Unit 60: Dynamics of Machines

Unit code: H/601/1411 QCF Level:4 Credit value:15

OUTCOME 2 – KINEMATICS AND DYNAMICS

TUTORIAL 1 CAMS

2 **Be able to determine the kinetic and dynamic parameters of mechanical systems**

Cams: radial plate and cylindrical cams; follower types; profiles to give uniform velocity; uniform acceleration and retardation and simple harmonic motion outputs; output characteristics of eccentric circular cams, circular arc cams and cams with circular arc and tangent profiles with flat-faced and roller followers

Plane mechanisms: determination of instantaneous output velocity for the slider-crank mechanism, the four-bar linkage and the slotted link and Whitworth quick return motions; construction of velocity vector diagrams; use of instantaneous centre of rotation

Resultant acceleration: centripetal, tangential, radial and Coriolis components of acceleration in plane linkage mechanisms; resultant acceleration and inertia force; use of Klein's construction for the slider crank mechanism

Gyroscopic motion: angular velocities of rotation and precession; gyroscopic reaction torque; useful applications e.g. gyro-compass and gyro-stabilisers

You should judge your progress by completing the self assessment exercises.

On completion of this short tutorial you should be able to do the following.

- Describe the purpose of cams.
- Describe the different types of cams and followers.
- Deduce the velocity and acceleration of cam followers for a specified motion.
- Deduce the cam profile for a specified motion.
- Explain the importance of inertia in cam mechanisms.

You will find a good general description of cams and animated video clips at
www.technologystudent.com/cams/camdex

1. INTRODUCTION

Cams precede the development of modern Mechatronics but still have their uses in a variety of devices. They convert the rotation of a shaft into a specified linear motion of a follower. A typical example is the overhead cam shaft in an internal combustion engine. Many applications have been replaced by computer controlled machines using hydraulic, pneumatic and electric actuators (Mechatronics).

The motion of the follower depends on the shape of the cam. The most basic motion might be a simple quick stroke (e.g. to knock something off a conveyer belt. A cam as shown in the diagram would be suitable.

A key part of your work is to understand how to produce the cam shape that produces the required motion. You should already know the relationship between displacement, velocity and acceleration. For a displacement x, the velocity is v = dx/dt = gradient of the displacement - time graph. This is clearly zero at any time where the profile is circular.

We must consider the inertia force produced by the sudden movement which produces acceleration and deceleration of the follower. The acceleration is a = dv/dt = the gradient of the velocity – time graph. It is normal to round the corners to avoid excessive wear.

Although most cam design would now be done with a suitable computer programme (CAD/CAM) it seems that this outcome requires you to understand the underlying principles.

2. TYPES OF CAMS and FOLLOWERS

DISC CAMS

This tutorial mainly deals with disc whose profile is on the edge of a disc. The motion of the follower also depends on the type of follower and a flat foot will produce a different result to a roller.

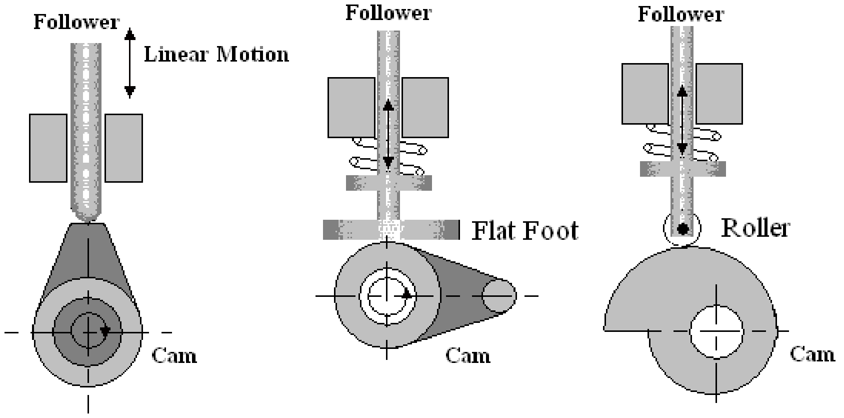


Figure 1

FLAT PLATE CAM – converts the sliding motion into an up/down motion. These can be turned into cylindrical cams as shown.

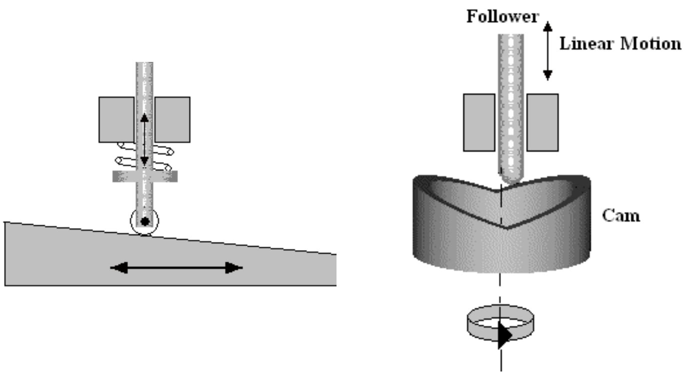


Figure 2

3. DISPLACEMENT, VELOCITY AND ACCELERATION OF FOLLOWER

Consider a simple cam with a pointed follower. The centre lines of the follower and cam are the same. The follower is restrained to move along the centre line by the guides. The displacement of the follower is x and a typical displacement – angle graph is shown.

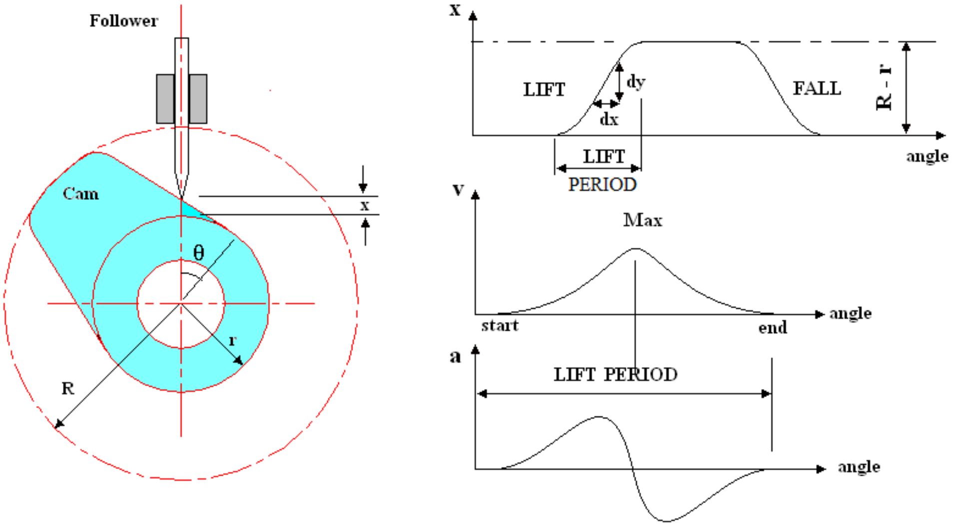


Figure 3

The velocity during the lift period clearly starts at zero and ends at zero. The acceleration, being the gradient, clearly has a positive and negative section during the lift period so it follows that the follower is first accelerated and then decelerated. This is illustrated on the diagram.

3. CONSTRUCTING CAMS FOR SPECIFIED MOVEMENT

3. 1 CONSTANT VELOCITY

If the velocity of the follower is constant, the time displacement graph must be a straight line since the gradient is the velocity. The following example illustrates this.

WORKED EXAMPLE No.1

A cam must produce a lift of 30 mm. This must occupy 60^o of rotation. The follower then dwells at this level for a further 30^o and the fall over the next 60^o. There is then 210^o of rotation to complete the cycle. Design a cam profile so that the velocity is constant when rising and falling. If the velocity is 15 mm/s what must be the speed of the rotation?

SOLUTION

The velocity – angle graph is easy to construct since this must be a straight line of constant gradient. The cam disc is marked out with radial lines at 10^o intervals. The lift at each corresponding angle is marked out and the profile drawn.
Speed = N rev/s. Time to revolve once = 1/N
Time of rise = t = (60/360) x 1/N = 1/6N
Velocity = 15 mm/s = 30/t so t = 2 seconds
The time taken to revolve once = 2 x 360/60 = 12 seconds
N = 1/12 rev/s = 5 rev/min

The diagram illustrates this.

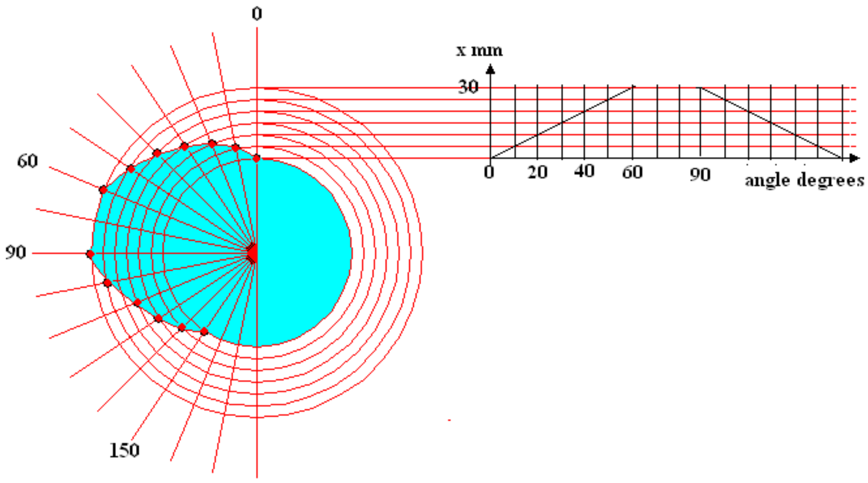


Figure 4

Speed = N rev/s. Time to revolve once = 1/N
Time of rise = t = (60/360) x 1/N = 1/6N
Velocity = 15 mm/s = 30/t so t = 2 seconds
The time taken to revolve once = 2 x 360/60 = 12 seconds
N = 1/12 rev/s = 5 rev/min

3.2 CONSTANT ACCELERATION

Remember that if a = constant then $x = \int v\,dt = at^2/2$ Knowing this we can plot the displacement – angle graph. The following example illustrates this.

WORKED EXAMPLE No.3

A cam must produce a constant acceleration of 1 m/s^2 over a rotation of 30^o followed by constant deceleration over the next 30^o. The rise over this 60^o period must be 16 mm. The cam follower then dwells for 60^o and this is followed by the fall. The fall is identical to the rise and returns the follower to the base level at 210^o. The base circle of the cam is 120 mm diameter. Construct the cam profile and deduce the speed of rotation needed.

SOLUTION

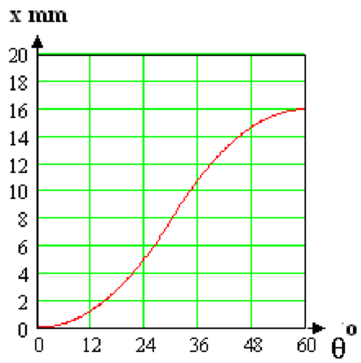
The displacement graph can be calculated for the acceleration period using $x = at^2/2$
The rise is 16 mm but at the change from acceleration to deceleration it is half way so 30^o corresponds to a lift of 8 mm. a = 1 000 mm/s^2
The time taken to move 8 mm is then t = √(2 x 8/1000) = 0.1265 seconds
This corresponds to 30^o of rotation so the time of one revolution is
1/N = 0.1265 x 360/30 = 1.518 s
The speed of the cam is hence 1/1.518 = 0.6588 rev/s or 39.53 rev/min
If we work out the results at 5^o intervals then the corresponding time periods are
(5/360) x 1.518 = 0.0211 s

θ deg	0	5	10	15	20	25	30
t	0	0.021	0.042	0.063	0.084	0.105	0.1265
a	1	1	1	1	1	1	1
$x = at^2/2$	0	0.2	0.9	2	3.6	5.6	8
radial length	60	60.2	60.9	62	63.6	65.6	68

The values during the deceleration period will show the incremental increases are the same working backwards. For example the next value is $x = 8 + (8 – 5.6) = 10.4$
The next value is $x = 10.4 + (5.6 – 3.6) = 12.4$

θ deg	35	40	45	50	55	60
t	0.148	0.169	0.19	0.21	0.23	0.253
x	10.4	12.4	14	15.1	15.8	16
radial length	73.4	72.4	74	75.1	75.8	76

The x – t graph is shown with the cam profile.



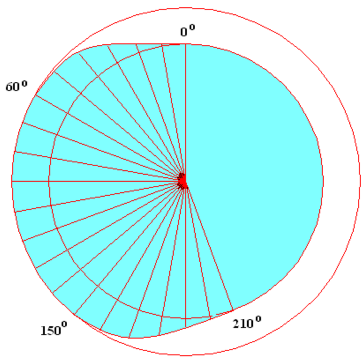


Figure 5

3.3 HARMONIC MOTION

Simple Harmonic Motion is when the shape of the displacement-time graph is sinusoidal.
Given that e is the amplitude, θ is the angle $\theta = \omega t$ and ω is the angular velocity.

Displacement $x = e \sin\theta = A \sin \omega t$

Velocity $v = dx/dt = \omega e \cos\theta = \omega e \cos \omega t$

Acceleration $a = dv/dt \ - \omega^2 e \sin \omega t = - \omega^2 x$

This is best achieved by using a cam in the form of an eccentric disc with a flat footed follower as explained in the following.

A Flat Foot will make it easier to maintain lubrication and reduce wear but cannot follow the profile accurately if it is concave at any point.

e is the eccentricity. The diagram illustrates that the follower must move in the constrained direction by $x = e \sin\theta$ so it is harmonic.

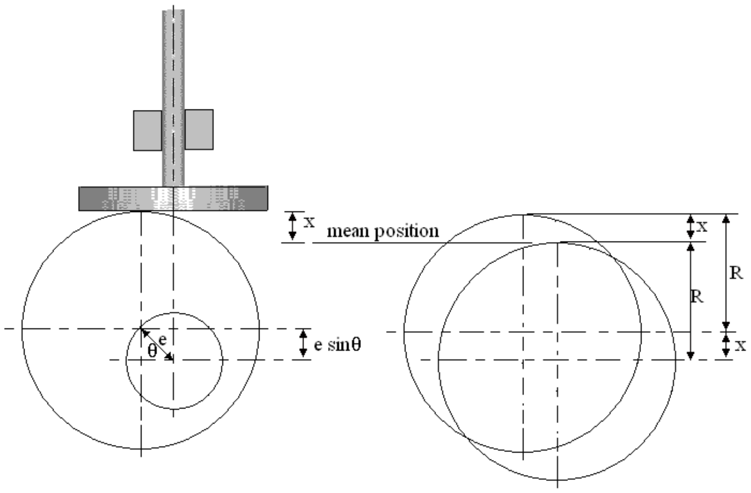


Figure 6

4. OTHER DESIGNS

4.1 ROLLER FOLLOWER

When the follower uses a roller, **it is the centre of the roller that must follow the profile** designed as for a pointed follower. The actual profile must then be reduced as shown to be tangential to the roller at every point. There is no simple way to calculate this and a graphical method should be used.

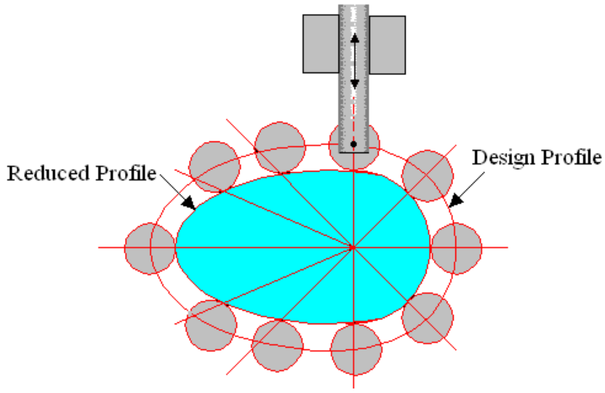


Figure 7

WORKED EXAMPLE No.4

Reproduce the cam in example 3 for use with a roller 10 mm diameter.

SOLUTION

The cam is plotted as before but at the end of each radial line a circle 10 mm diameter is drawn. The profile is then constructed to just touch the circles.

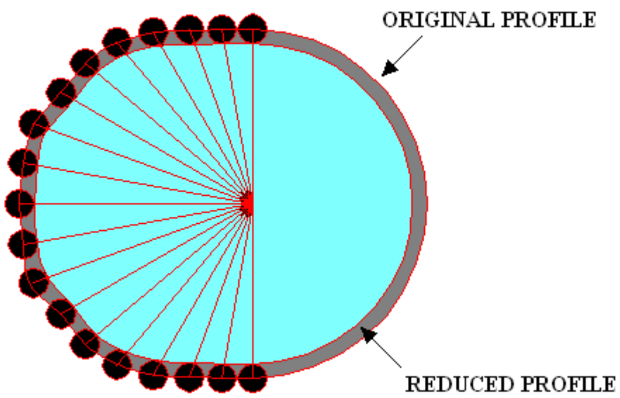


Figure 8

4.2 STRAIGHT SIDED CAM AND ROLLER

A common design for use with a roller is a cam with circular ends joined by straight tangents as shown. The lift occurs between points A and C.
d is the centre distance.
α is the angle rotated during the lift A to C.
θ is the angle rotated after commencement of lift at A.
With the dimensions shown on the diagram the following is used (without derivation)
Total Lift $L = d - r_1 + r_2 = d(1-\cos\alpha)$
When the roller is on the straight edge the following apply.

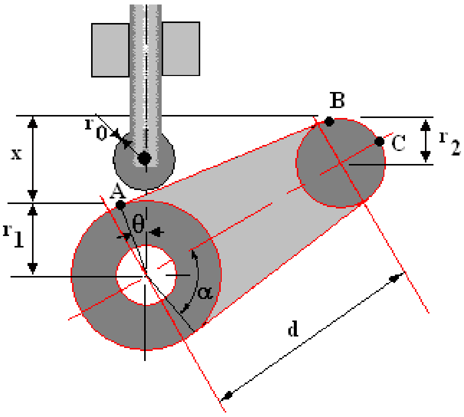


Figure 9

$x = (r_1 + r_o)(\sec\theta - 1)$ (n.b. sec = 1/cos)
$v = \omega(r_1 + r_o)(\sec\theta - 1)\sin\theta$
$a = \omega^2(r_1 + r_o)(1 + 2\tan^2\theta)\sec\theta$

The angle of contact is given by $\alpha = \cos^{-1}\left(\frac{r_1 - r_2}{d}\right)$

(This is the angle rotated as the contact moves from

The angle at the end of the straight edge is $\theta = \tan^{-1}\left(\frac{d\sin\alpha}{r_1 + r_o}\right)$

When the roller is on the nose the motion is more complex and this is not covered here.

WORKED EXAMPLE No. 5

A straight sided cam with a roller follower in line with the centre of rotation has the following dimensions:
$r_1 = 15$ mm $r_o = 10$ mm Total Lift = 9 mm Speed of revolution = 300 rev/min
The angle of contact α is to be 60°. Calculate the:
i) centre distance
ii) acceleration at the instant the lift starts.
iii) lift that occurs on the nose section.
iv) acceleration at the point of transition from the straight edge to the nose.

SOLUTION

$\omega = 2\pi N = 2\pi \times 300/60 = 10\pi$ rad/s
Total Lift = 9 mm = $d(1 - \cos\alpha) = d(1 - \cos 60) = 0.5d$ hence d = 18 mm
At the start of the lift θ = 0
$a = \omega^2(r_1 + r_o)(1 + 2\tan^2\theta)\sec\theta$
$a = \omega^2(r_1 + r_o)(1 + 2\tan^2 0)\sec 0 = (10\pi)^2(25)(1 + 0)\,1 = 24674$ mm/s^2 or 2.467 m/s^2
At the transition from the edge to the nose

$$\theta = \tan^{-1}\left(\frac{d\sin\alpha}{r_1 + r_o}\right) = \tan^{-1}\left(\frac{18\sin 60}{15 + 10}\right) = 32^o$$

$x = (r_1 + r_o)(\sec\theta - 1) = 25(\sec 32^o - 1) = 4.5$ mm
It follows that the lift on the nose section is 9 – 4.5 = 4.5 mm
The acceleration at this point is
$a = \omega^2(r_1 + r_o)(1 + 2\tan^2\theta)\sec\theta = (10\pi)^2(25)(1 + 2\tan^2 32^o)\sec 32^o = 51616$ mm/s^2

4.3. FLAT FOLLOWER

There is no simple way to deduce the dimensions of a cam used with a flat follower. At the point of contact the profile must have a tangent normal to the centreline of the follower. With specified constraints on the cam shape, it is possible to produce designs.

A common design is a circular heel radius r_1 and nose radius r_2 with centre distance d. They are joined by circular arcs radius R forming tangents to the circles as shown.
α is the angle rotated during the lift A to C.
θ is the angle rotated after commencement of lift at A.
β is the angle of the circular arc radius R

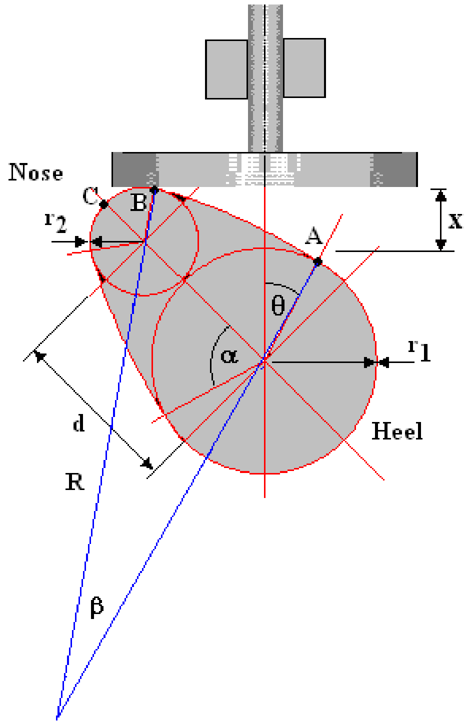


Figure 10

Total Lift = $d - r_1 + r_2$
When the follower is on the sides :
$x = (R - r_1)(1 - \cos\theta)$
$v = \omega(R - r_1)\sin\theta$
$a = \omega^2(R - r_1)\cos\theta$

When the cam is on the nose :
Displacement $x = d\cos(\alpha - \theta) + r_2 - r_1$

Velocity $v = d\,\omega\sin(\alpha - \theta)$

Acceleration $a = -d\,\omega^2\cos(\alpha - \theta)$

α is the angle of action given by

$$\alpha = \cos^{-1}\left(\frac{(R - r_2)^2 - d^2 - (R - r_1)^2}{2d(R - r_1)}\right) \text{ or rearranged } R = \frac{r_1^2 - r_2^2 + d^2 - 2r_1 d\cos\alpha}{2(r_1 - r_2 - d\cos\alpha)}$$

$$\beta = \sin^{-1}\left(\frac{d\sin\alpha}{(R - r_2)}\right)$$

4.4 INERTIA and SPRINGS

The acceleration of the cam follower and anything connected to it is important because the inertia force generated can be very large.

A spring is important to keep the follower in contact with the cam and must be capable of applying a force equal to the inertia force. The spring force will, however add to the force on the cam during the reverse direction.

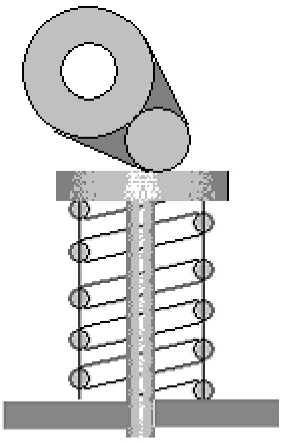


Figure 11

WORKED EXAMPLE No. 6

A cam with a flat follower has a round nose and heel with circular arcs for the sides. The dimensions are as follows.

r_1 = 10 mm r_2 = 2.5 mm Centre distance d = 12.5 mm

Speed of revolution = 600 rev/min. The angle of contact α is to be 75°. Calculate the:

i) radius of the sides.
ii) lift on the sides.
iii) lift on the nose.
iv) total lift.
v) acceleration at commencement of lift.
vi) acceleration at the point of transition to the nose.
vii) acceleration at the tip of the nose.

SOLUTION

ω = 2πN = 2π x 600/60 = 20π rad/s

$$R = \frac{r_1^2 - r_2^2 + d^2 - 2r_1 d\cos\alpha}{2(r_1 - r_2 - d\cos\alpha)} = \frac{10^2 - 2.5^2 + d^2 - 2 \text{ x } 12.5\cos 75^o}{2(10 - 2.5 - 12.5\cos 75^o)} = 21.724 \text{ mm}$$

$$\beta = \sin^{-1}\left(\frac{d\sin\alpha}{(R - r_2)}\right) = \sin^{-1}\left(\frac{12.5\sin 75^o}{(21.724 - 2.5)}\right) = 39.18^o$$

Lift on sides (A to B)

$x = (R - r_1)(1 - \cos\theta)$ and $\theta = \beta = 39.18^o$

$x = (21.724 - 10)(1 - \cos 39.18^o) = 2.636$ mm

Total Lift = $d - r_1 + r_2$ = 12.5 - 10 + 2.5 = 5

Lift on nose = 5 – 2.636 = 2.364 mm

Acceleration at commencement of lift (point A) θ = 0

$a = \omega^2(R - r_1)\cos\theta = (20\pi)^2(21.724 - 10)\cos\theta = 46284 \text{ mm/s}^2$

Acceleration at transition to nose (point B) θ = β

$a = \omega^2(R - r_1)\cos\theta$

$a = (20\pi)^2(21.724 - 10)\cos 39.18^o = 35880 \text{ mm/s}^2$

Acceleration just after the transition on the nose

$a = -d\omega^2\cos(\alpha - \theta) = -12.5(20\pi)^2\cos(75^o - 39.18^o) = -40013 \text{ mm/s}^2$

(Note the sudden change in acceleration)

At the tip of the nose (θ = α)

$a = -d\,\omega^2\cos(\alpha - \theta) = -12.5\,(20\pi)^2\cos(0^o) = -49348 \text{ mm/s}^2$

SELF ASSESSMENT EXERCISE No.1

1. A straight sided cam with a roller follower in line with the centre of rotation has the following dimensions:
   r_1 = 20 mm r_o = 8 mm Total Lift = 5 mm Speed of revolution = 3000 rev/min
   The angle of contact α is to be 70^o.
   Calculate the :
   i) centre distance (7.6)
   ii) acceleration at the instant the lift starts. (2763 m/s^2)
   iii) lift that occurs on the nose section. (4.1 mm)
   iv) acceleration at the point of transition from the straight edge to the nose. (3223 m/s^2)

2. A cam with a flat follower has a round nose and heel with circular arcs for the sides. The dimensions are as follows.
   r_1 = 15 mm r_2 = 5 mm Centre distance d = 20 mm
   Speed of revolution = 1000 rev/min. The angle of contact α is to be 80^o. Calculate the:
   i) radius of the sides. (37.98 mm)
   ii) lift on the sides. (4.57 mm)
   iii) total lift. (10 mm)
   iv) acceleration at commencement of lift. (252 m/s^2)
   v) acceleration at the point of transition to the nose. (201.9 m/s^2 and -159.8 m/s^2)
   vi) acceleration at the tip of the nose. (-219.3 m/s^2)

3. The diagram shows the time displacement profile of a cam.
   The profile follows the law $x = 0.5t^3$
   Produce the velocity – time graph and the acceleration – time graph.

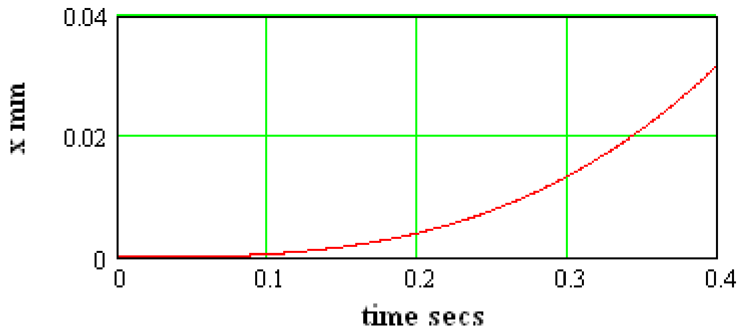


Figure 12

4. A cam must produce simple harmonic motion of the follower with amplitude 4 mm at a speed of 3000 rev/min. The follower has a mass of 30 g. Calculate the maximum acceleration and the minimum spring force needed to keep the follower in contact with the cam.
   (395 m/s^2 and 11.85 N)

5. The valves on an internal combustion engine must open for half of the revolution. The valves should be fully open for as much of this period as possible. Discuss the best form of cam and the limitations imposed by inertia on the shape.