SE 180 Earthquake Engineering

November 3, 2002

### STEP-BY-STEP PROCEDURE FOR SETTING UP A SPREADSHEET FOR USING Newmark's Method and Modal Analysis To Solve For The Response Of A Multi-Degree Of Freedom (MDOF) System

Start with the equation of motion for a linear multi-degree of freedom system with base ground excitation:

### $m\ddot{u} + c\dot{u} + ku = -m1\ddot{u}_{g}$

Using Modal Analysis, we can rewrite the original coupled matrix equation of motion as a set of un-coupled equations.

$$\ddot{q}_{i} + 2\zeta \omega \dot{q}_{i} + \omega_{i}^{2} q_{i} = -\frac{L_{i}}{M_{i}} \ddot{u}_{g}, i = 1, 2, ..., \text{NDOF}$$

with initial conditions of  $d_i(t = 0) = d_{i_0}$  and  $v_i(t = 0) = v_{i_0}$ Note that total acceleration or absolute acceleration will be  $\ddot{q}_{i_{abs}} = \ddot{q}_i + \ddot{u}_g$ 

We can solve each one separately (as a SDOF system), and compute histories of  $q_i$  and their time derivatives. To compute the system response, plug the q vector back into  $\mathbf{u} = \mathbf{\Phi} \mathbf{q}$  and get the u vector (and the same for the time derivatives to get velocity and acceleration).

The beauty here is that there is no matrix operations involved, since the matrix equation of motion has become a set of un-coupled equation, each including only one generalized coordinate  $q_n$ .

In the spreadsheet, we will solve each mode in a separate worksheet.

Step 1 - Define System Properties and Initial Conditions for First Mode

(A) Begin by setting up the cells for the Mass, Stiffness, and Damping of the SDOF System (Fig. 1). These values are known.

(B) Set up the cells for the modal participation factor  $\frac{L_i}{M_i}$  and mode shape  $\phi_i$  (Fig. 1). These values must be determined in advance using Modal Analysis.

(C) Calculate the Natural Frequency of the SDOF system using the equation

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$$\omega_i = \sqrt{K_i/M_i}$$
 (Equation 1)

*Note*: If the system damping is given in terms of the Modal Damping Ratio ( $\zeta_i$ ) then the Damping (c) can be calculated using the equation:

$$C_i = 2 \zeta_i \omega_i M_i$$
 (Equation 2)

(D) Set up the cells for the 2 Newmark Coefficients  $\alpha$  &  $\beta$  (Fig. 1), which will allow for performing

a) the Average Acceleration Method, use 
$$\alpha = \frac{1}{2}$$
 and  $\beta = \frac{1}{4}$ .

b) the Linear Acceleration Method, use  $\alpha = \frac{1}{2}$  and  $\beta = \frac{1}{6}$ .

(E) Set up cells (Fig. 1) for the initial displacement and velocity (d<sub>o</sub> and v<sub>o</sub> respectively)

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Figure 1: Spreadsheet After Completing Step 1

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Step 2 – Set Up Columns for Solving The Equation of Motion Using Newmark's Method

Figure 2: Spreadsheet After Completing Step 2

Place a cell (Fig. 2) for the time increment ( $\Delta t$ ).

Place columns (Fig. 2) for the time, base excitation, applied force divided by mass, relative acceleration, relative velocity, and relative displacement.

Step 3 – Enter the Time t & Applied Force f(t) into the Spreadsheet

 $t_{i+1} = t_i + \Delta t$  (Equation 3) (Fig. 3)

For the earthquake problem (acceleration applied to base of the structure), the applied force divided by the mass is calculated using:

$$\frac{f_i(t)}{M_i} = -\frac{L_i}{M_i} \ddot{u}_{g_i} \qquad (Equation 4) \quad (Fig. 3)$$

where,  $\ddot{u}_{g_i}$  is the applied base acceleration at step i. (Typically this is the base excitation time history)

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Check the units of the input motion file. They must be compatible with the units of the mass, stiffness, and damping!

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4	K1 =	276.52	N/m		0.01	0	-0.06282	0.046483259							
5		0.109758	N-s/m			0.01	-0.05914	0.043764854							
6	$L_1/M_1 =$	0.74				0.02	0.005203	-0.003850502							
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Figure 3: Spreadsheet After Completing Step 3

# Step 4 – Compute Initial Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration

(A) The Initial Relative Displacement and Relative Velocity are known from the initial conditions (Fig. 4).

 $q(t = 0) = d_0$  (Equation 5)

 $\dot{q}(t=0) = v_{o}$  (Equation 6)

(B) The Initial Relative Acceleration (Fig. 4) is calculated using

$$\ddot{q}(t=0) = -\frac{Li}{Mi}\ddot{u}_{g} - 2\zeta\omega v_{o} - \omega^{2}d_{o} \quad (\text{Equation 7})$$

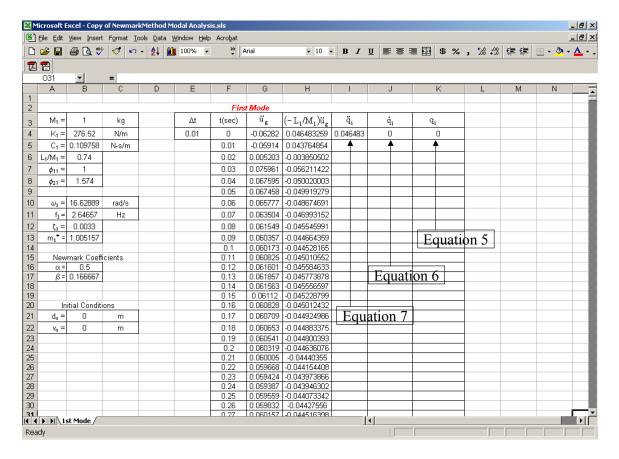


Figure 4: Spreadsheet After Completing Step 4

Step 5 – Compute Incremental Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration At Each Time Step (Fig. 5)

(A)

$$\ddot{q}_{i+1} = \frac{\left[-\frac{L_1}{M_1}\ddot{u}_{g_{i+1}} - C_1\left(\frac{\Delta t}{2}\ddot{q}_i + \dot{q}_i\right) - K_1\left(\frac{1}{2}\Delta t^2(1 - 2\beta)\ddot{q}_i + \Delta t\dot{q}_i + q_i\right)\right]}{m_1*}$$
(Equation 8)

$$\dot{q}_{i+1} = \ddot{q}_i \Delta t (1 - \alpha) + \ddot{q}_{i+1} \Delta t \alpha + \dot{q}_i$$
(Equation 9)

$$q_{i+1} = \ddot{q}_i \frac{\Delta t^2}{2} (1 - 2\beta) + \ddot{q}_{i+1} \Delta t^2 \beta + \dot{q}_i \Delta t + q_i$$
 (Equation 10)

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Where, the effective mass,  $m_1^* = M_1 + C_1 \Delta t \alpha + K_1 \Delta t^2 \beta$ 

Figure 5: Spreadsheet with values for the Relative Acceleration, Relative Velocity, and Relative Displacement at Time Step 1

(B) Then, highlight columns I, J, & K and rows 4 through to the last time step (in this example 4003) and "Fill Down" (Ctrl+D). See Figures 6 and 7.

Ahmed Elgamal Michael Fraser

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5	C1 =	0.109758	N-s/m			0.01	-0.05914	0.043764854	0.043089	0.00044786	2.26759E-06				
6	$L_1/M_1 =$	0.74				0.02	0.005203	-0.003850502							
7	φ <sub>11</sub> =	1				0.03	0.075961	-0.056211422	ĺ						
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3	M1 =	1	kg		∆t	t(sec)	üg	$(-L_1/M_1)\ddot{u}_g$	<b></b>		qi				
4	K1 =	276.52	N/m		0.01	0	-0.06282	0.046483259	0.046483	0	0				
5	C1 =	0.109758	N-s/m			0.01	-0.05914	0.043764854	0.043089	0.00044786	2.26759E-06				
6	$L_1/M_1 =$	0.74				0.02	0.005203	-0.003850502	-0.00615	0.00063253	8.0799E-06				
7	φ <sub>11</sub> =	1				0.03	0.075961	-0.056211422	-0.0599	0.00030229	1.32018E-05				
8	φ <sub>21</sub> =		1			0.04	0.067595	-0.050020003	-0.05368	-0.00026558	1.33335E-05				
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11	f <sub>1</sub> =	2.64657	Hz			0.07	0.063504	-0.046993152	-0.04191	-0.00174257	-1.76849E-05				
12	ζ1 =	0.0033				0.08	0.061549	-0.045545991	-0.03506	-0.00212741	-3.70919E-05				
13	m1 =	1.005157				0.09	0.060357	-0.044664359	-0.02781	-0.00244172	-5.99979E-05				
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991						39.87		-0.001385769		0.09148214	0.004283904				
992						39.88		-0.001274874		0.07835302					
993						39.89		-0.001182338		0.06308374	0.00584383				
994						39.9				0.04609653	0.006390976				
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998						39.94	0.00123	-0.000910024		-0.02922632	0.006740183				
999						39.95		-0.000875614		-0.04730093					
000						39.96		-0.000838991		-0.06405419					
001						39.97	0.001075	-0.0007955		-0.07902662					
002						39.98		-0.0007355	-1.15906	-0.07302882	0.004225339				
002										-0.10204963					
						39.99	0.000928	-0.000686691	-0.00923	<u>-0.10204963</u>	0.003253802	•			
004															
1005	▶ ▶  \1s	t Mode /							•						۰I
											Sum=-10.360835	541			
≀ead	y						0 (	<b>D</b> .11.	D		Bum=-10.360835	· · · · ·	<b>T</b>		

Figure 7: Spreadsheet After "Filling Down" Columns I through K

# Step 6 – Create Additional Worksheet for Second Mode

Make a copy of the "1<sup>st</sup> Mode" worksheet by right clicking on the "1<sup>st</sup> Mode" tab and selecting "<u>M</u>ove or Copy" (Fig. 8)

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1	~	U	U	U	L	-			-	J	N	L.	191	IN	
2						Fin	st Mode								
3	M1 =	1	kg		∆t	t(sec)	üg	$(-L_1/M_1)\ddot{u}_g$	äi	q <sub>i</sub>	qi				
4	K1 =	276.52	N/m		0.01	0	-0.06282	0.046483259	0.046483	0	0				
j	C1 =		N-s/m			0.01	-0.05914	0.043764854	0.043089	0.00044786	2.26759E-06				
;	L <sub>1</sub> /M <sub>1</sub> =	0.74				0.02	0.005203	-0.003850502	-0.00615	0.00063253	8.0799E-06				
,	φ <sub>11</sub> =	1				0.03	0.075961	-0.056211422	-0.0599	0.00030229	1.32018E-05				1
	φ <sub>21</sub> =	1.574				0.04	0.067595		-0.05368	-0.00026558	1.33335E-05				
,	Ψ21	1.014				0.04		-0.049919279	-0.05205	-0.00079422	8.02099E-06				
0	ω1 =	16.62889	rad/s			0.06	0.065777	-0.048674691	-0.04785	-0.00129374	-2.45378E-06				
1	f <sub>1</sub> =		Hz			0.07	0.063504	-0.046993152	-0.04191	-0.00174257	-1.76849E-05				
2	ζ1 =		112			0.08	0.061549	-0.045545991	-0.03506	-0.00212741	-3.70919E-05				
3	m1 =					0.09	0.060357	-0.044664359	-0.02781	-0.00244172	-5.99979E-05				
i	1111 -	1.003137				0.05		-0.044528165	-0.02054	-0.00268345					
5	New	/mark Coeffi	icients			0.11		-0.045010552		-0.00285281	-0.000113426				
5	α=					0.12	0.061601			-0.00294877	-0.000142496				
7	β=	0.166667				0.13	0.061857	-0.045773878	0.002153	-0.00296729	-0.000172143				
:						0.14	0.061563	-0.045556597	0.0105	-0.00290403					
9						0.15	0.06112			-0.00275823	-0.000229948				
I	lr	nitial Conditi	ons			0.16	0.060828	-0.045012432	0.026185	-0.00253401	-0.000256472				
	d <sub>o</sub> =	0	m			0.17	0.060709	-0.044924986	0.032855	-0.00223881	-0.000280392				
2	v <sub>o</sub> =	0	m			0.18	0.060653	-0.044883375	0.038567	-0.0018817	-0.000301042				
1						0.19	0.060541	-0.044800393	0.043254	-0.00147259	-0.000317853				
				_		0.2	0.060319	-0.044636076	0.046826	-0.00102219	-0.000330356				
1		Inse	rt			0.21	0.060005	-0.04440355	0.049174	-0.00054219	-0.000338198				
		Dele	te			0.22	0.059668	-0.044154408	0.050184	-4.5399E-05	-0.000341144				
1		Ren	ame			0.23	0.059424	-0.043973866	0.049743	0.00045424	-0.000339096				
			e or Copy			0.24	0.059387	-0.043946302	0.047782	0.00094186	-0.000332099				
)			ct All Sheets			0.25	0.059559	-0.044073342	0.044356	0.00140255	-0.000320349				
)				-		0.26	0.059832	-0.04427556		0.00182252	-0.000304184				
		st March	Code			0.27	0.060157	-0.044516398	0.033796	0.00218968	-0 000284074				Þ
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Figure 8: Creating a Copy of 1<sup>st</sup> Mode Worksheet

Then check the box for "Create a copy" and click on "OK" button (Fig. 9)

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_	15	•	= =(H5-:	\$B\$5*(\$E\$4	1/2*l4+J4)-:	\$B\$4*(\$E\$	4^2/2*(1-2*\$	B\$17)*l4+\$E\$4	*J4+K4))/\$E	3\$13					
	A	B	C	D	E	F	G	Н	L I	J	К	L	М	N	
1															_
2						Fir	st Mode	1		1					
3	M1 =	1	kg		Δt	t(sec)	üg	$(-L_1/M_1)\ddot{u}_g$	äi	ģ <sub>i</sub>	$q_i$				
4	K1 =	276.52	N/m		0.01	0	-0.06282	0.046483259	0.046483	0	0				
5	C1 =	0.109758	N-s/m			0.01	-0.05914	0.043764854	0.043089	0.00044786	2.26759E-06				
6	$L_1/M_1 =$	0.74				0.02	0.005203	-0.003850502	-0.00615	0.00063253	8.0799E-06				
7	φ <sub>11</sub> =	1				0.03	0.075961	-0.056211422	-0.0599	0.00030229	1.32018E-05				
8	$\phi_{21} =$	1.574				0.04	0.067595	-0.050020003	-0.05368	-0.00026558	1.33335E-05				
9	121					0.05	0.067458	-0.049919279	-0.05205	-0.00079422	8.02099E-06				
10	ω1 =	16.62889	rad/s			0.06	0.065777	-0.048674691	-0.04785	-0.00129374	-2.45378E-06				
11	f1 =	2.64657	Hz			0.07	0.063504	-0.046993152	-0.04191	-0.00174257	-1.76849E-05				
12	ζ1 =	0.0033				0.08	0.061549	-0.045545991	-0.03506	-0.00212741	-3.70919E-05				
13	m1 =	1.005157				0.09	0.060357	-0.044664359	-0.02781	-0.00244172	-5.99979E-05				
14						0.1	0.060173	-0.044528165	-0.02054	-0.00268345	-8.56843E-05				
15	Nev	vmark Coeff	icients			0.11	0.060825			-0.00285281	-0.000113426				
16	α=					0.12	0.061601	-0.045584633		-0.00294877	-0.000142496				
17 18	<u>β</u> =	0.166667				0.13	0.061857	-0.045773878 -0.045556597	0.002153	-0.00296729	-0.000172143 -0.000201569				
19	M	ove or Copy		? ×		0.14	0.061565	-0.045556597			-0.000201569				
20						0.16	0.060828				-0.000256472				
21	d, M	love selected	sheets			0.17	0.060709	-0.044924986	0.032855		-0.000280392				
22	1 V	o book:				0.10	0.060653	-0.044003375	0.038567	-0.0010017	-0.000301042				
23		Copy of Newm	harkMethod M	odal A 💌		0.19	0.060541	-0.044800393			-0.000317853				
24	B	efore sheet:				0.2	0.060319	-0.044636076	0.046826	-0.00102219	-0.000330356				
25 26		1st Mode				0.21	0.060005			-0.00054219	-0.000338198				
26		(move to end)				0.22	0.059668				-0.000341144				
27						0.23	0.059424				-0.000339096				
28 29				-		0.24	0.059387	-0.043946302			-0.000332099 -0.000320349				
30		Create a co	- D			0.25	0.059832			0.00140255	-0.000320349				
31			20			0.20		-0.044516398	0.033796,	ŋ nn218968					
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Figure 9: Creating a Copy of 1<sup>st</sup> Mode Worksheet

Rename this worksheet by right clicking on the "1st Mode (2)" tab and selecting "<u>R</u>ename". Rename this worksheet "2<sup>nd</sup> Mode" (Fig. 10)

Enter the appropriate values for M<sub>2</sub>, K<sub>2</sub>, C<sub>2</sub>,  $\frac{L_2}{M_2}$ ,  $\phi_2$ ,  $d_o$ , and  $v_o$  (Fig. 10).

M	licrosoft I	Excel - Newn	narkMetho	d Modal Ana	alysis.xls										_ 8 ×
	<u>Eile E</u> dit	⊻iew Insert	: F <u>o</u> rmat <u>T</u>	ools <u>D</u> ata y	<u>M</u> indow <u>H</u> e	elp Acro <u>b</u> at									_ 8 ×
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2						Seco	nd Mode								
3	M2 =	1	kg		∆t	t(sec)	üg	$(-L_1/M_1)\ddot{u}_g$			$q_i$				
4	K2 =	1951.652	N/m		0.01	0	-0.06282	0.016331956	0.016332	0	0				
5	C2 =	0.15152	N-s/m			0.01	-0.05914	0.01537684	0.013841	0.00015087	7.75087E-07				
6	$L_2/M_2 =$	0.26				0.02	0.005203	-0.001352879	-0.00653	0.00018744	2.63635E-06				
7	$\phi_{12} =$	1				0.03	0.075961	-0.019749959	-0.02725	1.8581E-05	3.83911E-06				
8	$\phi_{22} =$	-0.6356				0.04	0.067595	-0.017574596	-0.02288	-0.00023204	2.73543E-06				
9	422	0.0000				0.05		-0.017539206	-0.01627	-0.00042776	-6.1865E-07				
10	ω, =	44.17751	rad/s			0.06	0.065777	-0.017101918	-0.0062	-0.00054012	-5.54191E-06				
11		7.031068	Hz			0.07		-0.016511108		-0.00054532	-1.10638E-05				
12		0.001715				0.08		-0.016002645		-0.00044219	-1.60872E-05				
13		1.033285				0.09	0.060357	-0.015692883	0.02263	-0.00025174	-1.96166E-05				
14		1.000200				0.1		-0.015645031		-1.2284E-05	-2.09586E-05				
15	New	mark Coeff	icients			0.11		-0.015814518		0.00022855	-1.98576E-05				
16	α=	0.5				0.12	0.061601	-0.016016222	0.016197	0.00042407	-1.65387E-05				
17	β=	0.166667				0.13	0.061857	-0.016082714	0.00657	0.0005379	-1.16486E-05				
18						0.14	0.061563	-0.016006372	-0.00415	0.00055002	-6.11969E-06				
19						0.15	0.06112	-0.0158912	-0.01403	0.00045916	-9.91467E-07				
20	In	itial Conditi	ions			0.16	0.060828	-0.015815179	-0.02128	0.00028263	2.77794E-06				
21	d <sub>o</sub> =	0	m			0.17	0.060709	-0.015784454	-0.02455	5.3498E-05	4.48582E-06				
22	v <sub>o</sub> =	0	m			0.18	0.060653	-0.015769834	-0.02319	-0.00018519	3.81607E-06				
23						0.19	0.060541	-0.015740678	-0.01744	-0.00038833	9.00572E-07				
24						0.2	0.060319	-0.015682946	-0.00838	-0.00051741	-3.70365E-06				
25						0.21	0.060005	-0.015601247	0.002279	-0.0005479	-9.11897E-06				
26						0.22	0.059668	-0.015513711	0.012494	-0.00047403	-1.43137E-05				
27						0.23	0.059424	-0.015450277	0.02031	-0.00031002	-1.82991E-05				
28						0.24		-0.015440593	0.024227	-8.7328E-05	-2.03185E-05				
29						0.25		-0.015485228		0.00015136	-1.99924E-05				
30						0.26	0.059832	-0.015556278	0.018328	0.00036055	-1.73897E-05				
31						0.27	10.060157	-0 015640896 T / a2 / v2 / d2 ,	Lo nogeza,	9 00050057	-1 3012E-05				
		.st Mode ),2r	nd Mode /	1st Floor / 2	nd Floor	q1 <u>(</u> q2 <u>(</u> a1 )	v1 / d1 / a1	T <u>(a</u> 2 <u>(</u> v2 <u>(</u> d2 ,	( <u>a2</u> T/	•					
Rea	idy														

Figure 10: Worksheet for Second Mode

## Step 7 – Repeat Step 6 for Additional Modes

## Step 8 – Determine the Response at Each of the Floors

Determine the Response of the first floor using the equations:

For example for a 2DOF structure, the first floor response is

$\mathbf{u}_1 = \mathbf{\phi}_{11}\mathbf{q}_1 + \mathbf{\phi}_{12}\mathbf{q}_2$	(Equation 11)
$\dot{\boldsymbol{u}}_1 = \boldsymbol{\varphi}_{11} \dot{\boldsymbol{q}}_1 + \boldsymbol{\varphi}_{12} \dot{\boldsymbol{q}}_2$	(Equation 12)
$\ddot{\mathbf{u}}_1 = \boldsymbol{\varphi}_{11} \ddot{\mathbf{q}}_1 + \boldsymbol{\varphi}_{12} \ddot{\mathbf{q}}_2$	(Equation 13)

and the second floor response is

$$\mathbf{u}_2 = \mathbf{\phi}_{21}\mathbf{q}_1 + \mathbf{\phi}_{22}\mathbf{q}_2 \qquad (\text{Equation 14})$$

Ahmed Elgamal Michael Fraser

$\dot{\mathbf{u}}_2 = \mathbf{\phi}_{21}\dot{\mathbf{q}}_1 + \mathbf{\phi}_{22}\dot{\mathbf{q}}_2$	(Equation 15)
$\ddot{\mathbf{u}}_2 = \mathbf{\phi}_{21}\ddot{\mathbf{q}}_1 + \mathbf{\phi}_{22}\ddot{\mathbf{q}}_2$	(Equation 16)

The first floor absolute acceleration is  $\ddot{u}_1^T = \ddot{u}_1 + \ddot{u}_g$  (Equation 17)

The second floor absolute acceleration is  $\ddot{u}_2^T = \ddot{u}_2 + \ddot{u}_g$  (Equation 18)