

Section 2 - Composition, Information on Ingredients

CAS#	Chemical Name	Percent	EINECS/ELINCS
7647-14-5	Sodium chloride	100	231-598-3

Section 3 - Hazards Identification - EMERGENCY OVERVIEW

Appearance: colorless or white. Caution! May cause respiratory tract irritation.May cause eye and skin irritation. May cause digestive tract irritation with nausea, vomiting, and diarrhea.
Target Organs: None.
Potential Health Effects
Eye: May cause eye irritation.
Skin: May cause skin irritation.
Ingestion: Ingestion of large amounts may cause gastrointestinal irritation. Ingestion of large amounts may cause nausea and vomiting, rigidity or convulsions. Continued exposure can produce coma, dehydration, and internal organ congestion.
Inhalation: May cause respiratory tract irritation.
Chronic: Not available.

Section 4 - First Aid Measures

Eyes: Flush eyes with plenty of water for at least 15 minutes, occasionally lifting the upper and lower lids. Get medical aid.
Skin: Flush skin with plenty of soap and water for at least 15 minutes while removing contaminated clothing and shoes. Get medical aid if irritation develops or persists. Wash clothing before reuse.
Ingestion: If victim is conscious and alert, give 2-4 cupfuls of milk or water. Get medical aid.

Inhalation: Remove from exposure to fresh air immediately. If not breathing, give artificial respiration. If breathing is difficult, give oxygen. Get medical aid if cough or other symptoms appear.
Notes to Physician: None
Antidote: None reported

Section 5 – Fire fighting Measures

General Information: Wear appropriate protective clothing to prevent contact with skin and eyes. Wear a self-contained breathing apparatus (SCBA) to prevent contact with thermal decomposition products.
Extinguishing Media: For small fires, use water spray, dry chemical, carbon dioxide or chemical foam.
Auto-ignition Temperature: Not available.
Flash Point: Not available.(estimated) Health: 1; Flammability: 0; Reactivity: 0 Explosion Limits, Lower: Not available. Upper: Not available.

Section 6 - Accidental Release Measures

General Information: Use proper personal protective equipment as indicated in Section 8.
Spills/Leaks: Vacuum or sweep up material and place into a suitable disposal container. Avoid generating dusty conditions.

Section 7 - Handling and Storage

Handling: Use with adequate ventilation. Minimize dust generation and accumulation. Avoid contact with eyes, skin, and clothing. Do not ingest or inhale.
Storage: Store in a cool, dry place.

Section 8 - Exposure Controls, Personal Protection

Engineering Controls: Good general ventilation should be sufficient to control airborne levels.

Exposure Limits Chemical Name	ACGIH	NIOSH	OSHA - Final PELs
Sodium chloride	none listed	none listed	none listed

OSHA Vacated PELs: Sodium chloride: No OSHA Vacated PELs are listed for this chemical.
Personal Protective Equipment
Eyes: Wear safety glasses with side shields.
Skin: Wear appropriate gloves to prevent skin exposure.
Clothing: Wear appropriate protective clothing to minimize contact with skin.
Respirators: A NIOSH/MSHA approved air purifying dust or mist respirator or European Standard EN 149.

Section 9 - Physical and Chemical Properties

Physical State: Solid
Appearance: colorless or white
Odor: odorless
pH: Not available.
Vapor Pressure: Not available.
Vapor Density: Not available.
Evaporation Rate:
Viscosity: Not available.
Boiling Point: 2575 deg F

Freezing/Melting Point:1474 deg F
Decomposition Temperature:Not available.
Solubility: Soluble in water
Specific Gravity/Density:2.165
Molecular Formula:NaCl
Molecular Weight:

Section 10 - Stability and Reactivity

Chemical Stability: Stable.
Conditions to Avoid: High temperatures.
Incompatibilities with Other Materials: Reacts with most nonnoble metals such as iron or steel, building
materials (such as cement), bromine, or trifluoride. Potentially explosive reaction with dichloromaleic
anhydride + urea. Electrolysis of mixtures with nitrogen compounds may form explosive nitrogen trichloride.
Hazardous Decomposition Products: Toxic fumes of sodium oxide.
Hazardous Polymerization: Has not been reported.

Section 11 - Toxicological Information

RTECS#:
CAS# 7647-14-5: VZ4725000
LD50/LC50:
CAS# 7647-14-5:
Oral, mouse: LD50 = 4 gm/kg;
Oral, rat: LD50 = 3 gm/kg;
Carcinogenicity:
CAS# 7647-14-5: Not listed by ACGIH, IARC, NIOSH, NTP, or OSHA.
Epidemiology: No information reported.
Teratogenicity: An experimental teratogen.
Reproductive Effects: Human reproductive effects by intraplacental route: terminates pregnancy.
Experimental reproductive effects.
Neurotoxicity: No information reported.
Mutagenicity: Human mutation data reported.

Section 12 - Ecological Information

Ecotoxicity: No information found
Environmental Fate: No information reported.
Physical/Chemical: No information found
Other: No information found

Section 13 - Disposal Considerations

Dispose of in a manner consistent with federal, state, and local regulations.
RCRA D-Series Maximum Concentration of Contaminants: None listed.
RCRA D-Series Chronic Toxicity Reference Levels: None listed.
RCRA F-Series: None listed.
RCRA P-Series: None listed.
RCRA U-Series: None listed.

Section 14 - Transport Information

US DOT	IATA	RID/ADR	IMO	Canada TDG
Shipping Name: Hazard Class: UN Number:	No info	No info	No info	No info

Section 15 - Regulatory Information

US FEDERAL TSCA
CAS# 7647-14-5 is listed on the TSCA inventory.
Health & Safety Reporting List
None of the chemicals are on the Health & Safety Reporting List. Chemical Test Rules
None of the chemicals in this product are under a Chemical Test Rule.
Section 12b
None of the chemicals are listed under TSCA Section 12b.
TSCA Significant New Use Rule
None of the chemicals in this material have a SNUR under TSCA.
SARA
Section 302 (RQ)
None of the chemicals in this material have an RQ.
Section 302 (TPQ)
None of the chemicals in this product have a TPQ.
SARA Codes CAS # 7647-14-5: acute.
Section 313
No chemicals are reportable under Section 313.

Clean Air Act: This material does not contain any hazardous air pollutants. This material does not contain any Class 1 Ozone depletors. This material does not contain any Class 2 Ozone depletors.

Clean Water Act: None of the chemicals in this product are listed as Hazardous Substances under the CWA. None of the chemicals in this product are listed as Priority Pollutants under the CWA. None of the chemicals in this product are listed as Toxic Pollutants under the CWA.

OSHA: None of the chemicals in this product are considered highly hazardous by OSHA.

STATE CAS# 7647-14-5 is not present on state lists from CA, PA, MN, MA, FL, or NJ.
California No Significant Risk Level: None of the chemicals in this product are listed.
European/International Regulations
European Labeling in Accordance with EC Directives
Hazard Symbols: Not available.
Risk Phrases:
Safety Phrases:
WGK (Water Danger/Protection) CAS# 7647-14-5: 0

Canada CAS# 7647-14-5 is listed on Canada's DSL/NDSL List. This product has a WHMIS classification of Not controlled. CAS# 7647-14-5 is not listed on Canada's Ingredient Disclosure List.

Exposure Limits

Section 16 - Additional Information

MSDS Creation Date: 12/14/1994
Revision #18 Date: 12/12/1997

Appendix E

Rules for Significant Figures

A. Introduction

Unlike counting, measurement is never exact. You can *count* exactly ten M4 rifles in the arms room, but if you *measure* the length of the rifle 10 times, using a tape measure marked each cm, the values are likely to differ slightly. Table E-I presents such a set of measurements.

Measurement	Length(cm)
1	**82.6**
2	**82.5**
3	**82.5**
4	**82.7**
5	**82.4**
6	**82.6**
7	**82.5**
8	**82.5**
9	**82.6**
10	**82.7**

Table E-1. Length of a Rifle Measurements

First you must consider the uncertainty of the tape measure. Many quality measuring devices are marked with their uncertainty. If you cannot determine the uncertainty of a measuring device, a good rule of thumb is to use 50% of the smallest mark, which in this case gives the tape measure an uncertainty of ± 0.5 cm. Note that all ten measurements agree on the first two digits of the measurement; differences occur in the third digit. Which values are correct? The accuracy of measurement depends on many factors as discussed in Section 3A. Measured values are usually recorded with the last digit regarded as uncertain. The data in Table E-1, combined with the uncertainty data, allow us to state that the length of the rifle is between 81.9 and 83.2 cm, and should be recorded as the average plus the uncertainty, or 82.6 ± 0.5 cm. To determine the length with greater accuracy (e.g. 82.56 cm) would require a measuring device with less uncertainty. Reporting of numerical values is extremely important in all sciences. You will be required to be cognizant of reporting values correctly in all of your courses in mathematics, science, and engineering at the United States Military Academy.

B. Rules for Counting Significant Figures (Digits)

1. In any properly reported measurement, all nonzero digits are significant. Read the number from left to right and count all digits, starting with the first nonzero number.

2. The zero presents problems, however, because it can be used in two ways: to position the decimal point or to indicate a measured value. For zeros, follow these rules:

a) A zero between two other digits is always significant. Examples:

120**7** contains four significant figures.

40.00**3** contains five significant figures.

b) Zeros to the left of ***all*** nonzero digits are not significant. Examples:

$0.000\underbrace{16\mathbf{3}}_{\text{three numbers}}$ has three significant figures.

$0.0\underbrace{680\mathbf{1}}_{\text{four numbers}}$ has four significant figures.

c) Zeros that are *both* to the right of the decimal point *and* to the right of nonzero digits are significant. Examples:

0.4000	four significant figures.
0.05022**0**	five significant figures.
803.77**0**	six significant figures.

d) Zeros in numbers such as 50,000 (that is, zeros to the right of *all* nonzero digits in a number that is written **without** a decimal point) **may** or **may not** be significant. Without more information, we simply do not know whether 50,000 was measured to the nearest one, ten, hundred, thousand, or ten-thousand. To avoid this confusion, scientists use exponential notation for writing numbers. In exponential notation, 50,000 would be recorded as $\mathbf{5} \times 10^4$ or $5.\mathbf{0} \times 10^4$ or $5.000\mathbf{0} \times 10^4$ to indicate one, two, and five significant figures, respectively.

C. Rules for Significant Figures in Rounding Off Digits

Determine the correct number of digits in the final answer and round off to this number. In rounding off, you should increase the last significant figure by one if the following digit is 5 or greater (e.g. 549 rounded to two significant figures becomes 550). **When working through a problem with multiple calculations, only round off your FINAL answer. Rounding off during intermediate steps can introduce errors.**

D. Rules for Significant Figures in Addition and Subtraction

In addition or subtraction, the final result should contain no more place holder digits than the quantity that "ends" the earliest, when significant digits are read from left to right. Align the quantities to be added or subtracted on the decimal point, then perform the operation, assuming blank spaces are zeros.

EXAMPLE 1: Add the following numbers: 744, 2.6, 14.812

SOLUTION: Align the numbers on the decimal point and carry out the addition.

$$\begin{array}{l}
\texttt{744} \\
\texttt{~~2.6} \\
\underline{\texttt{~14.812}} \\
\texttt{76}\mathbf{1}\texttt{.412}
\end{array}$$

The correct answer is **761**. Since one of the digits to be added (744) goes no further than the ones place, the final answer must therefore be rounded so that it, too, contains **no** digits past the ones place.

EXAMPLE 2: Add the following numbers: 49.146, 72.13, 5.9432

SOLUTION: Align the numbers on the decimal point and carry out the addition.

$$\begin{array}{l}
\texttt{~49.14}\mathbf{6} \\
\texttt{~72.1}\mathbf{3} \\
\underline{\texttt{~~5.943}\mathbf{2}\texttt{~}} \\
\texttt{127.2}\mathbf{1}\texttt{92}
\end{array}$$

The correct answer is **127.22**. Read from left to right, the second quantity ends first, at the hundredths place. The final answer should also end at the hundredths place. Since the digit in the thousandths place is 5 or greater (in this case 9), the digit in the hundredths place is rounded up to 2.

EXAMPLE 3: Subtract 9.143 from 71.12496

SOLUTION: Align the numbers on the decimal point and carry out the subtraction.

$$\begin{array}{l}
\texttt{71.1248}\mathbf{6} \\
\underline{-\texttt{9.14}\mathbf{3}} \\
\texttt{61.98}\mathbf{18}\texttt{6}
\end{array}$$

The correct answer is **61.982**. Since the second quantity ends at the thousandths place, so must the final answer. Since the number in the ten-thousandths place is 5 or greater (in this case, 8), the number in the hundreds place is rounded up.

EXAMPLE 4: Subtract 0.055 from 7,700

SOLUTION: Align the numbers on the decimal point and carry out the subtraction.

$$\begin{array}{r} 7{,}700 \\ -\ \ 0.055 \\ \hline 7{,}699.945 \end{array}$$

The correct answer is **7,700** (*NOT 7,699.945*). Since the last significant figure in 7,700 is in the hundreds place, the final answer must also end at the hundreds place. Since the number in the tens place is 5 or greater (in this case, 9), the number in the hundreds place is rounded up.

E. Rules for Significant Figures in Multiplication and Division

In multiplication and division the rules are simpler. The answer can have no more significant figures than the factor that has the least total number of significant figures. In these operations, the *position* of the decimal point makes no difference.

EXAMPLE 1: Multiply 10.4 by 3.1416.

SOLUTION: 10.4 x 3.1416 = 32.672,64

The correct answer is 32.7. The answer has **only three** significant figures because the first term has **only three** significant figures.

EXAMPLE 2: Divide 500.0 by 34.408

SOLUTION: 500.0 / 34.408 = 14.5315043

The correct answer is 14.53 (*NOT* 14.531). The answer has **only four** significant figures because the first term has **only four** significant figures.

EXAMPLE 3: Divide 5,000 by 34.408 then multiply by 2.5

SOLUTION: 5,000 / 34.408 * 2.5 = 363.2876075331318

The correct answer is 400 (no decimal place). The answer has **only one** significant figure because the number 5,000 has **only one** significant figure.

COMBINED EXAMPLE: 5,324.2 + (4.0 * 33)

SOLUTION: 4.0 * 33 = 132 (two significant figures, don't round until final answer)

$$\begin{array}{r} 132 \\ +5{,}324.2 \\ \hline 5{,}456.2 \end{array}$$

The correct answer is 5,460 (no decimal point). Because 132 has two significant figures (determined by the multiplication of 4.0), the significant figures end at the tens place, so the significant figure in the final answer must also end at the tens place. Note that if you rounded 132 to 130 (2 significant figures) before the addition step, you would get an <u>in</u>correct final answer of 5,450. When performing a series of calculations, you may find it useful to place a small horizontal line above or below the last significant digit in an intermediate step.

F. Rules for Significant Figures in Logarithmic Operations

Measurements such as pH, radioactive decay and earthquakes are commonly discussed using a logarithmic scale for easier comparison of values that span a very wide range. In this type of notation, the number before the decimal place in the answer is the power term and is equal to one less than the number of digits before the decimal place in the original number. The power term is **not** included when counting significant figures. The Mantissa is a positive number less than 1 (i.e. the number after the decimal place). The Mantissa contains as many significant figures as the number whose log was found. Using reverse logic, the antilogarithm of a number has the same significant figures as the mantissa of the number whose antilogarithm was found.

EXAMPLE 1: Find the log (base ten) of 52 (two significant figures)

SOLUTION: $\text{Log}(52) = \underset{\uparrow\ \textbf{power term}}{1}.\underbrace{716003}_{\textbf{mantissa}}$

The correct final answer is **1.72** (NOT 1.7). The answer has two significant figures <u>after</u> the decimal place, the same number of significant figures that were in the original number.

EXAMPLE 2: Find the natural log (base *e*) of 4.45 (3 significant figures)

SOLUTION: $\mathbf{ln\ 4.45} = \underset{\nearrow\ \textbf{power term}}{\mathbf{1}}.\underbrace{\mathbf{492904}}_{\textbf{mantissa}}$

The correct answer is **1.493** (NOT 1.49). The answer has three significant figures <u>after</u> the decimal place, the same number of significant figures that were in the original number.

EXAMPLE 3: Find the antilog (base *e*) of 1.1234

SOLUTION: $\underset{\nearrow\ \textbf{power term}}{\boldsymbol{e}}^{\underbrace{\mathbf{1.1234}}_{\textbf{mantissa}}} = \mathbf{3.0752924}$

The correct answer is **3.075** (NOT 3.0753). The answer has four significant figures, the same number of significant figures that were in the mantissa of the number whose antilogarithm was found.

EXAMPLE 4: Find the pH of a solution with a $[H^+]$ of 2.8×10^{-13} M

SOLUTION: The digits to the left of the decimal point in a pH represent the power of 10, therefore, only the digits to the right of the decimal are significant. Keeping in mind that 2.8×10^{-13} M is the same as 0.00000000000028 M,

$$\begin{aligned} pH &= -\log (2.8 \text{ X } 10^{-13}) \\ &= -\log (2.8) + (-\log 10^{-13}) \\ &= -0.45 + 13.00 \\ &= 12.55 \end{aligned}$$

The correct answer is **12.55** (NOT 13). There are two digits to the right of the decimal in 12.55 because the H^+ concentration had two significant figures.

Appendix F

Uncertainty Theory and Special Cases

A. Mathematical Details for the Theory of Partial Derivatives[10].

We will address the general case of a function of one variable first, (e.g.$f(x) = x$)where x has some known absolute uncertainty $\pm\delta x$. Graphically, this case can be represented by Figure F-1.

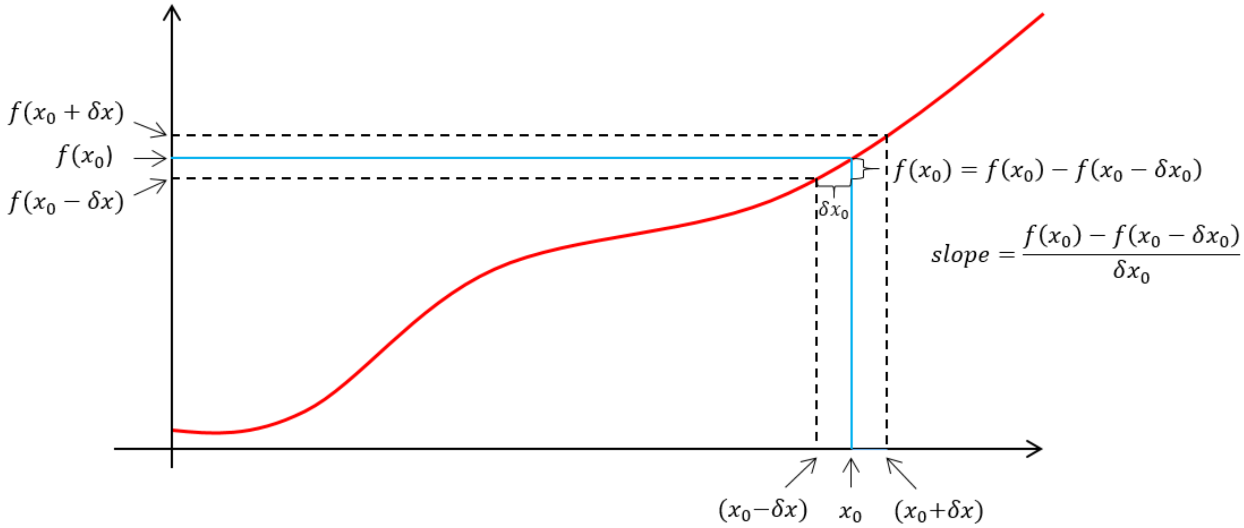


Figure F-1. Graphical Representation of a Function of One Variable.

If we know the functional form of $f(x)$, then the absolute uncertainty in f, δf, is given by:

$$\delta f(x_0) = |f(x_0) - f(x_0-\delta x_0)|.$$

If $\delta x_0 \ll x_0$:

$$\left.\frac{\Delta f}{\Delta x}\right|_{x_0} = \lim_{\delta x\to 0}\frac{f(x_0)-f(x_0-\delta x_0)}{\delta x_0}$$

$$\therefore f(x_0) - f(x_0-\delta x_0) = \left.\frac{df}{dx}\right|_{x_0}\delta x_0 \qquad \text{(F-1)}$$

---

[10] Taylor, John, R. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. 2nd ed. Sausalito, California: University Science Books, 1997, p.63

Therefore,

$$\delta f = \left|\frac{df}{dx}\right|_{x_0} \delta x_0 \qquad \text{(F-2)}$$

To generalize to functions with more than one variable, the derivative $\frac{df}{dx}$ in Equation F-2 becomes the partial derivative $\frac{\partial f}{\partial x}$ and the equation can be generalized to

$$\delta f = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \delta x_i\right)^2} \qquad \text{(F-3)}$$

B. Special Cases

1. Uncertainty in Sums and Differences

Suppose that our function of interest is solely made up of sums and differences of other measured quantities:

$$f = \sum_{i=1}^{N} A_i x_i \qquad \text{(F-4)}$$

where A_i are fixed parameters (positive for sum terms and negative for difference terms) and x_i are measured quantities each with associated uncertainty δx_i. To find δf, we can substitute Equation F-4 into F-3:

$$\delta f = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial}{\partial x_i} \left(\sum_{j=1}^{N} A_j x_j \right) \delta x_i \right)^2} \qquad \text{(F-5)}$$

Since $\frac{\partial}{\partial x_i} A_j x_j = 0$ if $i \neq j$ and $\frac{\partial}{\partial x_i} A_j x_j = A_i$ if $i = j$, Equation F-5 becomes:

$$\delta f = \sqrt{\sum_{i=1}^{N} \left(A_i \delta x_i \right)^2} \ . \qquad \text{(F-6)}$$

Put another way, **when our function involves only sums and differences, our resulting uncertainty is the sum in quadrature of the absolute uncertainties (multiplied by any factors the measured numbers are multiplied by)**.

2. Uncertainty in Products and Quotients.

Suppose that our function of interest is solely made up of products of other measured quantities:

$$f = A\prod_{i=1}^{N} x_i^{B_i} \qquad \text{(F-7)}^{*}$$

where A and B_i are fixed parameters and x_i are measured quantities each with associated uncertainty δx_i. To find δf, we can substitute Equation F-7 into F-3:

$$\delta f = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial}{\partial x_i}\left(A\prod_{j=1}^{N} x_j^{B_j}\right)\delta x_i\right)^2} \qquad \text{(F-8)}$$

Since $\frac{\partial}{\partial x_i}x_i^{B_i} = \frac{B_i x_i^{B_i}}{x_i}$ (as long as $B_i \neq 0$), Equation F-8 becomes

$$\delta f = \sqrt{\sum_{i=1}^{N}\left(\frac{B_i}{x_i}\left(A\prod_{j=1}^{N} x_j^{B_j}\right)\delta x_i\right)^2}$$

$$\delta f = \sqrt{\left(A\prod_{j=1}^{N} x_j^{B_j}\right)^2 \sum_{i=1}^{N}\left(\frac{B_i}{x_i}\delta x_i\right)^2}\ . \qquad \text{(F-9)}$$

Dividing both sides of Equation F-9 by F-7, we get

$$\frac{\delta f}{f} = \frac{\left(A\prod_{j=1}^{N} x_j^{B_j}\right)\sqrt{\sum_{i=1}^{N}\left(\frac{B_i}{x_i}\delta x_i\right)^2}}{A\prod_{j=1}^{N} x_j^{B_j}}$$

$$\Rightarrow \frac{\delta f}{f} = \sqrt{\sum_{i=1}^{N}\left(B_i\frac{\delta x_i}{x_i}\right)^2} \qquad \text{(F-10)}$$

Put another way, **when our function involves only multiplications and divisions (B_i = -1) and raising to powers, our resulting uncertainty is the sum in quadrature of the relative uncertainties (multiplied by any powers the measured numbers are raised to).**

---

[*] Note that "Π" is the product operator similar to the summation operator "Σ." Just as Equation H-4 represented a function $f = A_1x_1 + A_2x_2 + \ldots + A_Nx_N$, Equation H-7 represents a function $f = A \cdot x_1^{B_1} \cdot x_2^{B_2} \cdot \ldots \cdot x_N^{B_N}$.

Appendix G

Error Propagation: Intermediate Method Examples

Below you will find three examples of how to apply the method of partial derivatives as explained in Chapter 3, Section 3.2.

Example #1: Calculate $g(x)$ and its absolute uncertainty

Given:
$x = (5.01 \pm 0.03)$ m
$g(x) = 2x$

Solve:
$g = 2(5.01)$
$g = 10.02$

Apply Equation (3-6) with g(x) and take the partial derivative

$$\delta g(x) = \sqrt{\left[\frac{\partial g}{\partial x}\delta x\right]^2} = \sqrt{\left[\frac{\partial (2x)}{\partial x}\delta x\right]^2} = \sqrt{[2\delta x]^2} = 2\delta x$$

Multiply the absolute uncertainty of x

$\delta g(x) = (2)(0.03 \text{ m}) = 0.06$ m

Combine the uncertainty of g(x) and g(5.01) into a confidence interval

$g(x) = (10.02 \pm 0.06)$ m

Example #2: A ball falls from rest with uniform acceleration (5.32 ± 0.02) cm in (0.103 ± 0.001) s. Calculate the ball's acceleration and its absolute uncertainty.

Given:

$$x = (0.0532 \pm 0.0002)\ \text{m},\ t = (0.103 \pm 0.001)\ \text{s}$$

Physics: The position, acceleration, and time in uniform acceleration are related by $x - x_0 = \frac{1}{2}at^2 + v_0t$. If we set our origin ($x_0$) at zero and start from rest ($v_0 = 0$), our acceleration will be $a = \frac{2x}{t^2}$.

Solve:

$$a = \frac{2(0.0532\ \mathrm{m})}{(0.103\ \mathrm{s})^2}$$

$$a = 10.02922\ \ \mathrm{m/s^2}$$

For a function of two (or more) variables, we calculate the partial derivatives with respect to each variable, and then add each term in quadrature in order to calculate the absolute uncertainty of our function.

Apply Equation 3-6 to a(x,t)

$$\delta a(x,t) = \sqrt{\left[\frac{\partial a}{\partial x}\delta x\right]^2 + \left[\frac{\partial a}{\partial t}\delta t\right]^2} = \sqrt{(\delta a_x)^2 + (\delta a_t)^2}$$

Take the partial derivatives with respect to x and t

$$\delta a_x = \frac{\partial(-\frac{2x}{t^2})}{\partial x}\delta x = \left|\frac{2}{t^2}\right|\delta x \quad and \qquad \delta a_t = \frac{\partial(-\frac{2x}{t^2})}{\partial t}\delta t = \left|-\frac{4x}{t^3}\right|\delta t$$

Substitute values into the variables

$$\delta a_x = \left|\frac{2}{(0.103\ s)^2}\right|(0.0002\ m) \quad and \qquad \delta a_t = \left|-\frac{4(0.0532\ m)}{(0.0103\ s)^3}\right|(0.001\ s)$$

Evaluate the terms

$$\delta a_x = \ 0.03770\ m/s^2 \quad and \qquad \delta a_t = \ 0.195\ m/s^2$$

Substitute the term values into the quadrature equation

$$\delta a(x,t) = \sqrt{(0.03770)^2 + (0.195)^2} = 0.199\,\frac{m}{s^2} = \ 0.2\ m/s^2$$

Combine the uncertainty and the calculated acceleration for a confidence interval

$\underline{a = (10.0 \pm 0.2)\ \mathrm{m/s^2}}$

Example #3: Calculate the density of a sphere of mass (6.90 ± 0.05) g and diameter (7.35 ± 0.03) cm and report the density as a confidence interval.

Given: Mass $m = 6.90 \times 10^{-3}$ kg with uncertainty $\delta m = 0.05 \times 10^{-3}$ kg and diameter $d = 7.35 \times 10^{-2}$ m with uncertainty $\delta d = 0.03 \times 10^{-2}$ m.

Task: Density ρ as a confidence interval.

Physics: Definition of density: $\rho = \frac{m}{V}$; volume of a sphere: $V = \frac{4}{3}\pi r^3$; relationship between diameter and radius: $d = 2r$; method of partial derivatives

$$\delta f = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\delta x_i\right)^2}.$$

Solve:

$$\rho = \frac{m}{V}$$

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

Substitute volume as a function of diameter into the density equation

$$\Rightarrow V = \frac{1}{6}\pi d^3$$

$$\Rightarrow \rho = \frac{6m}{\pi d^3}$$

Substitute values into the density function

$$\Rightarrow \rho = \frac{6(0.00690 \text{ kg})}{\pi (0.0735 \text{ m})^3} = 33.18856 \text{ kg/m}^3$$

Isolate the two variables for the density function for quadrature

$$\delta\rho = \sqrt{\left(\frac{\partial\rho}{\partial m}\delta m\right)^2 + \left(\frac{\partial\rho}{\partial d}\delta d\right)^2}$$

Take the respective partial derivatives

$$\frac{\partial\rho}{\partial m} = \frac{\partial}{\partial m}\left(\frac{6m}{\pi d^3}\right) \qquad \frac{\partial\rho}{\partial d} = \frac{\partial}{\partial d}\left(\frac{6md^{-3}}{\pi}\right)$$

$$\Rightarrow \frac{\partial\rho}{\partial m} = \frac{6}{\pi d^3} \quad \text{and} \quad \Rightarrow \frac{\partial\rho}{\partial d} = \frac{-18d^{-4}}{\pi}$$

Substitute the algebraic the partial derivatives and values and propagate the error

$$\delta\rho = \sqrt{\left(\frac{6}{\pi d^3}\delta m\right)^2 + \left(\frac{-18m}{\pi d^4}\delta d\right)^2}$$

$$\Rightarrow \delta\rho = \sqrt{\left(\frac{6}{\pi\left(0.0735\ \mathrm{m}\right)^3}\left(0.00005\ \mathrm{kg}\right)\right)^2 + \left(\frac{-18\left(0.00690\ \mathrm{kg}\right)}{\pi\left(0.0735\ \mathrm{m}\right)^4}\left(0.0003\ \mathrm{m}\right)\right)^2}$$

$$\Rightarrow \delta\rho = 0.47222\ \ \mathrm{kg/m^3}$$

Rounding the uncertainty to one significant figure$\delta\rho$, we get $\delta\rho = 0.5$ kg/m^3. Round ρ to the same decimal place (tenths) and get a density of 33.2 kg/m^3. Reporting the density as a confidence interval, we get:

$$\underline{\rho = (33.2 \pm 0.5)\ \mathrm{kg/m^3}}_{\text{Answer}}$$

Appendix H

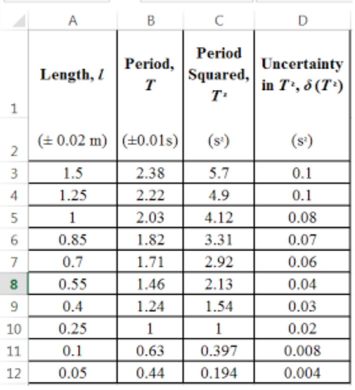
Graphing, Linear Regression, and Curve Fitting

A. Creating a graph using *Excel*

Many experiments will involve measuring one quantity (a dependent variable) while varying another quantity (the independent variable). *Excel* can perform a graphical and numerical analysis of the relationship between these variables.

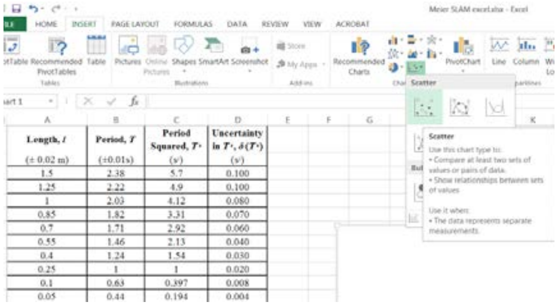
Follow these instructions with your own computer and this data to gain familiarity with *Excel*. Consider a very basic set of data in Table 3-7, entered into an *Excel* spreadsheet as shown in Fig. H-1.

The independent variable is length, *l*, and is measured in meters (m), and the dependent variable is period, *T*, measured in seconds. In order to display the relationship (or trend) between these two variables, create a graph of period vs. length.



	A	B	C	D
1	**Length, *l***	**Period, *T***	**Period Squared, T^2**	**Uncertainty in T^2, $\delta(T^2)$**
2	(± 0.02 m)	(±0.01s)	(s^2)	(s^2)
3	1.5	2.38	5.7	0.1
4	1.25	2.22	4.9	0.1
5	1	2.03	4.12	0.08
6	0.85	1.82	3.31	0.07
7	0.7	1.71	2.92	0.06
8	0.55	1.46	2.13	0.04
9	0.4	1.24	1.54	0.03
10	0.25	1	1	0.02
11	0.1	0.63	0.397	0.008
12	0.05	0.44	0.194	0.004

Figure H-1. Sample *Excel* Data



A	B	C	D
Length, *l*	**Period, *T***	**Period Squared, T^2**	**Uncertainty in T^2, $\delta(T^2)$**
(± 0.02 m)	(±0.01s)	(s^2)	(s^2)
1.5	2.38	5.7	0.100
1.25	2.22	4.9	0.100
1	2.03	4.12	0.080
0.85	1.82	3.31	0.070
0.7	1.71	2.92	0.060
0.55	1.46	2.13	0.040
0.4	1.24	1.54	0.030
0.25	1	1	0.020
0.1	0.63	0.397	0.008
0.05	0.44	0.194	0.004

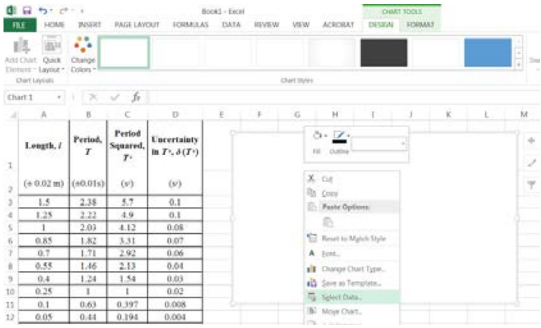
Scatter
Use this chart type to:
- Compare at least two sets of values or pairs of data.
- Show relationships between sets of values.

Use it when:
- The data represents separate measurements.

Figure H-2. Choosing an XY Scatter Plot

Insert a new chart by clicking the "Insert" tab on the *Excel* ribbon, clicking on "Insert Scatter or Bubble Chart," and choosing "Scatter" (the upper left-hand choice) as in Figure H-2. This procedure will cause a blank chart area to appear.

The second step is to select the data that will be used as the source for the chart. Right click anywhere on the chart area and highlight "Select Data." Click on "Select Data" as in Fig. H-3.

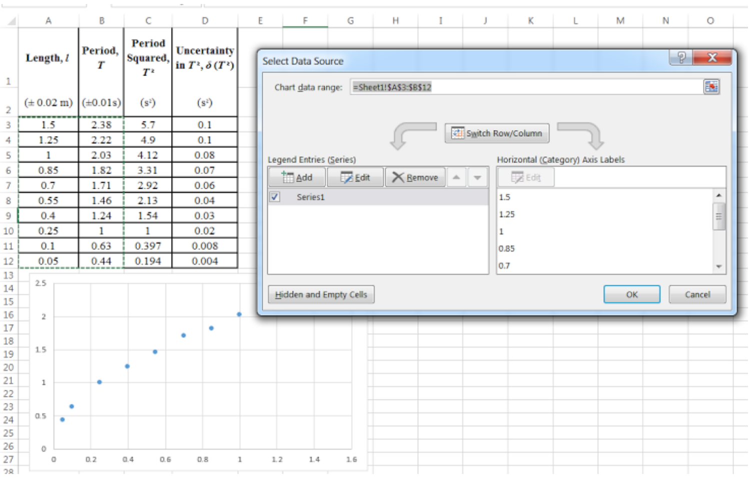


Length, l	Period, T	Period Squared, T^2	Uncertainty in T^2, $\delta(T^2)$
(± 0.02 m)	(±0.01s)	(s^2)	(s^2)
1.5	2.38	5.7	0.1
1.25	2.22	4.9	0.1
1	2.03	4.12	0.08
0.85	1.82	3.31	0.07
0.7	1.71	2.92	0.06
0.55	1.46	2.13	0.04
0.4	1.24	1.54	0.03
0.25	1	1	0.02
0.1	0.63	0.397	0.008
0.05	0.44	0.194	0.004

Figure H-3. Defining the Plot's Data.

Click the "Add" button and then type in a name for the "Series Name" ("Data" may work). Then click into the "Series X Values." Highlight the range of your *x* values in your *Excel* table. Click into "Series Y Values." Highlight the range of your *y* values in your *Excel* table. Click "OK" twice. If your data is plotted with a solid line connecting each data point, right click anywhere in the chart area and click on "Change Chart Type." Highlight the "Scatter" option and click. This should eliminate the solid line. See Fig H-4.

If you wish to have the chart on its own sheet, you can move it to its own sheet by right clicking on the chart's edge and selecting "Move Chart," clicking the "New Sheet" button, and clicking "OK."



Length, l	Period, T	Period Squared, T^2	Uncertainty in T^2, $\delta(T^2)$
(± 0.02 m)	(±0.01s)	(s^2)	(s^2)
1.5	2.38	5.7	0.1
1.25	2.22	4.9	0.1
1	2.03	4.12	0.08
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0.7	1.71	2.92	0.06
0.55	1.46	2.13	0.04
0.4	1.24	1.54	0.03
0.25	1	1	0.02
0.1	0.63	0.397	0.008
0.05	0.44	0.194	0.004

Figure H-4. Plotting the Data.

You must give titles to both of the axes and the graph (see Fig. H-5). The axis labels consist of the measured quantity and its units. The title takes a little thought. While the chart is a graphical representation of the dependent variable (*y*-axis) versus the independent variable (*x*-axis), this should not be the title of your graph! The title should show some level of insight into the result that you will obtain from the chart. If *Excel* hasn't put in a placeholder title on your chart, click on "Add Chart Element" on the *Excel* ribbon within the "Design" tab for the chart, click on "Chart Title" and choose on "Above Chart." You can now type your title. If *Excel* has put in a placeholder title on your chart, you can click on it and type in your title. To title your axes, click on "Axes Titles" and highlight "Primary Horizontal Axis." Then click on "Title Below Axis" and type your title, including units and uncertainty. Title your vertical axis by click on "Axes Titles" and highlight "Primary Vertical Axis" and then click on "Rotated Title." Type your title, including units and uncertainty.

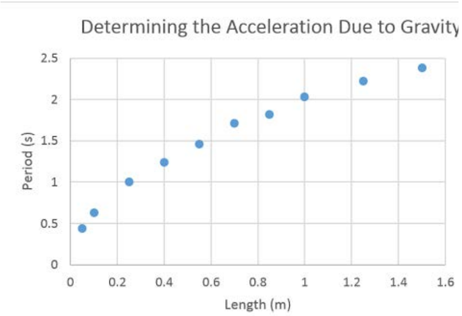
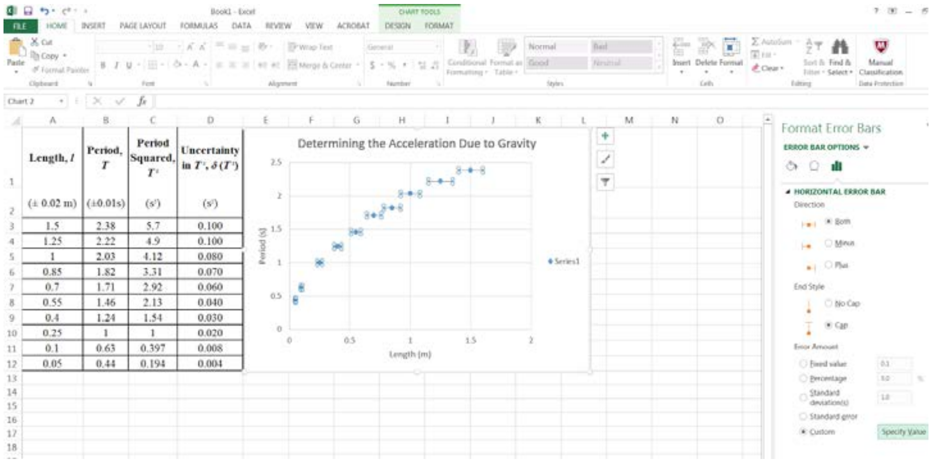


Figure H-5. Chart Title and Named Axis

Your instructor may require you to put error bars on your data points. If your data points have different uncertainties, you must create a column with the uncertainty values. If neither of the quantities have the same uncertainty, you will have to create a column for the uncertainty values shown in column D of Fig. H-1. To create error bars on your chart, select your chart and go to the *Excel* ribbon, choose “Design,” select “Add Chart Element.” and click on “Error Bars.” Choose “Error Bars with Standard Error.” Right-click on one of the horizontal error bars and choose “Format Error Bars.” If each of your independent values has the same uncertainty, choose “Fixed Value” and type in the appropriate value in the box. If each of your independent values has a different uncertainty, choose “Custom,” click on the “Specify Value” button, choose the appropriate cells for both positive and negative error bars, and click “OK.” Click “Close” to finish your horizontal error bars. Right click on a vertical error bar and choose “Format Error Bars.” Repeat the process for your dependent variable.

After completing these tasks, your graph should look similar to Fig. H-6. You can make any modifications to the chart by clicking on the chart area and using the “Chart Tools” of “Design,” “Layout,” or “Format.” Refer to the Microsoft Excel User’s Guide for further guidance on formatting a chart. You may find that it is easier to save the chart as an object within the current sheet rather than as its own sheet so that you can view the tabular and graphical representations of your data at the same time. If you create your chart in this manner and wish to just print out the chart, select the chart and then print.



Length, l	Period, T	Period Squared, T^2	Uncertainty in T^2, $\delta(T^2)$
(± 0.02 m)	(±0.01s)	(s^2)	(s^2)
1.5	2.38	5.7	0.100
1.25	2.22	4.9	0.100
1	2.03	4.12	0.080
0.85	1.82	3.31	0.070
0.7	1.71	2.92	0.060
0.55	1.46	2.13	0.040
0.4	1.24	1.54	0.030
0.25	1	1	0.020
0.1	0.63	0.397	0.008
0.05	0.44	0.194	0.004

Format Error Bars
ERROR BAR OPTIONS
HORIZONTAL ERROR BAR
Direction
● Both
○ Minus
○ Plus
End Style
○ No Cap
● Cap
Error Amount
○ Fixed value 0.1
○ Percentage 5.0 %
○ Standard deviation(s) 1.0
○ Standard error
● Custom Specify Value

Figure H-6. Custom Error Bars Chart

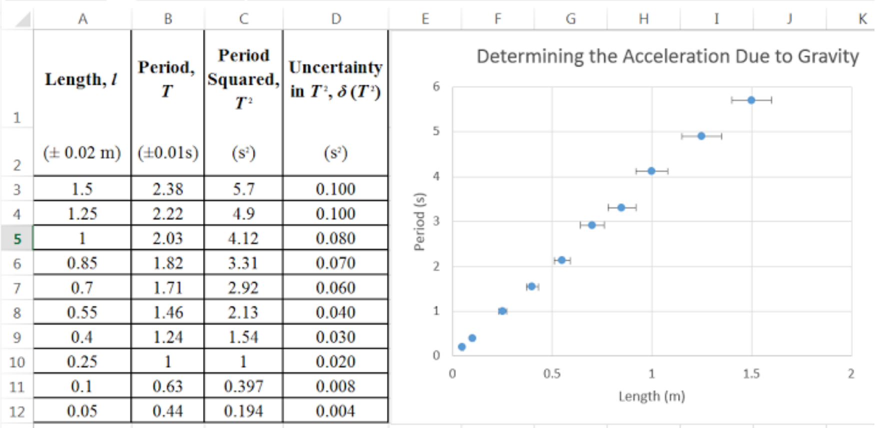
B. Linear Regression Using *Excel*

1. Curve Fitting. The most common type of relationship you will be expected to graph is a linear relationships. The process of fitting a curve, in this case a line, to your experimental data is called linear regression. The method of linear regression (also known as least squares fit to a line) determines the line for which the sum of the distance from the data points to the line, when added in quadrature, is minimized. This is called the best-fit line. The equation for a straight line can be generalized as:

$$y = mx + b \tag{H-1}$$

where m is the slope of the line and b is the intercept on the y-axis.

2. Creating a Trendline. The previous section covered the basic information needed to obtain a graphical representation of data. All of this information is important and forms the basis for the quantitative analysis of data. This section will cover the implementation of linear regression in *Excel*. It is important to note that while there are many types of regression fits, most of which are available through this option, the one you will most commonly use at this stage in your experimental development is **linear regression**. Therefore, since you have already created a chart of your data, you need to perform a sanity check and ask, “Is this data linear?” In the case of the example given in Figure H-5, this is not true. You should not attempt to perform a *linear* regression on a graph that is *not* linear. *For the purposes of this manual*, we can create a linear graph (see Figure H-7) by squaring the independent variable resulting in the following data and chart (if your graph is not linear, you must review the theory and equations governing the relationship and determine what the independent quantity must be to make the relationship linear).



Length, l (± 0.02 m)	Period, T (±0.01s)	Period Squared, T^2 (s^2)	Uncertainty in T^2, $\delta(T^2)$ (s^2)
1.5	2.38	5.7	0.100
1.25	2.22	4.9	0.100
1	2.03	4.12	0.080
0.85	1.82	3.31	0.070
0.7	1.71	2.92	0.060
0.55	1.46	2.13	0.040
0.4	1.24	1.54	0.030
0.25	1	1	0.020
0.1	0.63	0.397	0.008
0.05	0.44	0.194	0.004

Figure H-7. Linear Relationship Chart

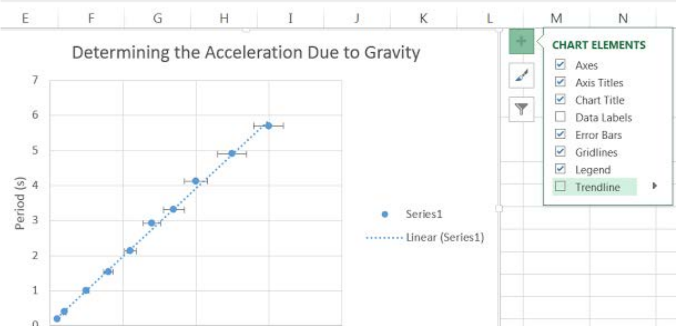
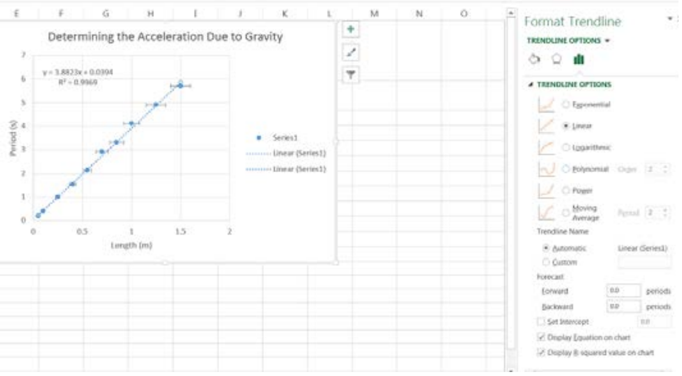


Figure H-8. Adding a Trendline

Data Regression is accessed using the trendline. This tool is found by returning to the chart and right-clicking on one of the data points (this action should highlight all of the data points). The right-click will reveal the menu shown in Fig. H-8. Within this menu, highlight the option “**Add Trendline**” Note that there is more than one type of Trend/Regression analysis. Use the "**L****inear**" regression type. Do not choose any of the other types that may be listed in that particular sub-menu. Ensure the "**Linear**" type is selected and click the buttons for "**Display Equation on Chart**" and "**Display R-Squared Value on Chart**" as shown in Fig. H-9.

Once completed, your graph should look like Figure H-10.



TRENDLINE OPTIONS

○ Exponential
● Linear
○ Logarithmic
○ Polynomial Order 2
○ Power
○ Moving Average Period 2

Trendline Name
● Automatic Linear (Series1)
○ Custom

Forecast
Forward 0.0 periods
Backward 0.0 periods
☐ Set Intercept 0.0
☑ Display Equation on chart
☑ Display R-squared value on chart

Figure H-9. Format Trendline Menu

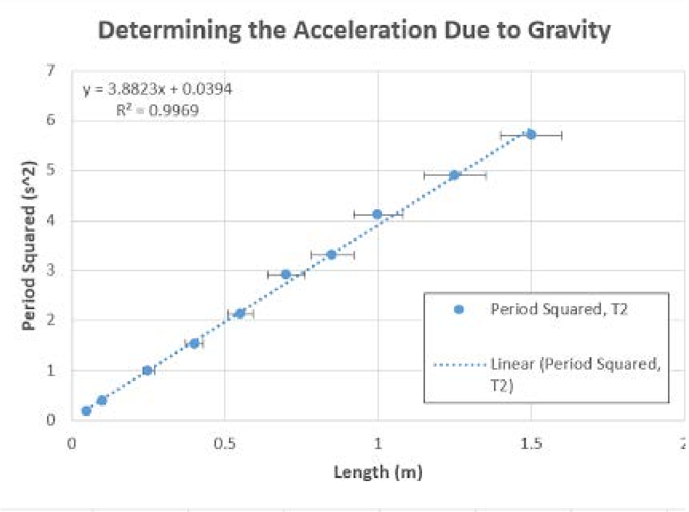


Figure H-10. Graph of Experimental Data

3. Using the Data Analysis Function. In order to obtain more information about the regression, such as the uncertainty in the slope of the best fit line, you will use the Data Analysis Function. Since this is probably the first time you have been exposed to this function of *Excel*, you need to ensure that it is loaded and operational within the program. The Regression program resides in the Data button of the *Excel* ribbon. If "Data Analysis" is present on this tab (see Figure H-11), then your computer is ready to perform linear regression.

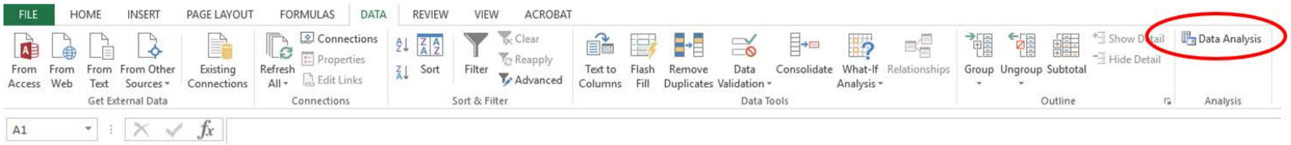


Figure H-11. Data Tab of *Excel*

If this option is missing, you need to add this option to *Excel*. This addition is accomplished by clicking on the File button on the *Excel* ribbon. Select “Options” and click on the Add-Ins tab. Finally you will select “Excel Add-ins” from the drop down menu and click “Go…” (see Figure H-12). This option will open the window listing the options that are present (see Figure H-13) but not loaded in the *Excel* program. Click the box next to “Analysis Tool Pak.” Click “OK” (see Figure H-13).

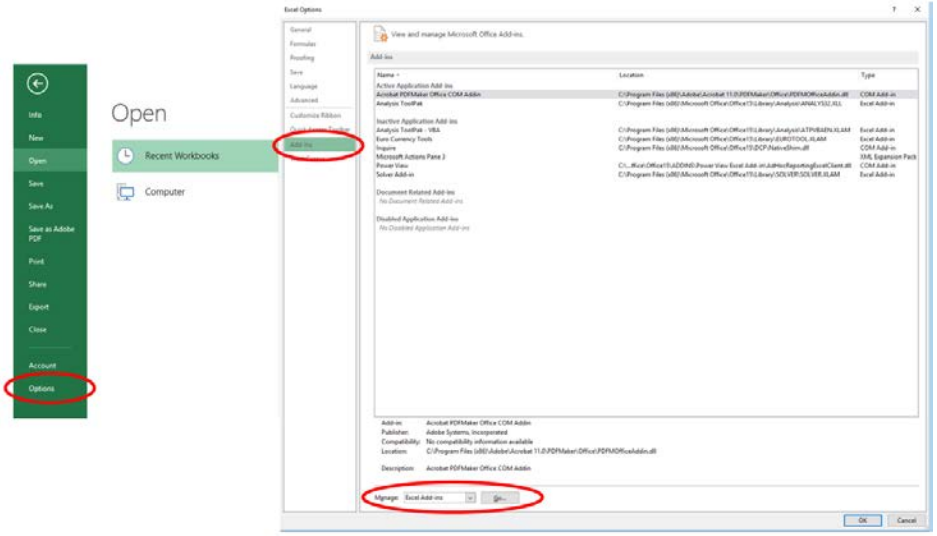
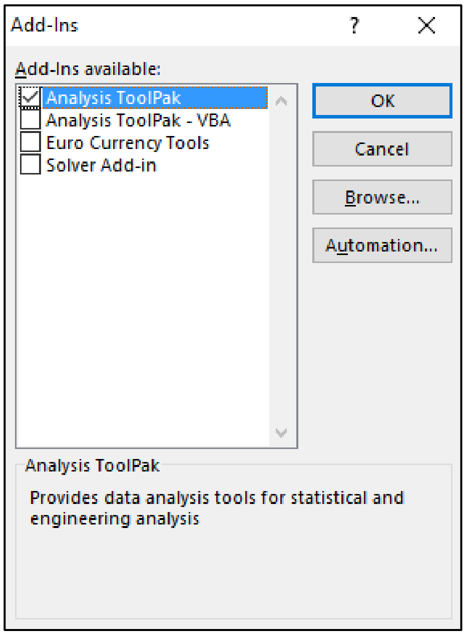


Figure H-12. Adding *Excel* Options

The Analysis Toolpak is the first one in the list and is the only one by which you need to place a check in the box. Be advised there may be other boxes that already have checks; this window does not show that situation. Do not “uncheck” any boxes that the program has already marked for installation during the initial set up of the program.



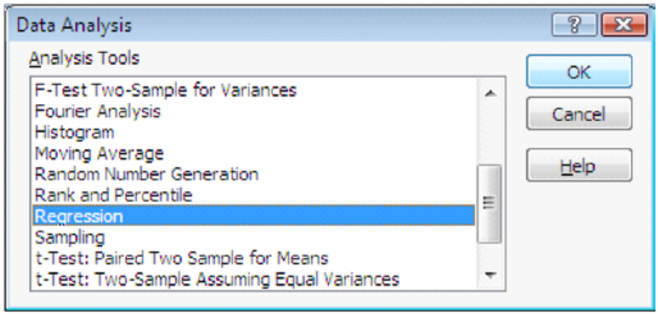
☑ Analysis ToolPak
☐ Analysis ToolPak - VBA
☐ Euro Currency Tools
☐ Solver Add-in

Analysis ToolPak

Provides data analysis tools for statistical and engineering analysis

Figure H-13. Selecting the Analysis ToolPak

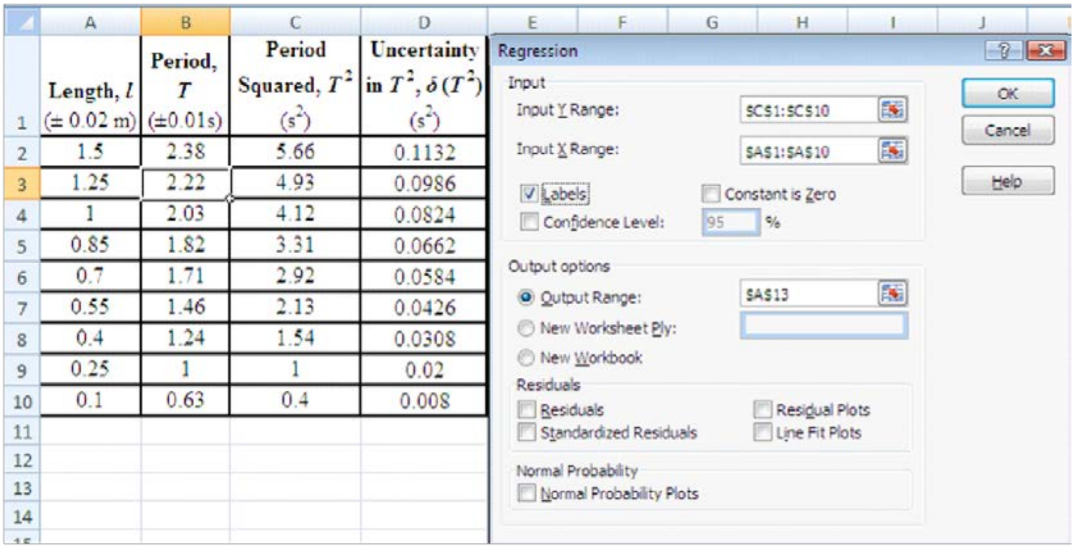
Once the ToolPak has been loaded, click “Data” on the *Excel* ribbon. Click “Data Analysis.” The data analysis toolbox should appear. Scroll down and highlight “Regression.” Click “OK.”
(See Figure H-14)



F-Test Two-Sample for Variances
Fourier Analysis
Histogram
Moving Average
Random Number Generation
Rank and Percentile
Regression
Sampling
t-Test: Paired Two Sample for Means
t-Test: Two-Sample Assuming Equal Variances

Figure H-14. Regression Options for Data Analysis

Insert your *x* and *y* values by highlighting each column just like above when you entered your data to plot (Be careful as the Input Y Range is listed first). For the summary output to appear on the same worksheet as the data (recommended), click the **"Output Range"** button and then click on the upper left-hand cell where the output is to be placed. For the program to create a new worksheet with the output, leave the **"New Worksheet Ply"** button checked. Leaving the box blank will create a worksheet named "Sheet *N*+1" where *N* is the number of sheets currently in your Excel workbook (See Figure H-15).



	A	B	C	D
1	Length, l (± 0.02 m)	Period, T (±0.01s)	Period Squared, T^2 (s^2)	Uncertainty in T^2, $\delta(T^2)$ (s^2)
2	1.5	2.38	5.66	0.1132
3	1.25	2.22	4.93	0.0986
4	1	2.03	4.12	0.0824
5	0.85	1.82	3.31	0.0662
6	0.7	1.71	2.92	0.0584
7	0.55	1.46	2.13	0.0426
8	0.4	1.24	1.54	0.0308
9	0.25	1	1	0.02
10	0.1	0.63	0.4	0.008

Regression

Input

Input Y Range: $C$1:$C$10

Input X Range: $A$1:$A$10

☑ Labels
☐ Constant is Zero
☐ Confidence Level: 95 %

Output options

● Output Range: $A$13
○ New Worksheet Ply:
○ New Workbook

Residuals

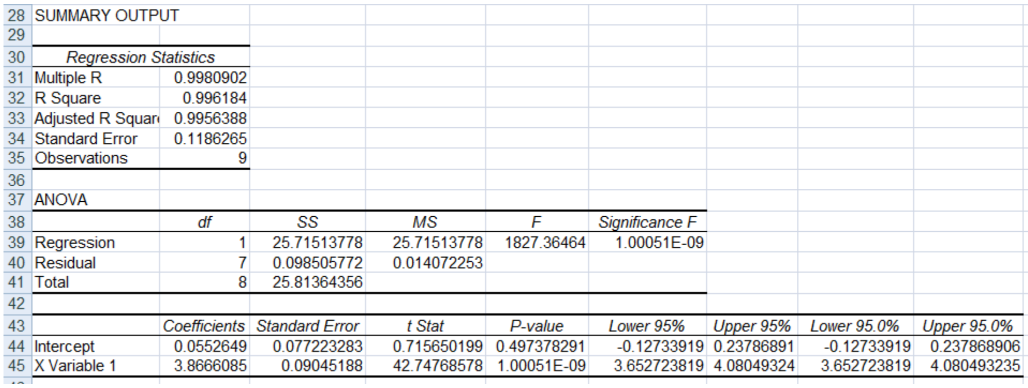
☐ Residuals
☐ Residual Plots
☐ Standardized Residuals
☐ Line Fit Plots

Normal Probability

☐ Normal Probability Plots

Figure H-15. Regression Menu and Data Selection

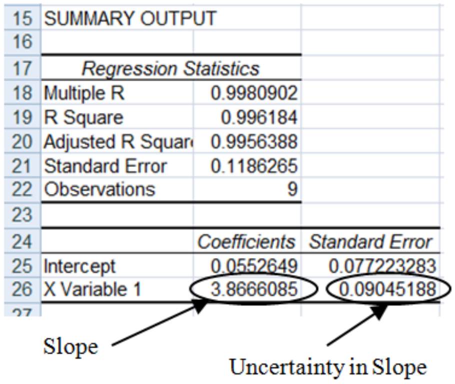
Leave all other boxes unselected and click "**OK**." The summary output should appear where you selected it to go and look like Figure H-16.



28	SUMMARY OUTPUT								
29									
30	*Regression Statistics*								
31	Multiple R	0.9980902							
32	R Square	0.996184							
33	Adjusted R Squar	0.9956388							
34	Standard Error	0.1186265							
35	Observations	9							
36									
37	ANOVA								
38		*df*	*SS*	*MS*	*F*	*Significance F*			
39	Regression	1	25.71513778	25.71513778	1827.36464	1.00051E-09			
40	Residual	7	0.098505772	0.014072253					
41	Total	8	25.81364356						
42									
43		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
44	Intercept	0.0552649	0.077223283	0.715650199	0.497378291	-0.12733919	0.23786891	-0.12733919	0.237868906
45	X Variable 1	3.8666085	0.09045188	42.74768578	1.00051E-09	3.652723819	4.08049324	3.652723819	4.080493235

Figure H-16. Summary output from *Excel* linear regression

There is an abundance of information on this worksheet, on which only a portion is applicable to PH205 and PH206. Consequently, for all regression output, you must delete the lines for ANOVA and delete all the columns to the right of *Standard Error*. "Dress up" your output so that it looks similar to Figure H-17. This information can now be combined with the final graph and data for presentation. However, while this information is needed, it is not yet merged with your previous results. This combination of qualitative and quantitative representations is the reason for going further than a graphical analysis of the data. The Summary Output shown here includes statistical data on the graph itself to include numerical values for the slope and *y*-intercept of the graph. As you know, measurements made in laboratory are not exact and are therefore subject to a degree of uncertainty. Linear Regression takes this into account and through the Summary Output presents the standard error of the slope and *y*-intercept as well! This information is used to report these values as confidence intervals.

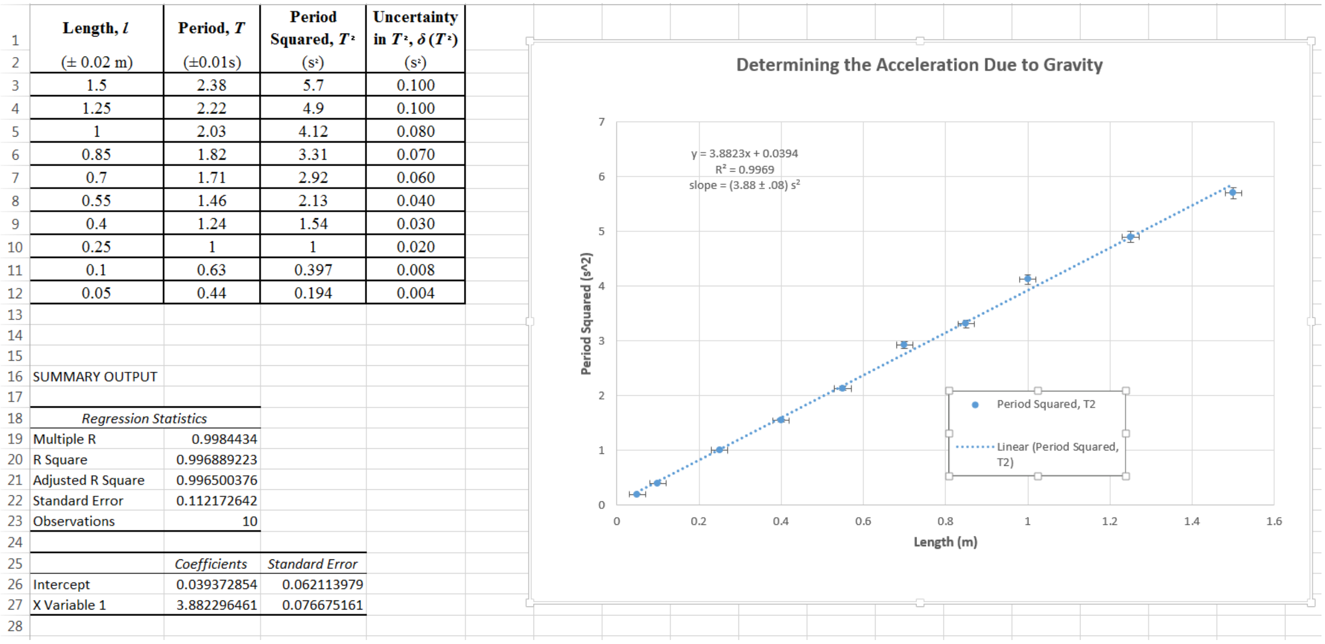


15	SUMMARY OUTPUT		
16			
17	*Regression Statistics*		
18	Multiple R	0.9980902	
19	R Square	0.996184	
20	Adjusted R Squar	0.9956388	
21	Standard Error	0.1186265	
22	Observations	9	
23			
24		*Coefficients*	*Standard Error*
25	Intercept	0.0552649	0.077223283
26	X Variable 1	3.8666085	0.09045188

Figure H-17. Summary Output Modified to PH205/206 Standards

Since this output provides us with the confidence interval for out slope, this value and absolute uncertainty is often used to determine the objective quantity in experiments (see Linearization in Appendix I) and used in the propagation of error in that calculation as covered in Chapter 3, Section B.

The final step is to update your graph by **adding the confidence interval for slope** below your R^2 value. Ensure to include any units associated with your slope. All of the information from tabular, graphical and numerical analysis of data can be combined together in a clear and concise format as shown in Figure H-18. This information now becomes an invaluable tool for you to present your data and results and support your conclusions.



1	**Length, *l***	**Period, *T***	**Period Squared, T^2**	**Uncertainty in T^2, $\delta(T^2)$**
2	(± 0.02 m)	(±0.01s)	(s^2)	(s^2)
3	1.5	2.38	5.7	0.100
4	1.25	2.22	4.9	0.100
5	1	2.03	4.12	0.080
6	0.85	1.82	3.31	0.070
7	0.7	1.71	2.92	0.060
8	0.55	1.46	2.13	0.040
9	0.4	1.24	1.54	0.030
10	0.25	1	1	0.020
11	0.1	0.63	0.397	0.008
12	0.05	0.44	0.194	0.004
13				
14				
15				
16	SUMMARY OUTPUT			
17				
18	*Regression Statistics*			
19	Multiple R	0.9984434		
20	R Square	0.996889223		
21	Adjusted R Square	0.996500376		
22	Standard Error	0.112172642		
23	Observations	10		
24				
25		*Coefficients*	*Standard Error*	
26	Intercept	0.039372854	0.062113979	
27	X Variable 1	3.882296461	0.076675161	
28				

Figure H-18. Final Graph with Slope and Uncertainty for Submission

C. Curve Fitting and a Least Squares Fit.

A common method used to analytically find the best curve fit for a series of experimental points is the *least-squares fit*. A least-squares fit is used to create a mathematical model for a combination of two or more measurements, which are known to be related by a theoretical concepts. The error minimized in the curve is the sum of the squared difference, X^2, between the mathematical model's prediction, $f(x)$ and the experimental results, $y(x)$, shown by Equation H-2.

$$X^2 = \sum(f(x) - y(x))^2 \qquad \text{(H-2)}$$

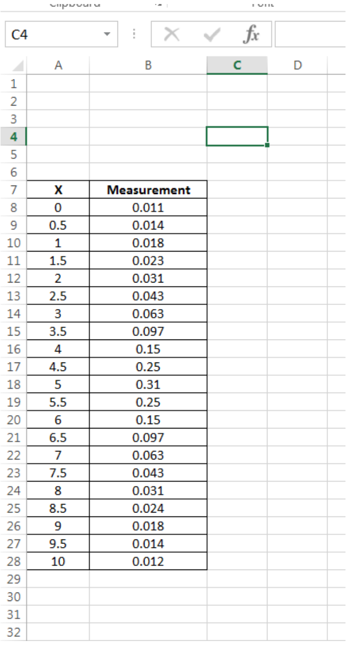
Any mathematical model can be applied utilizing this technique to find a best fit curve once the X^2 is minimized.

Lorentzian Model Example

In the example below, a step by step process is provided for how a Lorentzian curve is fit to a set of experimental data. The equation for a Lorentzian curve is:

$$f(x) = \frac{1}{\pi}\left(\frac{\left(\frac{1}{2}\right)k}{(x-x_0)^2 - \left(\left(\frac{1}{2}\right)k\right)^2}\right) \qquad \text{(H-3)}$$

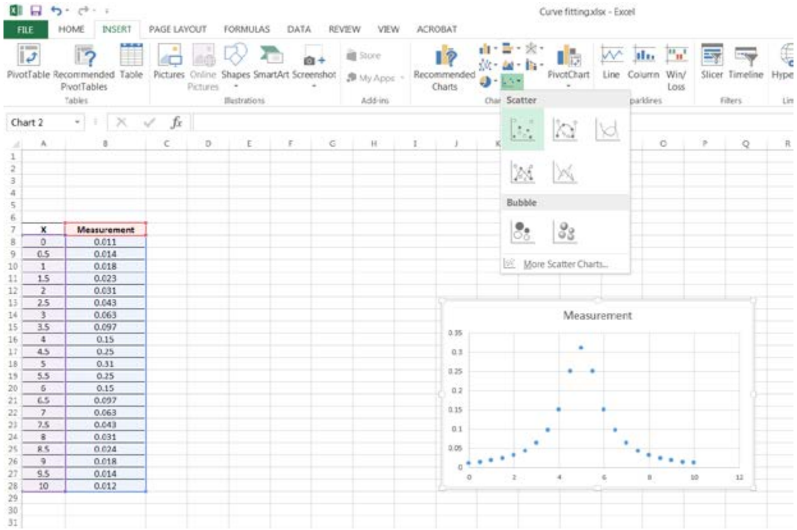
1. Input the experimental data set into the excel file.



X	Measurement
0	0.011
0.5	0.014
1	0.018
1.5	0.023
2	0.031
2.5	0.043
3	0.063
3.5	0.097
4	0.15
4.5	0.25
5	0.31
5.5	0.25
6	0.15
6.5	0.097
7	0.063
7.5	0.043
8	0.031
8.5	0.024
9	0.018
9.5	0.014
10	0.012

Figure H-19. Experimental Data Table Creation

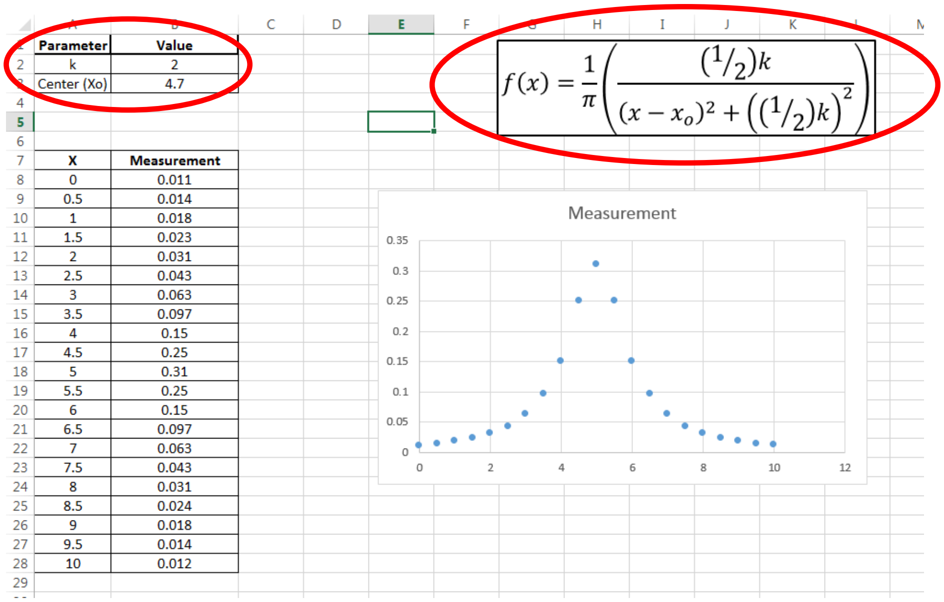
2. Select the Scatter Plot to graph the data.



X	Measurement
0	0.011
0.5	0.014
1	0.018
1.5	0.023
2	0.031
2.5	0.043
3	0.063
3.5	0.097
4	0.15
4.5	0.25
5	0.31
5.5	0.25
6	0.15
6.5	0.097
7	0.063
7.5	0.043
8	0.031
8.5	0.024
9	0.018
9.5	0.014
10	0.012

Figure H-20. Scatter Plot Creation

3. Select the model, Lorentzian Curve for this example, to fit to the data to, identify parameters, and insert an educated guess for each parameter.



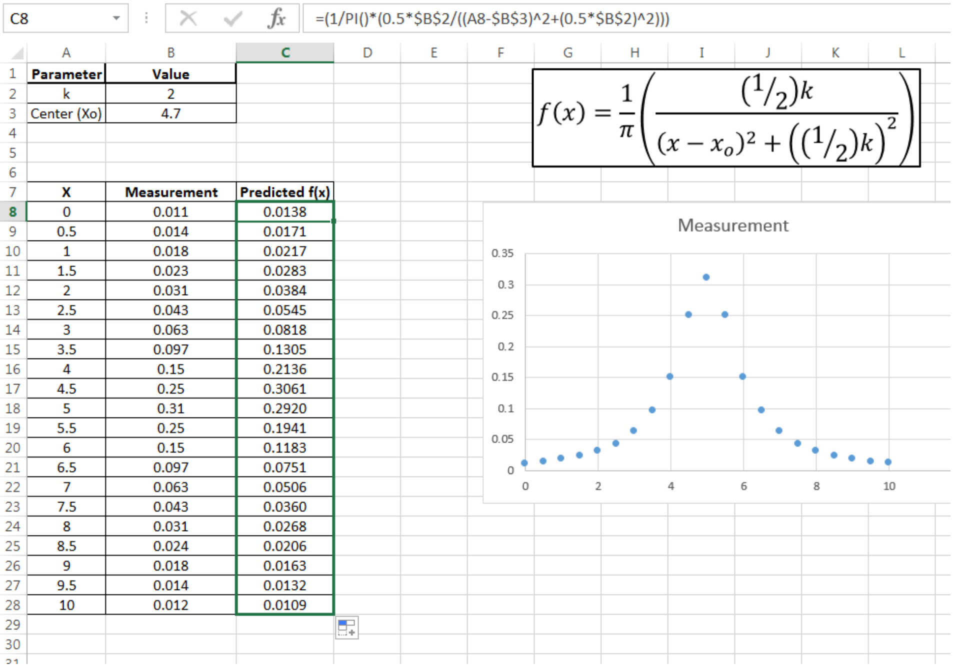
Parameter	Value
k	2
Center (Xo)	4.7

$$f(x) = \frac{1}{\pi}\left(\frac{\left(1/2\right)k}{(x - x_o)^2 + \left(\left(1/2\right)k\right)^2}\right)$$

X	Measurement
0	0.011
0.5	0.014
1	0.018
1.5	0.023
2	0.031
2.5	0.043
3	0.063
3.5	0.097
4	0.15
4.5	0.25
5	0.31
5.5	0.25
6	0.15
6.5	0.097
7	0.063
7.5	0.043
8	0.031
8.5	0.024
9	0.018
9.5	0.014
10	0.012

Figure H-21. Building Model Equation into Spreadsheet

4. Build an additional column where the independent variable(s), X in this example, is the input for the model to predict the dependent variable.



C8 =(1/PI()*(0.5*$B$2/((A8-$B$3)^2+(0.5*$B$2)^2)))

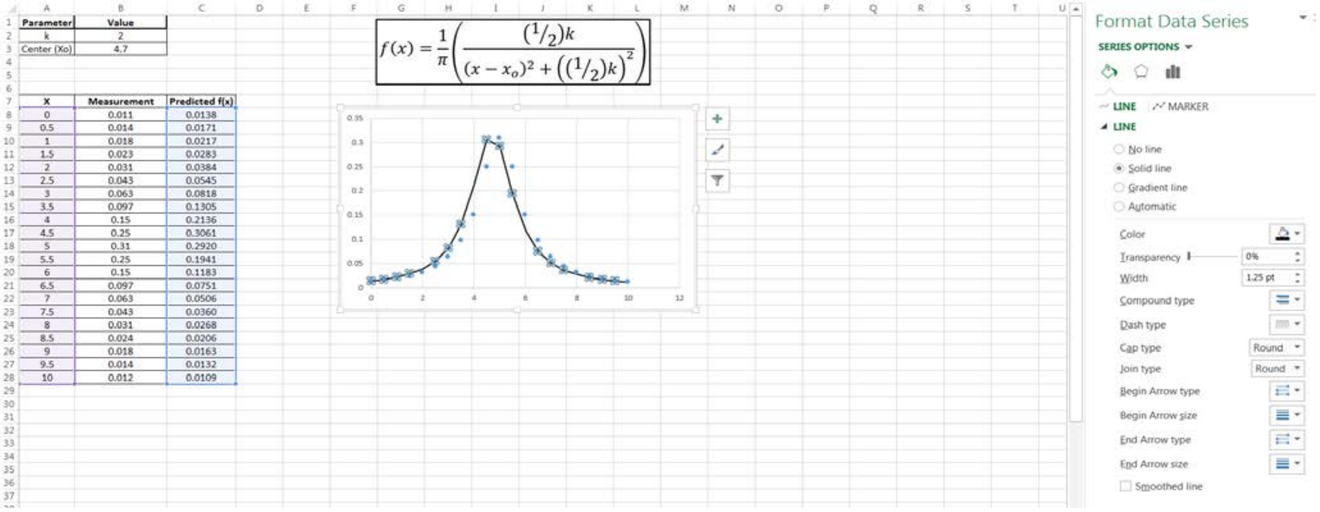
$$f(x) = \frac{1}{\pi}\left(\frac{(1/2)k}{(x-x_o)^2 + \left((1/2)k\right)^2}\right)$$

Parameter	Value
k	2
Center (Xo)	4.7

X	Measurement	Predicted f(x)
0	0.011	0.0138
0.5	0.014	0.0171
1	0.018	0.0217
1.5	0.023	0.0283
2	0.031	0.0384
2.5	0.043	0.0545
3	0.063	0.0818
3.5	0.097	0.1305
4	0.15	0.2136
4.5	0.25	0.3061
5	0.31	0.2920
5.5	0.25	0.1941
6	0.15	0.1183
6.5	0.097	0.0751
7	0.063	0.0506
7.5	0.043	0.0360
8	0.031	0.0268
8.5	0.024	0.0206
9	0.018	0.0163
9.5	0.014	0.0132
10	0.012	0.0109

Figure H-22. Model Prediction Column

5. Plot the predicted values as a line in the scatter plot in order to build the model's trendline using the Select Data Tool. The predicted data points can be formatted into a trendline by removing the markers and formatting the data series to a solid line with the format data series tool.



$$f(x) = \frac{1}{\pi}\left(\frac{(1/2)k}{(x-x_o)^2 + \left((1/2)k\right)^2}\right)$$

Parameter	Value
k	2
Center (Xo)	4.7

X	Measurement	Predicted f(x)
0	0.011	0.0138
0.5	0.014	0.0171
1	0.018	0.0217
1.5	0.023	0.0283
2	0.031	0.0384
2.5	0.043	0.0545
3	0.063	0.0818
3.5	0.097	0.1305
4	0.15	0.2136
4.5	0.25	0.3061
5	0.31	0.2920
5.5	0.25	0.1941
6	0.15	0.1183
6.5	0.097	0.0751
7	0.063	0.0506
7.5	0.043	0.0360
8	0.031	0.0268
8.5	0.024	0.0206
9	0.018	0.0163
9.5	0.014	0.0132
10	0.012	0.0109

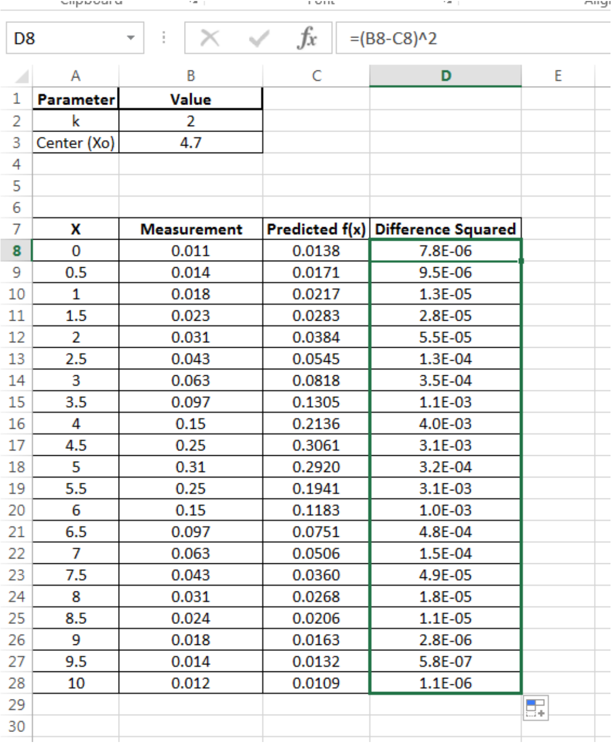
Format Data Series

○ No line
● Solid line
○ Gradient line
○ Automatic

Color
Transparency 0%
Width 1.25 pt
Cap type Round
Join type Round
☐ Smoothed line

Figure H-23. Model Trendline Creation

6. Construct a Difference Squared column to calculate the difference squared, where the difference between each experimental and predicted data is squared.

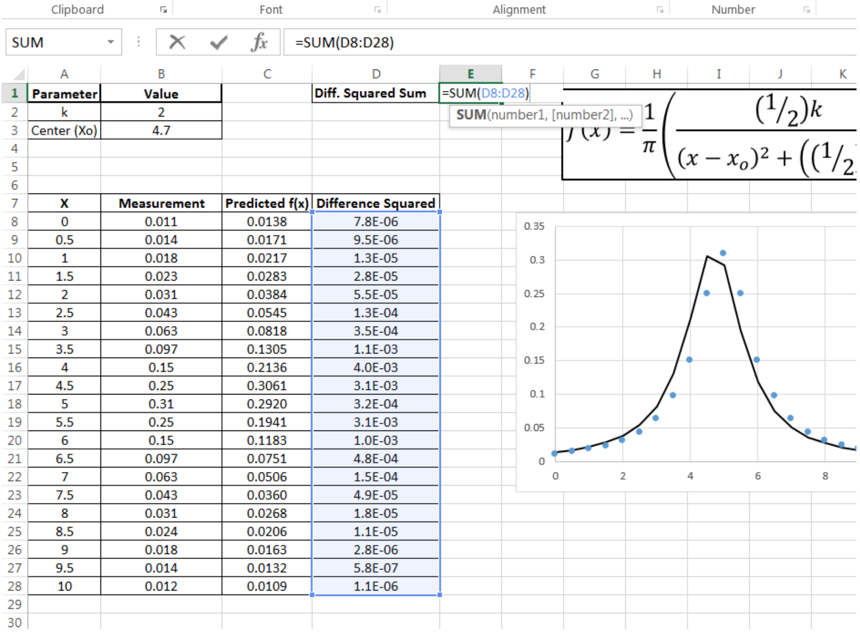


Parameter	Value
k	2
Center (Xo)	4.7

X	Measurement	Predicted f(x)	Difference Squared
0	0.011	0.0138	7.8E-06
0.5	0.014	0.0171	9.5E-06
1	0.018	0.0217	1.3E-05
1.5	0.023	0.0283	2.8E-05
2	0.031	0.0384	5.5E-05
2.5	0.043	0.0545	1.3E-04
3	0.063	0.0818	3.5E-04
3.5	0.097	0.1305	1.1E-03
4	0.15	0.2136	4.0E-03
4.5	0.25	0.3061	3.1E-03
5	0.31	0.2920	3.2E-04
5.5	0.25	0.1941	3.1E-03
6	0.15	0.1183	1.0E-03
6.5	0.097	0.0751	4.8E-04
7	0.063	0.0506	1.5E-04
7.5	0.043	0.0360	4.9E-05
8	0.031	0.0268	1.8E-05
8.5	0.024	0.0206	1.1E-05
9	0.018	0.0163	2.8E-06
9.5	0.014	0.0132	5.8E-07
10	0.012	0.0109	1.1E-06

Figure H-24. Difference Squared Column Creation

7. Sum the difference squared data to observe the variance between the experimental data and the model’s fit.

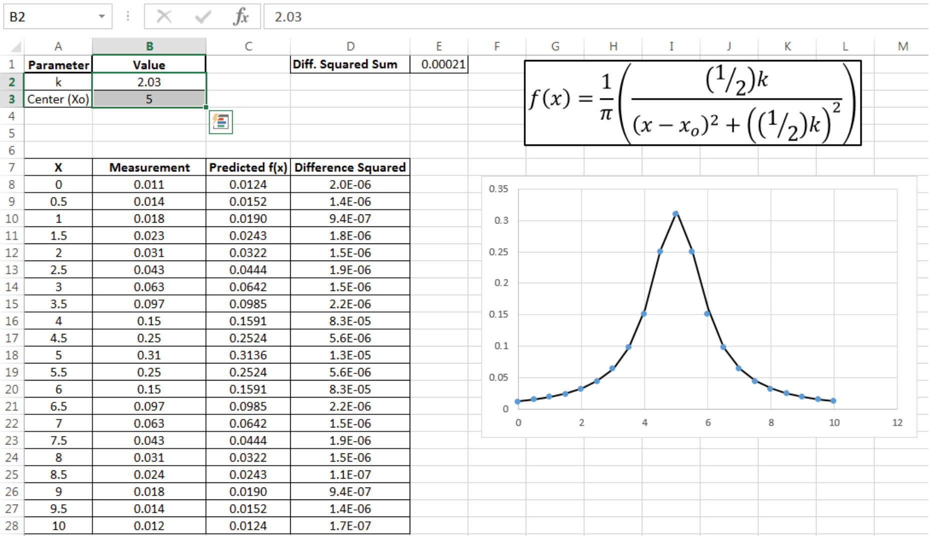


Parameter	Value		Diff. Squared Sum	=SUM(D8:D28)
k	2			
Center (Xo)	4.7			

X	Measurement	Predicted f(x)	Difference Squared
0	0.011	0.0138	7.8E-06
0.5	0.014	0.0171	9.5E-06
1	0.018	0.0217	1.3E-05
1.5	0.023	0.0283	2.8E-05
2	0.031	0.0384	5.5E-05
2.5	0.043	0.0545	1.3E-04
3	0.063	0.0818	3.5E-04
3.5	0.097	0.1305	1.1E-03
4	0.15	0.2136	4.0E-03
4.5	0.25	0.3061	3.1E-03
5	0.31	0.2920	3.2E-04
5.5	0.25	0.1941	3.1E-03
6	0.15	0.1183	1.0E-03
6.5	0.097	0.0751	4.8E-04
7	0.063	0.0506	1.5E-04
7.5	0.043	0.0360	4.9E-05
8	0.031	0.0268	1.8E-05
8.5	0.024	0.0206	1.1E-05
9	0.018	0.0163	2.8E-06
9.5	0.014	0.0132	5.8E-07
10	0.012	0.0109	1.1E-06

Figure H-25. Summation of Squared Differences

8. Roughly alter the parameters to get the trendline to match the experimental data. Do not fully optimize the trendline yet, only a rough adjustment.



Parameter	Value
k	2.03
Center (Xo)	5

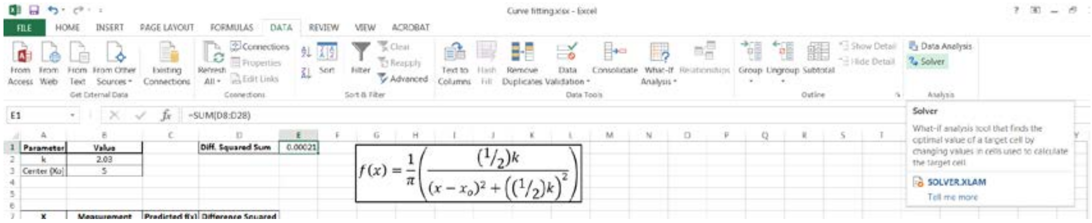
Diff. Squared Sum: 0.00021

$$f(x) = \frac{1}{\pi}\left(\frac{(1/2)k}{(x-x_o)^2+\left((1/2)k\right)^2}\right)$$

X	Measurement	Predicted f(x)	Difference Squared
0	0.011	0.0124	2.0E-06
0.5	0.014	0.0152	1.4E-06
1	0.018	0.0190	9.4E-07
1.5	0.023	0.0243	1.8E-06
2	0.031	0.0322	1.5E-06
2.5	0.043	0.0444	1.9E-06
3	0.063	0.0642	1.5E-06
3.5	0.097	0.0985	2.2E-06
4	0.15	0.1591	8.3E-05
4.5	0.25	0.2524	5.6E-06
5	0.31	0.3136	1.3E-05
5.5	0.25	0.2524	5.6E-06
6	0.15	0.1591	8.3E-05
6.5	0.097	0.0985	2.2E-06
7	0.063	0.0642	1.5E-06
7.5	0.043	0.0444	1.9E-06
8	0.031	0.0322	1.5E-06
8.5	0.024	0.0243	1.1E-07
9	0.018	0.0190	9.4E-07
9.5	0.014	0.0152	1.4E-06
10	0.012	0.0124	1.7E-07

Figure H-26. Rough Adjustment of Model Parameters

9. Fully optimize the trendline by using the Solver Tool in the Data Tab (see the following section for activating the Solver Tool in Excel). Select the Sum of the Differences Squared cell as the "set objective" to be minimized by changing the parameter cells. Figure H-27



=SUM(D8:D28)

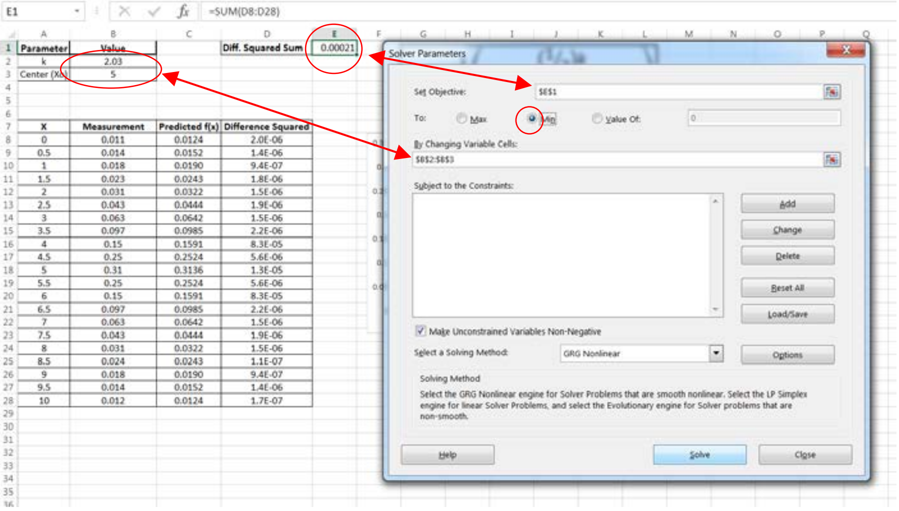
Solver

What-if analysis tool that finds the optimal value of a target cell by changing values in cells used to calculate the target cell

Parameter	Value
k	2.03
Center (Xo)	5

Diff. Squared Sum: 0.00021

Figure H-27. Solver Tool Selection



=SUM(D8:D28)

Parameter	Value
k	2.03
Center (Xo)	5

Diff. Squared Sum: 0.00021

X	Measurement	Predicted f(x)	Difference Squared
0	0.011	0.0124	2.0E-06
0.5	0.014	0.0152	1.4E-06
1	0.018	0.0190	9.4E-07
1.5	0.023	0.0243	1.8E-06
2	0.031	0.0322	1.5E-06
2.5	0.043	0.0444	1.9E-06
3	0.063	0.0642	1.5E-06
3.5	0.097	0.0985	2.2E-06
4	0.15	0.1591	8.3E-05
4.5	0.25	0.2524	5.6E-06
5	0.31	0.3136	1.3E-05
5.5	0.25	0.2524	5.6E-06
6	0.15	0.1591	8.3E-05
6.5	0.097	0.0985	2.2E-06
7	0.063	0.0642	1.5E-06
7.5	0.043	0.0444	1.9E-06
8	0.031	0.0322	1.5E-06
8.5	0.024	0.0243	1.1E-07
9	0.018	0.0190	9.4E-07
9.5	0.014	0.0152	1.4E-06
10	0.012	0.0124	1.7E-07

Set Objective: $E$1

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $B$2:$B$3

Subject to the Constraints:

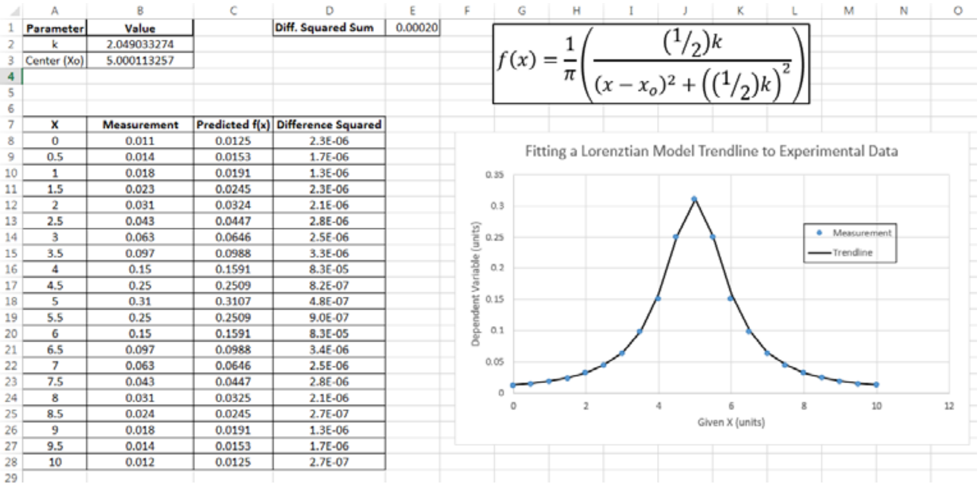
☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure H-28. Selecting Parameters and Objective



Parameter	Value
k	2.049033274
Center (Xo)	5.000113257

Diff. Squared Sum 0.00020

$$f(x) = \frac{1}{\pi}\left(\frac{(1/2)k}{(x-x_o)^2 + \left((1/2)k\right)^2}\right)$$

X	Measurement	Predicted f(x)	Difference Squared
0	0.011	0.0125	2.3E-06
0.5	0.014	0.0153	1.7E-06
1	0.018	0.0191	1.3E-06
1.5	0.023	0.0245	2.3E-06
2	0.031	0.0324	2.1E-06
2.5	0.043	0.0447	2.8E-06
3	0.063	0.0646	2.5E-06
3.5	0.097	0.0988	3.3E-06
4	0.15	0.1591	8.3E-05
4.5	0.25	0.2509	8.2E-07
5	0.31	0.3107	4.8E-07
5.5	0.25	0.2509	9.0E-07
6	0.15	0.1591	8.3E-05
6.5	0.097	0.0988	3.4E-06
7	0.063	0.0646	2.5E-06
7.5	0.043	0.0447	2.8E-06
8	0.031	0.0325	2.1E-06
8.5	0.024	0.0245	2.7E-07
9	0.018	0.0191	1.3E-06
9.5	0.014	0.0153	1.7E-06
10	0.012	0.0125	2.7E-07

Figure H-29. Finalized Model Fit Example

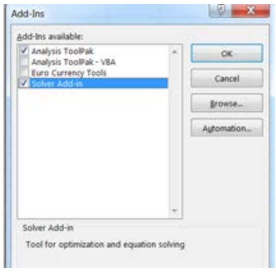
The Lorentzian model fit is now fully optimized with a Sum of the Differences Squared value of 0.00029. The above steps can be utilized for fitting any curve, or model, to a set of experimental data with properly adjusting the theoretical model, predicted values, and parameters.

Activating the Solver Function.

In order to optimize models, the Solver Function will be used. The Solver Function is usually not activated in most Excel programs. The following steps are to be followed in order to activate the add-in.

1. Select the File tab in Excel and select the Options menu, where a pop-up will be initiated (see Figure H-12). Select the Add-Ins menu and click the "Go…" button at the bottom of the pop-up in order to manage the Excel Add-ins.

2. Click the selection box to activate the Solver Add-in. Click the "OK" button finalize the activation of the Solver Function, and the Solver Function button should now be present in the Data tab.



Add-Ins available:

☑ Analysis ToolPak
☐ Analysis ToolPak - VBA
☐ Euro Currency Tools
☑ Solver Add-in

Solver Add-in

Tool for optimization and equation solving

Figure H-30. Excel Add-In Window

Appendix I

Linearization

A. Overview

Conducting an experiment in which a variable is adjusted with each trial allows an experimenter to gain additional information about the suitability of the model equation to describe the experimental setup as well as solve for an objective quantity.

Linearization is the process of taking a model equation and using algebra to put it in slope-intercept form, $y = (m)x + b$, where y is the dependent quantity (the physical parameter that the experimenter measures changes in), m is the slope of a graph to be generated, x is the independent quantity (the physical parameter that the experimenter alters), and b is the y-intercept of the graph to be generated. The slope and y-intercept terms will contain control variables (variables that do not change during the experiment).

B. Why Linearize?

1. Linearization simplifies and clarifies complex relationships between variables.

2. Linearization allows visualization of how the dependent and independent variables interact. The direction and steepness of the slope show how much changing one variable should effect the other. If the results fit the line tightly they support to the model equation and underlying theory.

3. Linearization allows for testing a wide variety of values for the independent quantity to see if the model equation holds under a host of different conditions.

4. Using linearization makes it possible to determine uncertainty in the objective quantity, and can also reduce the uncertainty in said quantity.

5. The y-intercept value can be used to validate the theory or help isolate systematic errors in measurement.

C. Linearization Process

1. Identify fundamental physics equations for the experiment.

2. Apply physics concepts and equations to build a model equation that describes the relationship between the independent and dependent variables.

3. Identify the independent and dependent variables in the model equation.

4. Use algebra to rearrange the model equation into slope-intercept form, $y = (m)x + b$. **This is the linearized model equation.**

5. Identify the independent and dependent quantities in the linearized model equation.

6. Identify the objective quantity and control variables.

7. Write an expression for the slope of your linearized model equation in terms of the control variables and the objective quantity (ie. *slope* = a combination of the objective quantity and control variables).

8. Rearrange the expression for the slope and solve for the objective quantity. **This is the objective quantity equation.**

9. Conduct the experiment. Collect data. Graph the dependent quantity vs. the independent quantity. Follow the steps outlined in Appendix H of the SLAM to format your graph, add a trendline and conduct linear regression to determine the slope and the uncertainty in the slope of the trendline.

10. Use the propagation of uncertainty equation on the objective quantity equation from step 8 to calculate the uncertainty of the objective quantity. The linear regression from step 9 provides the absolute uncertainty for the slope term.

D. Sample Problems

a. Example 1 – General

Task: Determine the parameter *A*, where during the experiment parameter *B* was varied and as a result measured changes were recorded in parameter *C*. All other parameters (*D* and *E*) were held constant during the experiment.

1. Identify fundamental physics equations for the experiment.
    a. Since this is a general example we do not have any principles or laws to apply.
2. Apply physics concepts and equations to build a model equation that describes the relationship between the independent and dependent variables.
    a. Since this is a general example our model equation is just given.
    b. Model Equation: $4DC^2 = \frac{AE}{B^2}$
3. Identify the independent and dependent variables in the model equation.
    a. Independent variable – B
    b. Dependent variable – C

4. Use algebra to rearrange the model equation into slope-intercept form, $y = (slope)x + b$. **This is the linearized model equation.**

$$C^2 = \frac{AE}{4DB^2}$$

$$\boldsymbol{C^2 = \left(\frac{AE}{4D}\right)\left(\frac{1}{B^2}\right) + 0}$$

$$y = (slope)(x) + b$$

5. Identify the independent and dependent quantities in the linearized model equation.
   a. Independent quantity – $\frac{1}{B^2}$
   b. Dependent quantity – C^2
6. Identify the objective quantity and control variables.
   a. Objective quantity – A
   b. Control variables – D, E
7. Write an expression for the slope of your linearized model equation in terms of the control variables and the objective quantity.

$$slope = \frac{AE}{4D}$$

8. Rearrange the expression for the slope and solve for the objective quantity. This is the objective quantity equation.

$$A = \frac{4D(slope)}{E}$$

9. Conduct the experiment. Collect data. Graph the dependent quantity vs. the independent quantity. Follow the steps outlined in Appendix H to format your graph, add a trendline, and conduct linear regression to determine the slope and the uncertainty in the slope of the trendline.

Use the propagation of uncertainty equation on the objective quantity equation from step 8 to calculate the uncertainty of the objective quantity.

b. Example 2 – Conservation of Mechanical Energy

Task: Determine the spring constant of spring using a cart and mass by varying the mass (independent variable) and measuring changes in maximum speed (dependent variable). (Assume no losses due to friction or drag.)

Control Variables (Given): displacement of spring
Measured quantities: maximum speed of cart, mass of cart (including extra added masses)

1. Identify fundamental physics equations for the experiment.
   a. Conservation of Energy
   b. Elastic Potential Energy
   c. Translational Kinetic Energy

2. Apply physics concepts and equations to build a model equation that describes the relationship between the independent and dependent variables.
    a. Conduct Conservation of Energy table
    b. List energies in initial (spring stretched, cart at rest) and final (spring relaxed, cart moving) states.
    c. Model Equation: $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
3. Identify the independent and dependent variables in the model equation.
    a. Independent variable – *m*
    b. Dependent variable – *v*
4. Use algebra to rearrange the model equation into slope-intercept form, *y* = (*slope*)*x* + *b*. **This is the linearized model equation.**

$$mv^2 = kx^2$$

$$v^2 = \frac{kx^2}{m}$$

$$\boldsymbol{v^2 = (kx^2)\left(\frac{1}{m}\right) + 0}$$

$$y = (slope)(x) + b$$

(note than *m* represents mass, not *slope*)

5. Identify the independent and dependent quantities in the linearized model equation.
    a. Independent quantity – $\frac{1}{m}$
    b. Dependent quantity – v^2
6. Identify the objective quantity and control variables.
    a. Objective quantity – *k*
    b. Control variables – *x*
7. Write an expression for the slope of your linearized model equation in terms of the control variables and the objective quantity.

$$slope = (kx^2)$$

8. Rearrange the expression for the slope and solve for the objective quantity. This is the objective quantity equation.

$$k = \frac{(slope)}{x^2}$$

9. Conduct the experiment. Collect data. Graph the dependent quantity vs. the independent quantity. Follow the steps outlined in Appendix H to format your graph, add a trendline, and conduct linear regression to determine the slope and the uncertainty in the slope of the trendline.

Use the propagation of uncertainty equation on the objective quantity equation from step 8 to calculate the uncertainty of the objective quantity.

SLAM
Revised AY 18-1

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