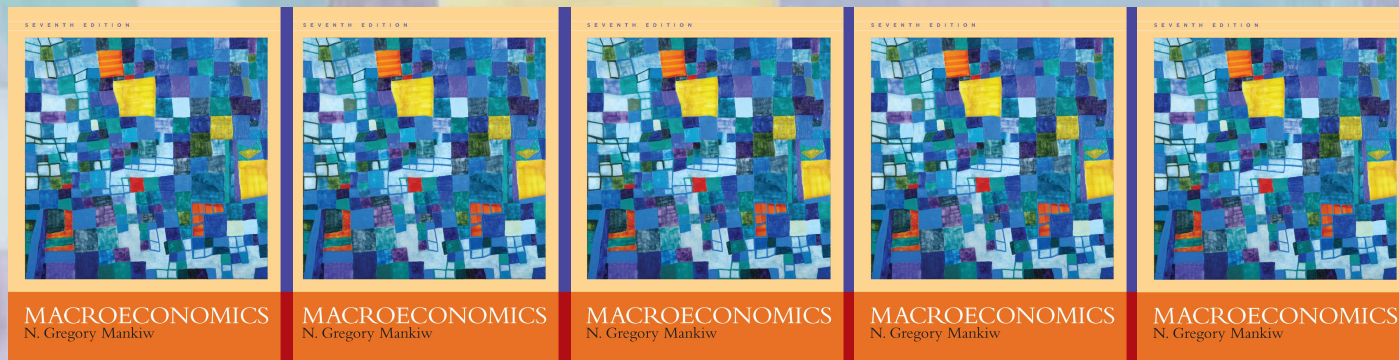


MANKIW'S MACROECONOMICS MODULES[®]

CHAPTER 7

Economic Growth I: Capital Accumulation and Population Growth



A PowerPoint[™] Tutorial

To Accompany

MACROECONOMICS, 7th. Edition

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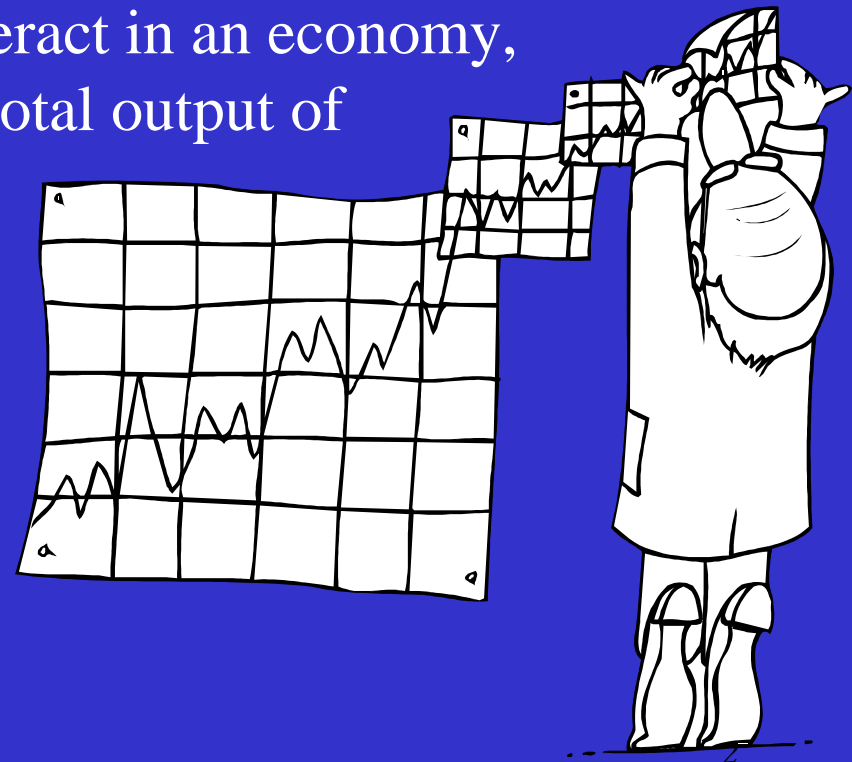
M.P.A., Harvard University Kennedy School of Government

M.B.A., Massachusetts Institute of Technology (MIT) Sloan School of Management

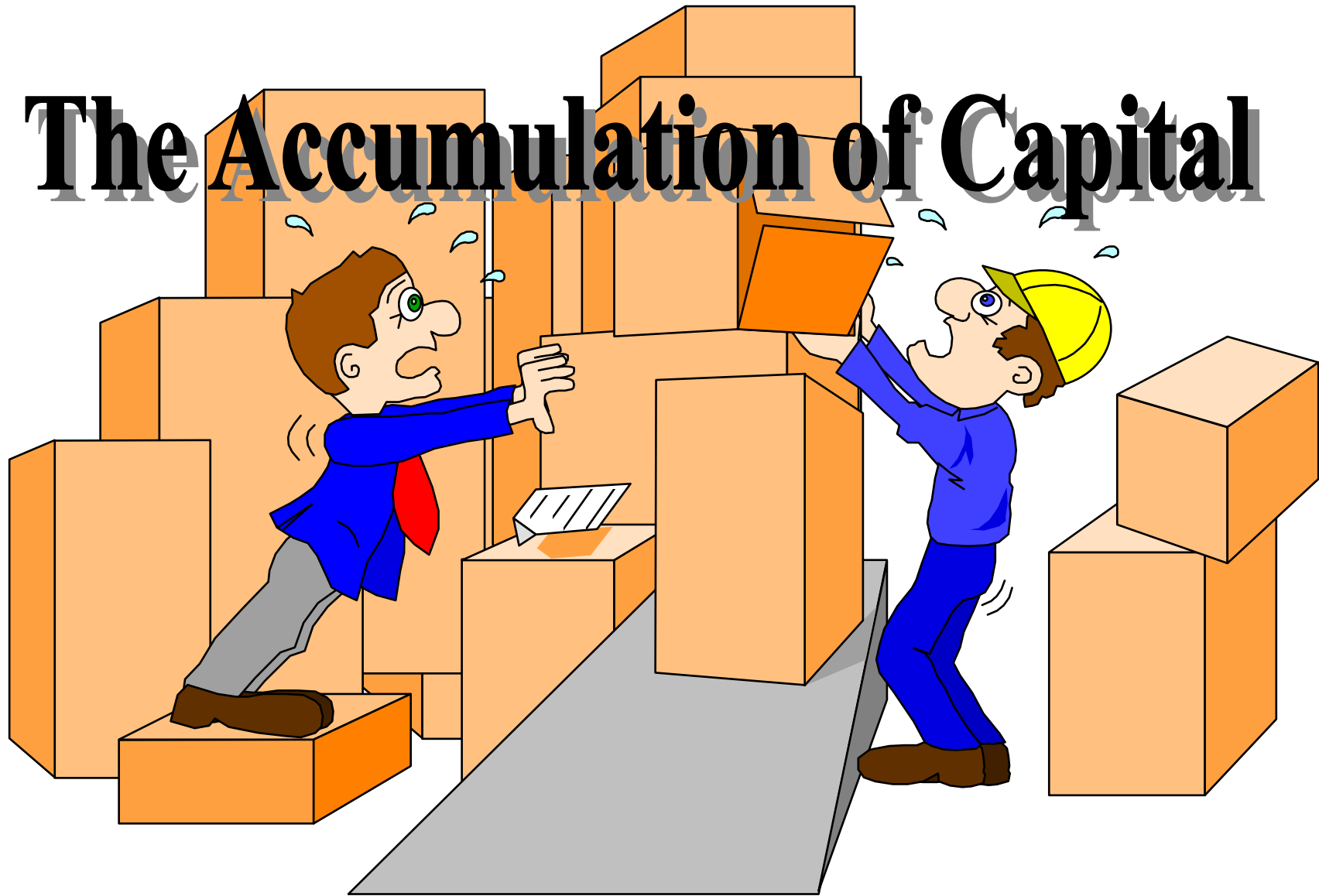
The Solow Growth Model

The **Solow Growth Model** is designed to show how growth in the capital stock, growth in the labor force, and advances in technology interact in an economy, and how they affect a nation's total output of goods and services.

Let's now examine how the model treats the accumulation of capital.



The Accumulation of Capital



Let's analyze the supply and demand for goods, and see how much output is produced at any given time and how this output is allocated among alternative uses.



The Production Function

The production function represents the transformation of **inputs** (labor (L), capital (K), production technology) into **outputs** (final goods and services for a certain time period).

The algebraic representation is:

$$zY = F(zK, zL)$$

Income is some function of our given inputs₄

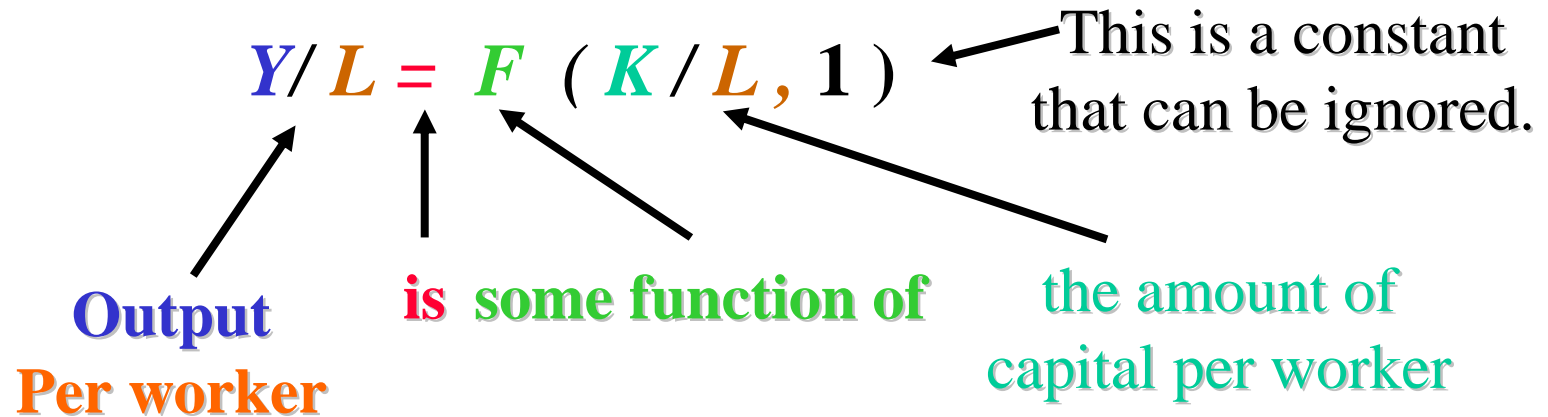
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Key Assumption: The Production Function has constant returns to scale.



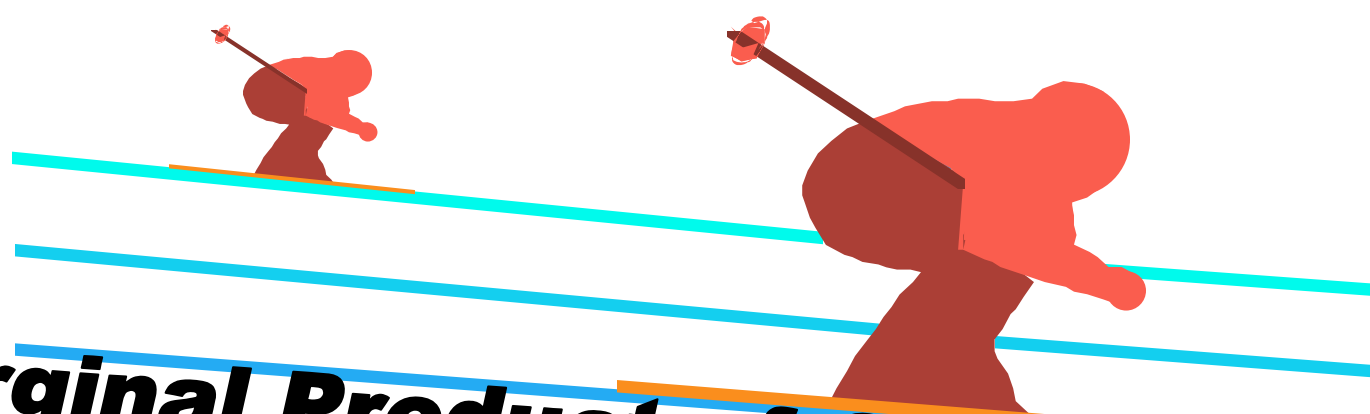
Constant Returns to Scale

This assumption lets us analyze all quantities relative to the size of the labor force. Set $z = 1/L$.



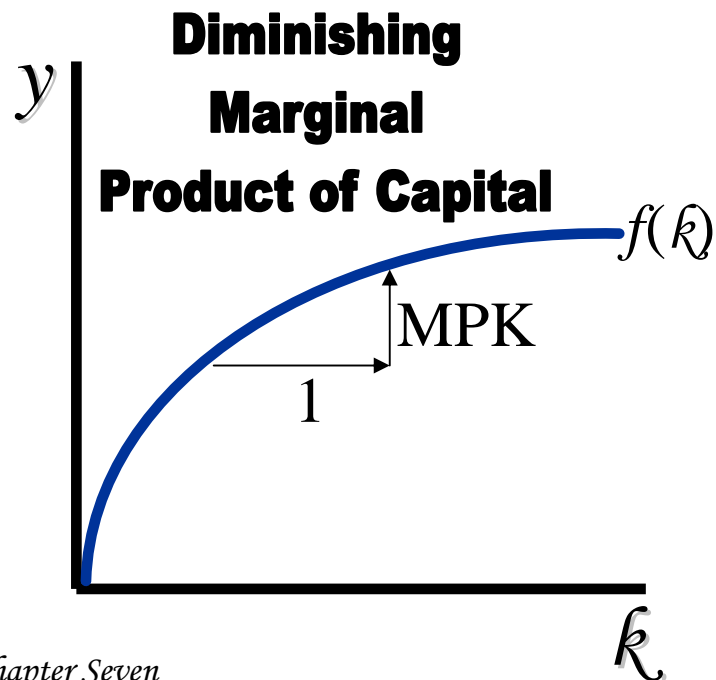
Constant returns to scale imply that the size of the economy as measured by the number of workers does not affect the relationship between output per worker and capital per worker. So, from now on, let's denote all quantities in per worker terms in lower case letters.

Here is our production function: $y = f(k)$, where $f(k) = F(k, 1)$.



Marginal Product of Capital (MPK): The Slope of the Production Function

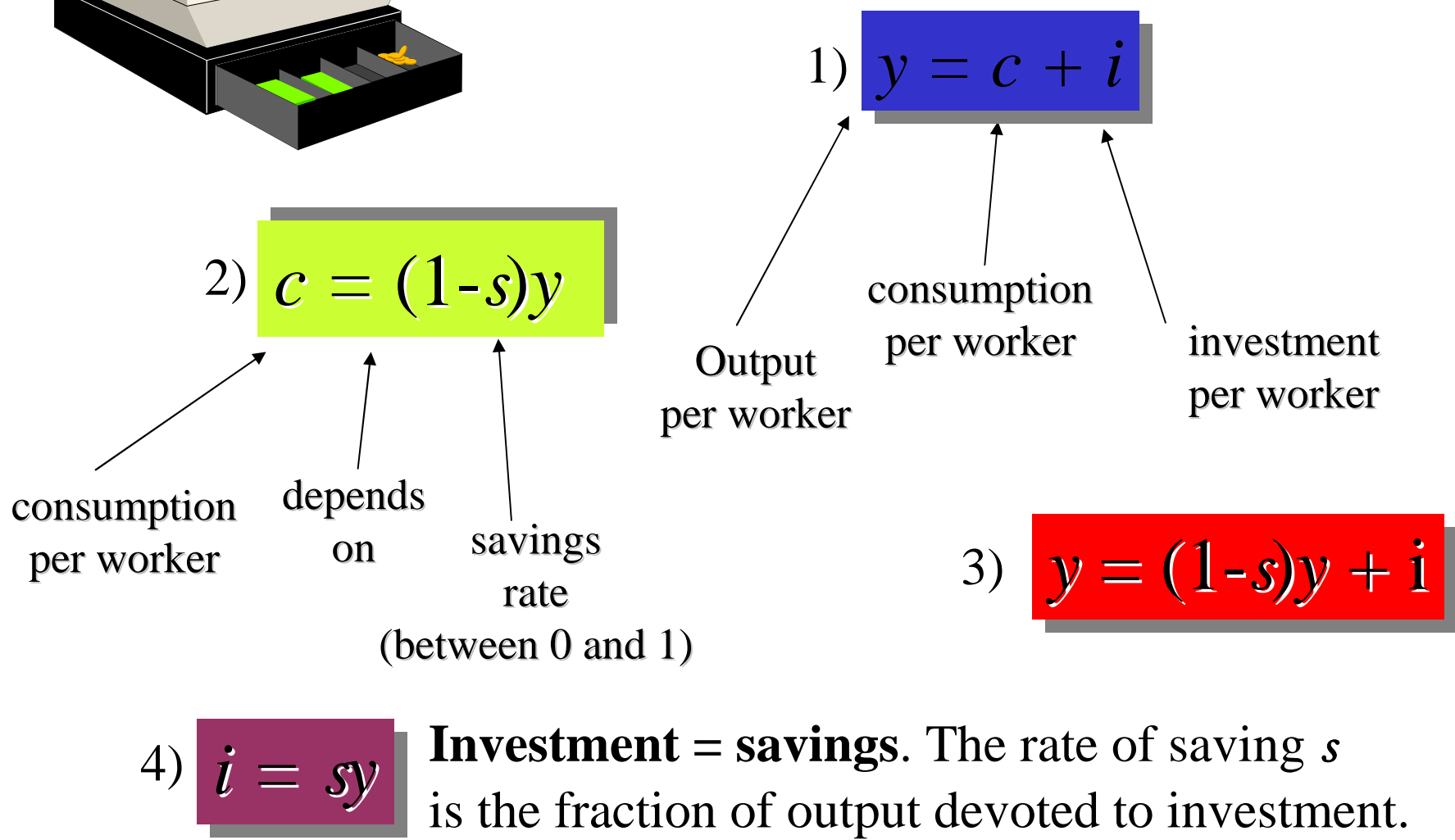
$$MPK = f(k + 1) - f(k)$$



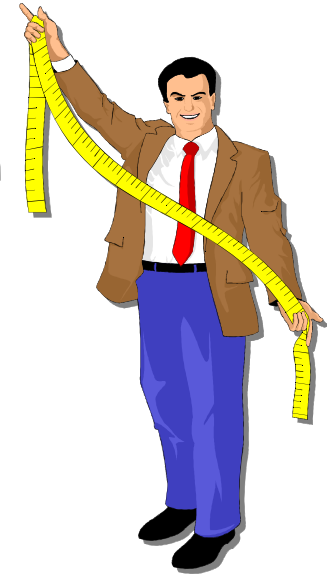
The production function shows how the amount of capital per worker k determines the amount of output per worker $y = f(k)$. The **slope of the production function** is the marginal product of capital: if k increases by 1 unit, y increases by MPK units.



The Demand for Goods and the Consumption Function



Growth in the Capital Stock and the Steady State



Here are two forces that influence the capital stock:

- **Investment:** expenditure on plant and equipment.
- **Depreciation:** wearing out of old capital; causes capital stock to fall.

Recall investment per worker $i = sy$.

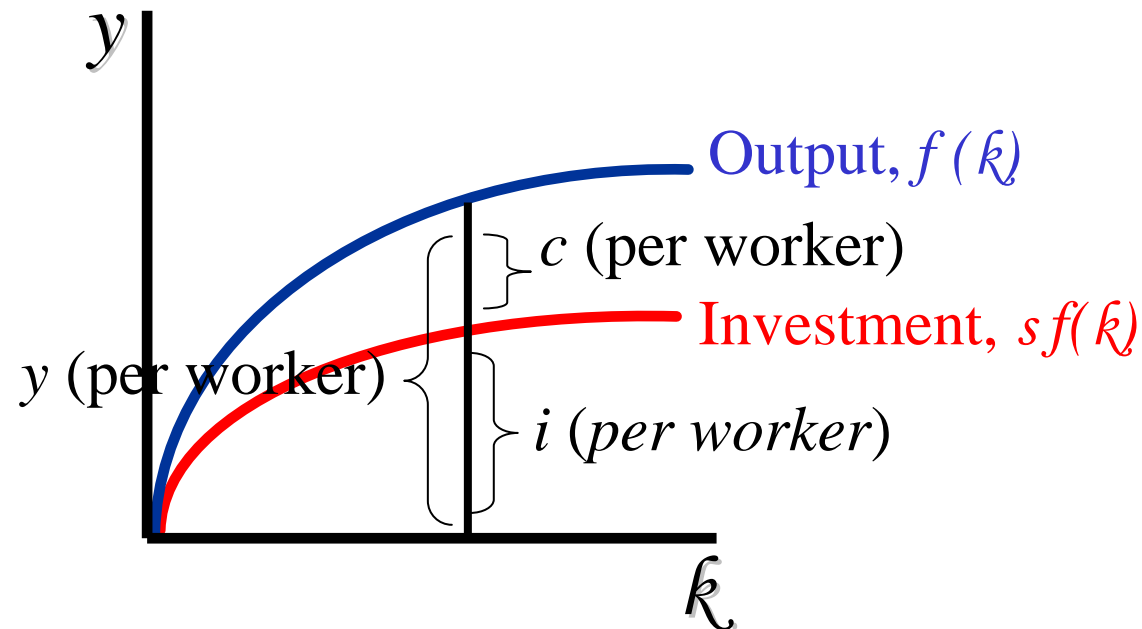
Let's substitute the production function for y , we can express investment per worker as a function of the capital stock per worker:

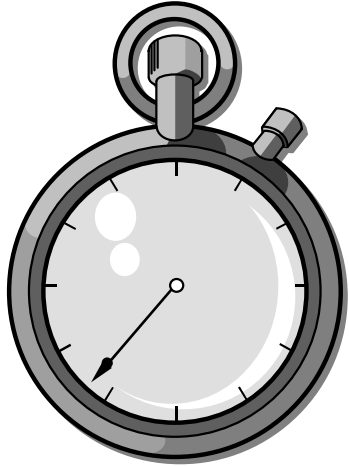
$$i = sf(k)$$

This equation relates the existing stock of capital k to the accumulation of new capital i .

Output, Consumption, and Investment

The saving rate s determines the allocation of output between consumption and investment. For any level of k , output is $f(k)$, investment is $sf(k)$, and consumption is $f(k) - sf(k)$.





Depreciation

Impact of investment and depreciation on the capital stock: $\Delta k = i - \delta k$

Change in
capital stock

Investment

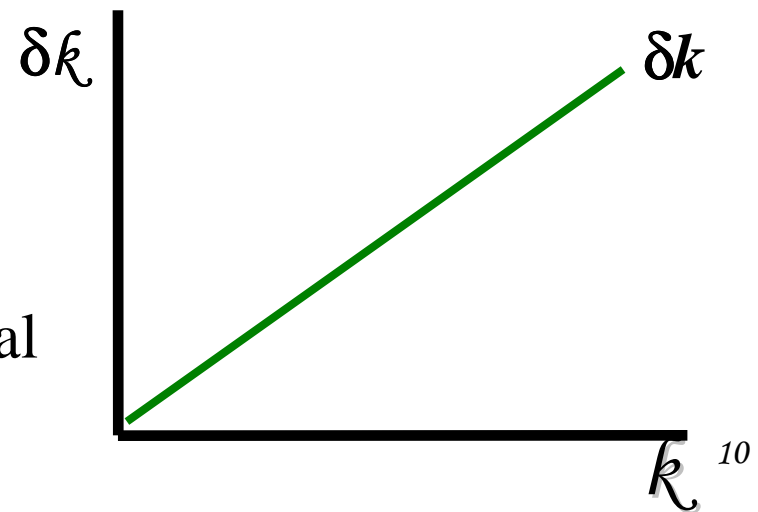
Depreciation

Remember investment equals
savings so, it can be written:

$$\Delta k = sf(k) - \delta k$$

Depreciation is therefore proportional
to the capital stock.

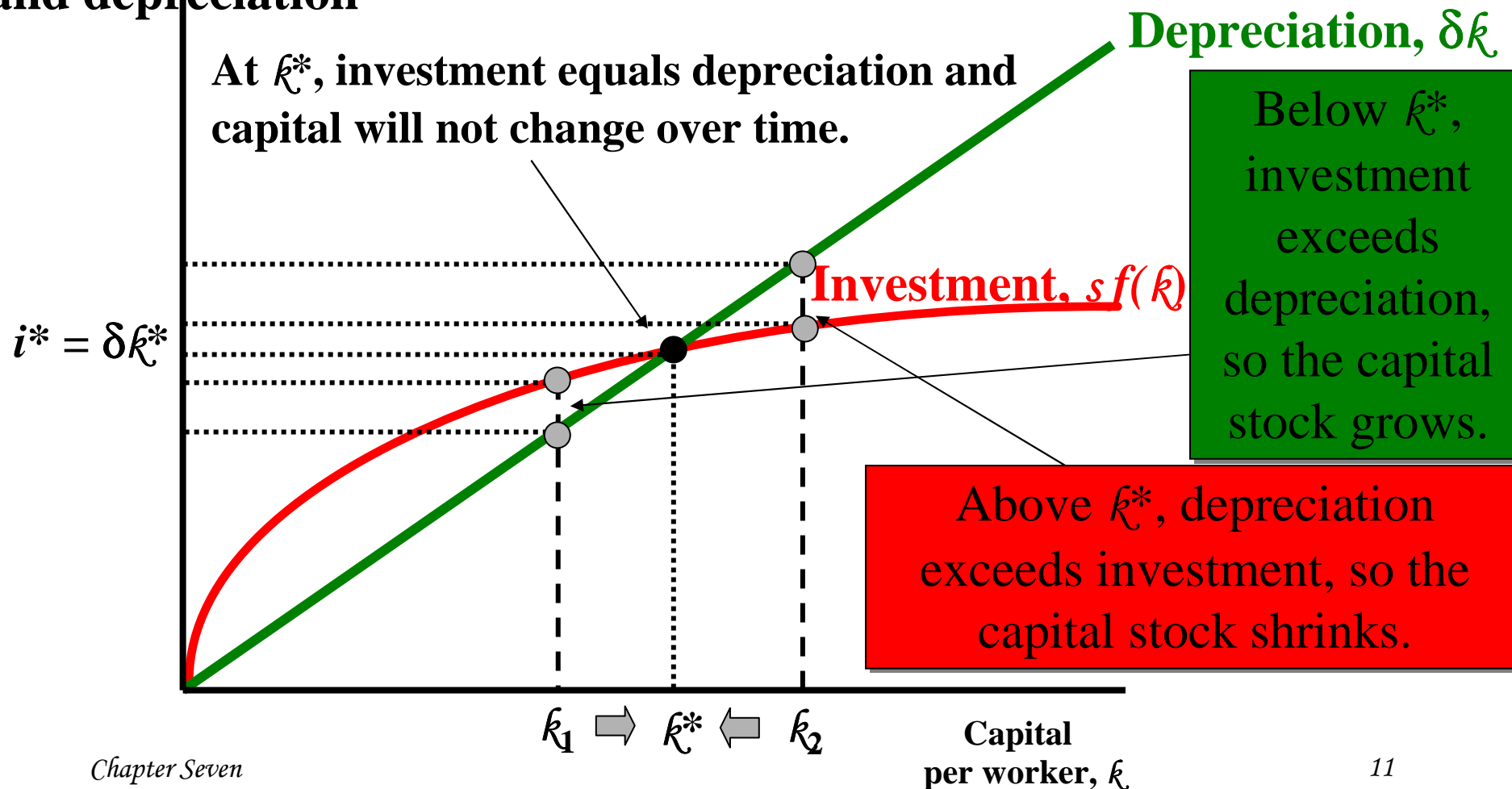
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The Steady State, k^*

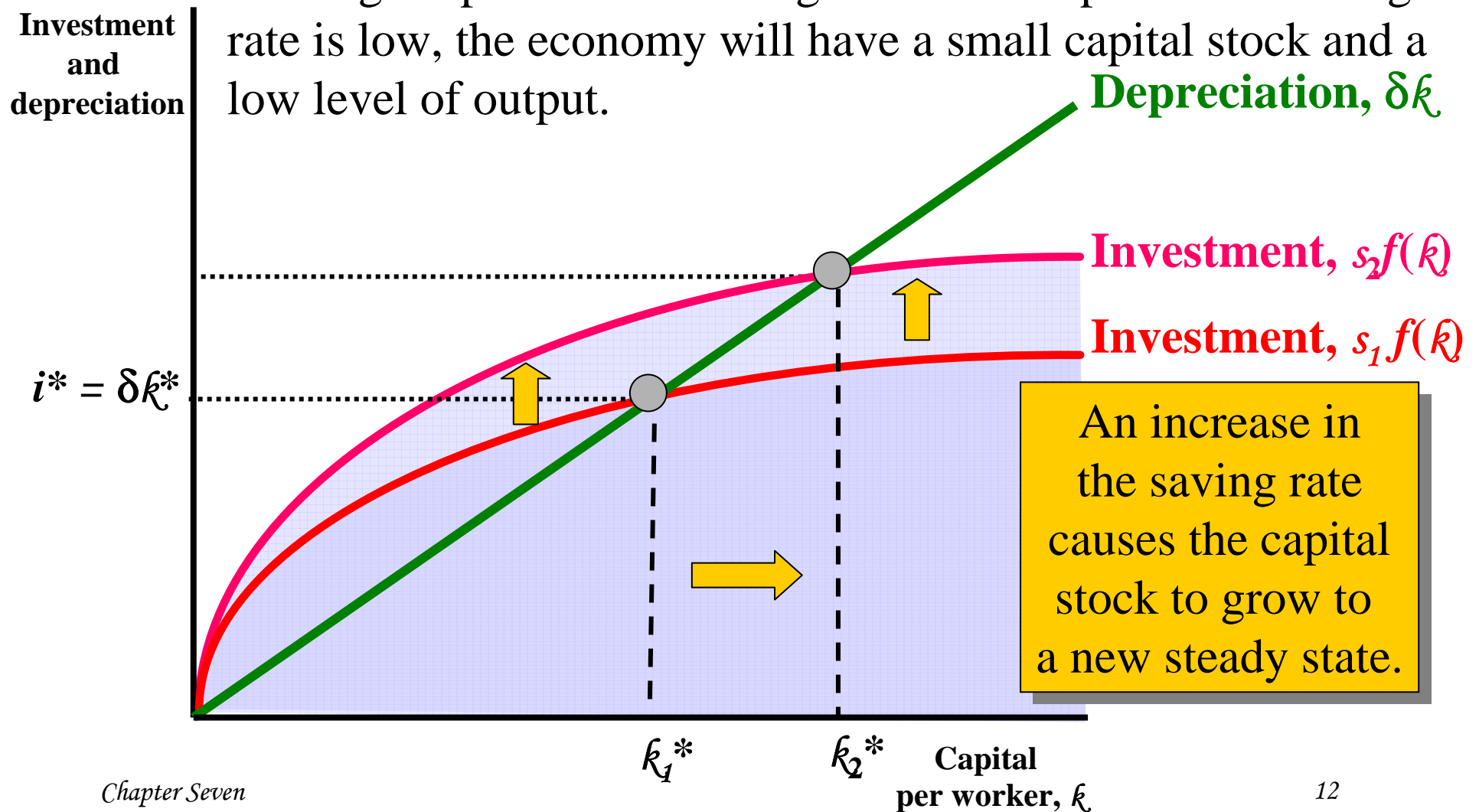
The long-run equilibrium of the economy

Investment
and depreciation



How Saving Affects Growth

The Solow Model shows that if the saving rate is high, the economy will have a large capital stock and high level of output. If the saving rate is low, the economy will have a small capital stock and a low level of output.



The Golden Rule Level of Capital

The steady-state value of k that maximizes consumption is called the *Golden Rule Level of Capital*. To find the steady-state consumption per worker, we begin with the national income accounts identity:

$$y - c + i$$

and rearrange it as:

$$c = y - i.$$

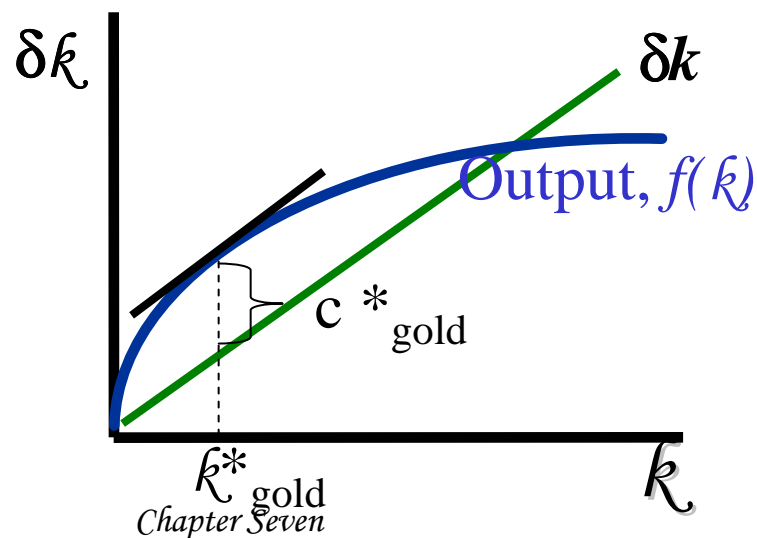
This equation holds that consumption is output minus investment. Because we want to find steady-state consumption, we substitute steady-state values for output and investment. Steady-state output per worker is $f(k^*)$ where k^* is the steady-state capital stock per worker. Furthermore, because the capital stock is not changing in the steady state, investment is equal to depreciation δk^* . Substituting $f(k^*)$ for y and δk^* for i , we can write steady-state consumption per worker as:

$$c^* = f(k^*) - \delta k^*.$$

Steady-state Consumption

$$c^* = f(k^*) - \delta k^*$$

According to this equation, steady-state consumption is what's left of steady-state output after paying for steady-state depreciation. It further shows that an increase in steady-state capital has two opposing effects on steady-state consumption. On the one hand, more capital means more output. On the other hand, more capital also means that more output must be used to replace capital that is wearing out.



The economy's output is used for consumption or investment. In the steady state, investment equals depreciation. Therefore, steady-state consumption is the difference between output $f(k^*)$ and depreciation δk^* . Steady-state consumption is maximized at the **Golden Rule** steady state. The **Golden Rule** capital stock is denoted k^*_{gold} , and the **Golden Rule** consumption is c^*_{gold} .

Let's now derive a simple condition that characterizes the **Golden Rule** level of capital. Recall that the slope of the production function is the marginal product of capital MPK . The slope of the δk^* line is δ . Because these two slopes are equal at k^*_{gold} , the **Golden Rule** can be described by the equation: $MPK = \delta$.

At the **Golden Rule** level of capital, the marginal product of capital equals the depreciation rate.

Keep in mind that the economy does not automatically gravitate toward the **Golden Rule** steady state. If we want a particular steady-state capital stock, such as the **Golden Rule**, we need a particular saving rate to support it.

Population Growth

The basic Solow model shows that capital accumulation, alone, cannot explain sustained economic growth. High rates of saving lead to high growth temporarily, but the economy eventually approaches a steady state in which capital and output are constant.

To explain the sustained economic growth, we must expand the Solow model to incorporate the other two sources of economic growth.

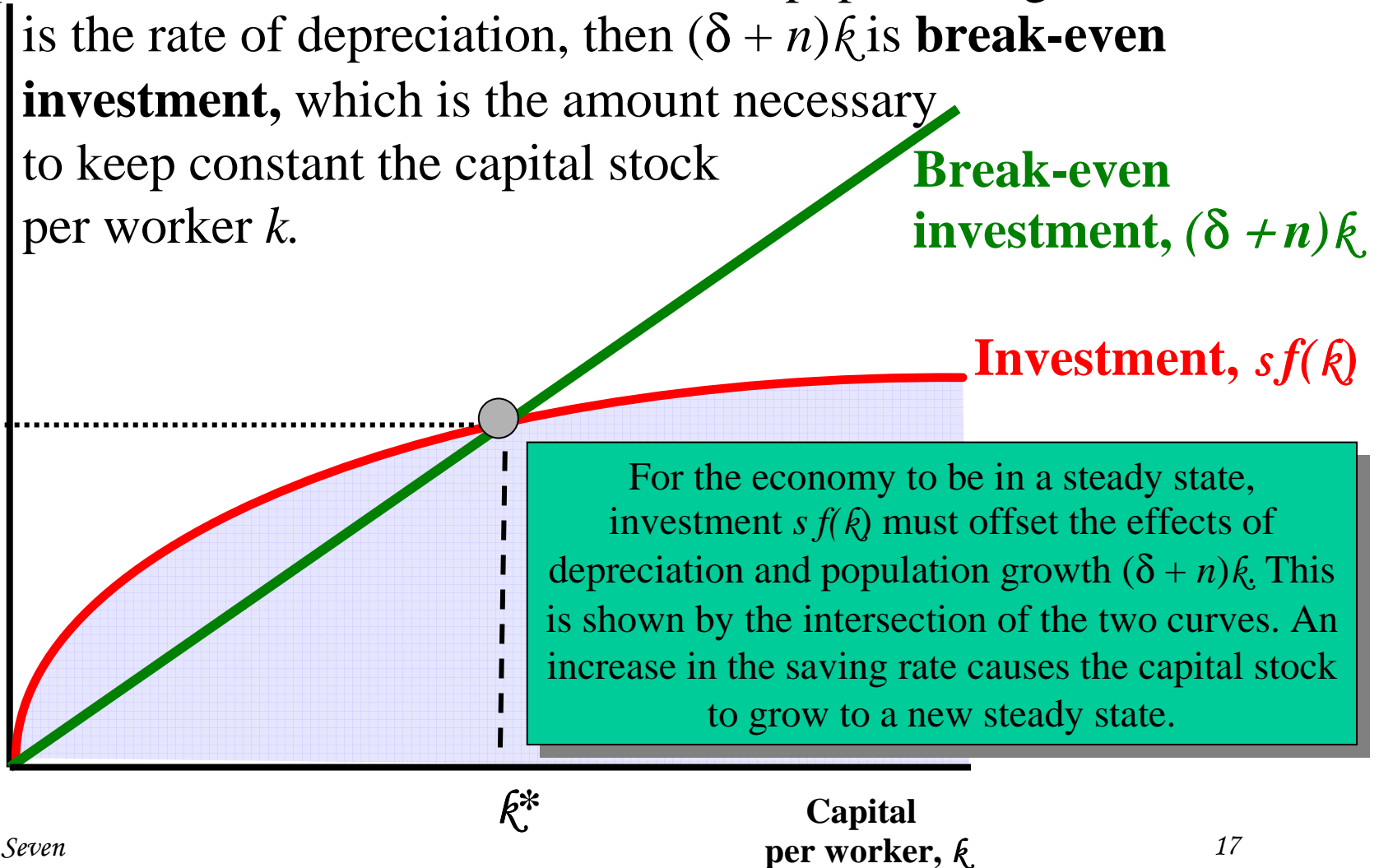
So, let's add population growth to the model. We'll assume that the population and labor force grow at a constant rate n .

The Steady State with Population Growth

Like depreciation, population growth is one reason why the capital stock per worker shrinks. If n is the rate of population growth and δ

Investment,
break-even
investment

is the rate of depreciation, then $(\delta + n)k$ is **break-even investment**, which is the amount necessary to keep constant the capital stock per worker k .

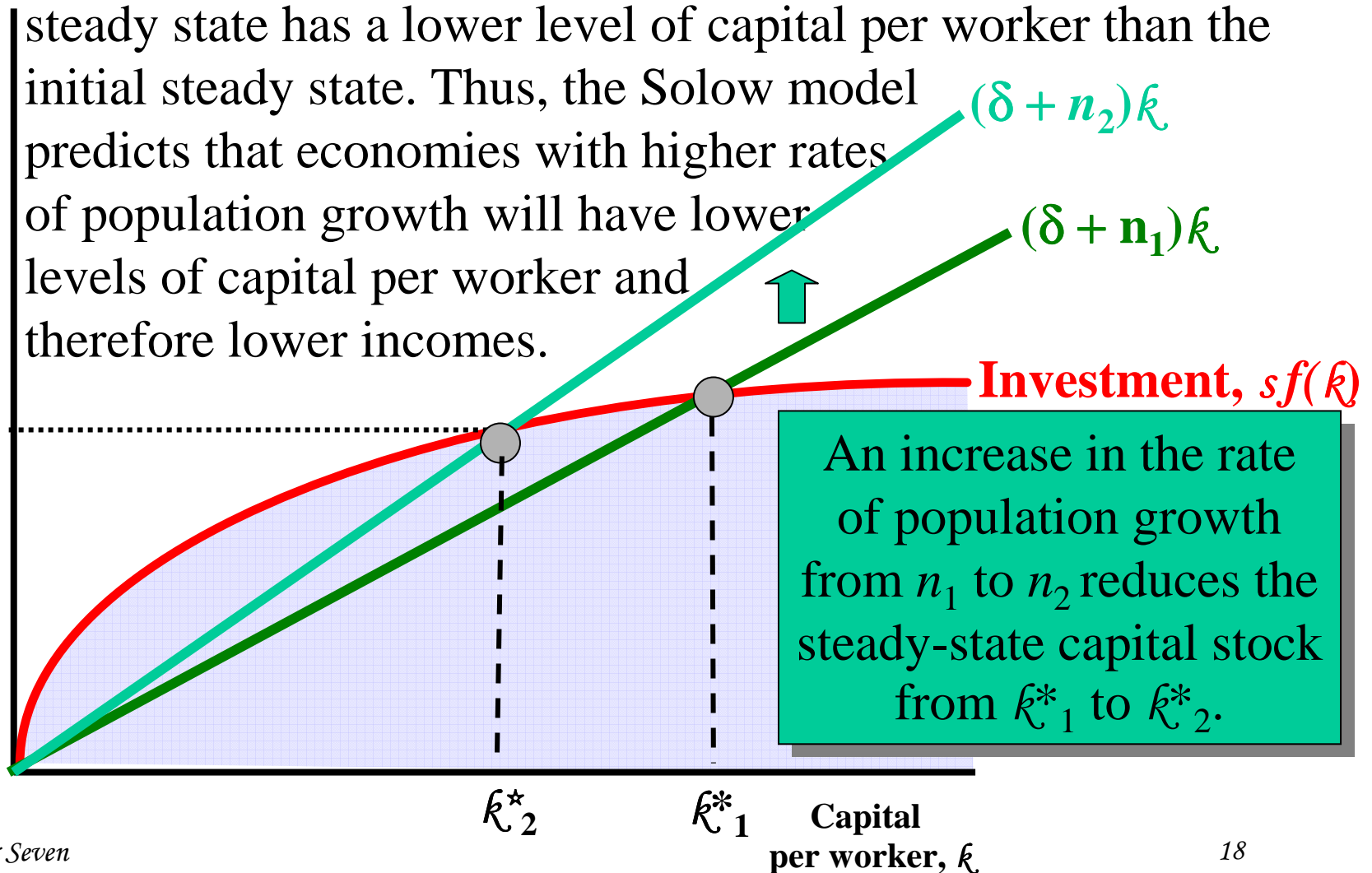


The Impact of Population Growth

An increase in the rate of population growth shifts the line representing population growth and depreciation upward. The new

Investment,
break-even
investment

steady state has a lower level of capital per worker than the initial steady state. Thus, the Solow model predicts that economies with higher rates of population growth will have lower levels of capital per worker and therefore lower incomes.



Population Growth (n)

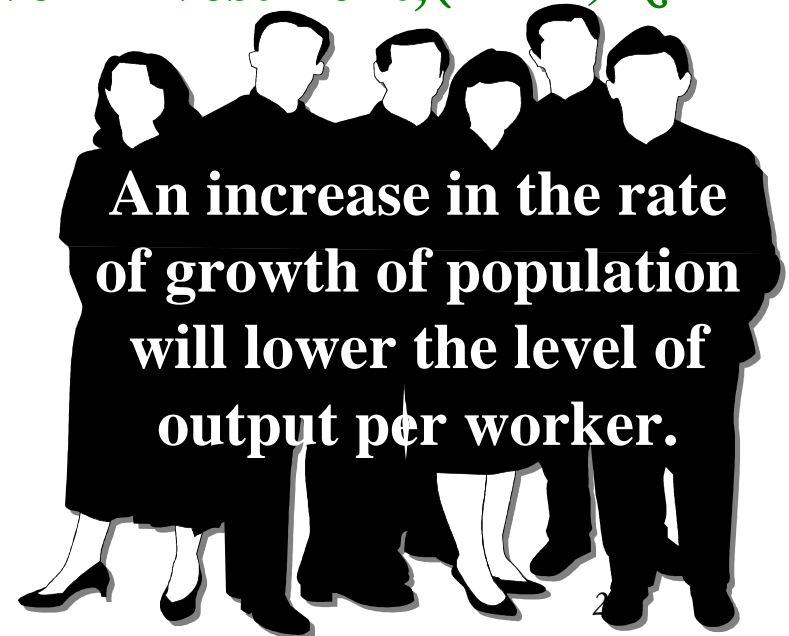
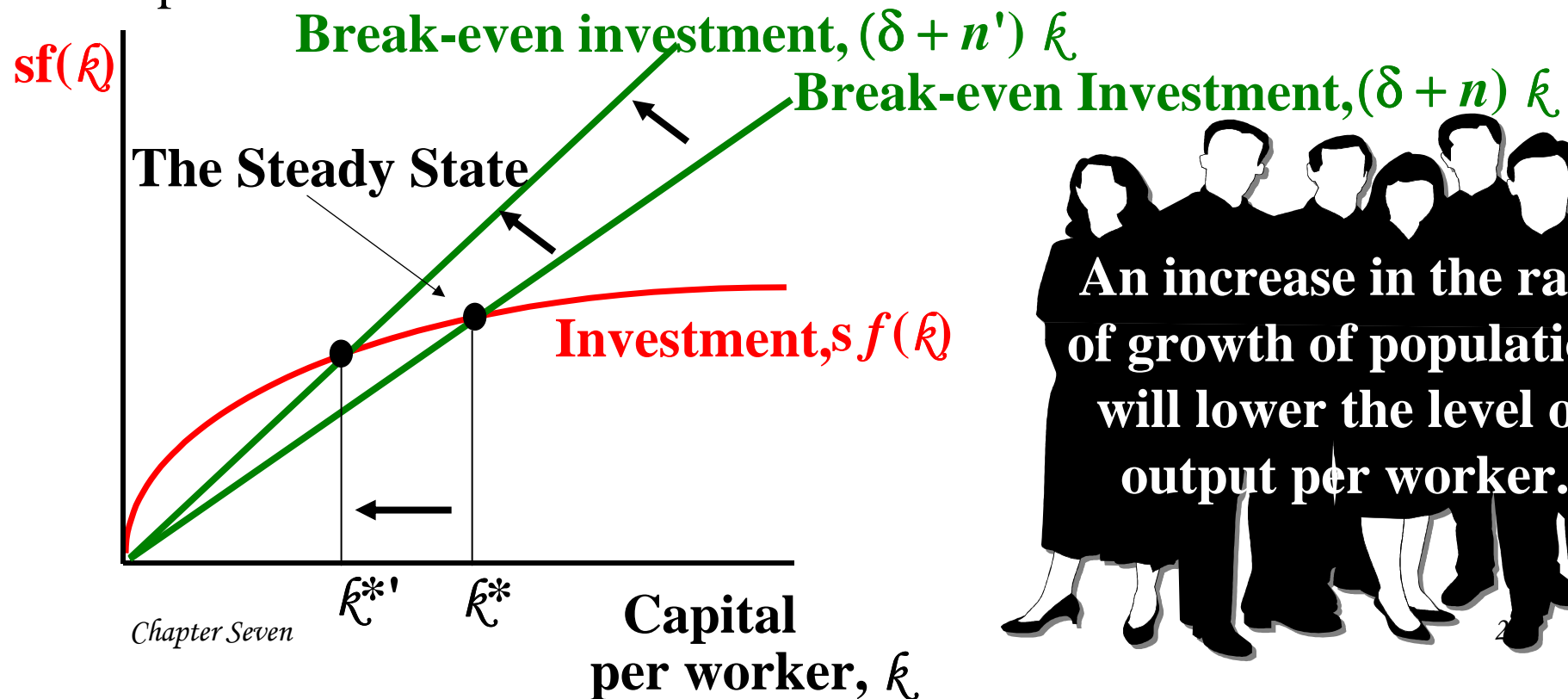
The change in the capital stock per worker is: $\Delta k = i - (\delta + n)k$

Now, let's substitute $sf(k)$ for i : $\Delta k = (sf(k) - (\delta + n)k)$

This equation shows how new investment, depreciation, and population growth influence the per-worker capital stock. New investment increases k , whereas depreciation and population growth decrease k . When we did not include the “ n ” variable in our simple version—we were assuming a special case in which the population growth was 0.

In the steady state, the positive effect of investment on the capital per worker just balances the negative effects of depreciation and population growth. Once the economy is in the steady state, investment has two purposes:

- 1) Some of it, (δk^*) , replaces the depreciated capital,
- 2) The rest, $(n k^*)$, provides new workers with the steady state amount of capital.



Final Points on Saving

- In the long run, an economy's saving determines the size of k and thus y .
- The higher the rate of saving, the higher the **stock** of capital and the higher the level of y .
- An increase in the rate of saving causes a period of rapid growth, but eventually that growth slows as the new steady state is reached.

Conclusion: although a high saving rate yields a high steady-state level of output, **saving by itself cannot generate persistent economic growth.**

Key Concepts of Chapter 7

Solow growth model

Steady state

Golden Rule level of capital