Section 5.5 Text Answers and Solutions for Selected Exercises

Answers to Selected Exercises of Section 5.5

1. $\sin(A) = \frac{5\sqrt{41}}{41}, \cos(A) = \frac{4\sqrt{41}}{41}, \tan(A) = \frac{5}{4}$

$\sec(A) = \frac{\sqrt{41}}{5}, \csc(A) = \frac{\sqrt{41}}{4}, \cot(A) = \frac{4}{5}$

3. $c = 14,\ b = 7\sqrt{3},\ B = 60°$

5. $a = 5.3171,\ c = 11.3257,\ A = 28°$

7. $a = 9.0631,\ b = 4.2262,\ B = 25°$

9. 32.4987 ft

11. 836.2698 ft

13. 460.4069 ft

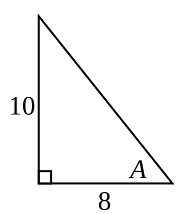
15. 660.35 feet

17. 28.025 ft

19. 143.0427

21. 86.6685

Worked Solutions to Selected Exercises of Section 5.5

1. 

hypotenuse^2 = $10^2 + 8^2 = 164$ => hypotenuse = $\sqrt{164} = 2\sqrt{41}$

Therefore, sin (A) = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{10}{2\sqrt{41}} = \frac{5}{\sqrt{41}}$

cos (A) = $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{2\sqrt{41}} = \frac{4}{\sqrt{41}}$

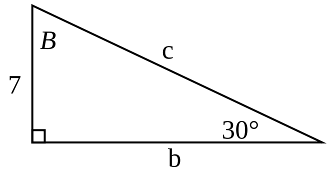
tan (A) = $\frac{\sin(A)}{\cos(A)} = \frac{\frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}}} = \frac{5}{4}$ or tan (A) = $\frac{\text{opposite}}{\text{adjacent}} = \frac{10}{8} = \frac{5}{4}$

sec (A) = $\frac{1}{\cos(A)} = \frac{1}{\frac{4}{\sqrt{41}}} = \frac{\sqrt{41}}{4}$

csc (A) = $\frac{1}{\sin(A)} = \frac{1}{\frac{5}{\sqrt{41}}} = \frac{\sqrt{41}}{5}$

and cot (A) = $\frac{1}{\tan(A)} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$.

3.



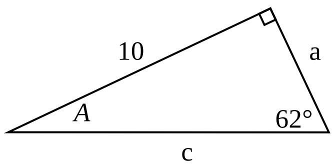
$\sin(30°) = \frac{7}{c}$ => $c = \frac{7}{\sin(30°)} = \frac{7}{\frac{1}{2}} = 14$

$\tan(30°) = \frac{7}{b}$ => $b = \frac{7}{\tan(30°)} = \frac{7}{\frac{1}{\sqrt{3}}} = 7\sqrt{3}$

or $7^2 + b^2 = c^2 = 14^2$ => $b^2 = 14^2 - 7^2 = 147$ => $b = \sqrt{147} = 7\sqrt{3}$

$\sin(B) = \frac{b}{c} = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$ => B = 60° or B = 90° - 30° = 60°.

5.

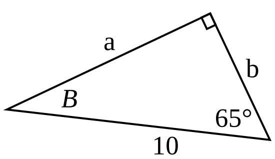


$\sin(62°) = \frac{10}{c}$ => $c = \frac{10}{\sin(62°)} \approx 11.3257$

$\tan(62°) = \frac{10}{a}$ => $a = \frac{10}{\tan(62°)} \approx 5.3171$

A = 90° - 62° = 28°

7.

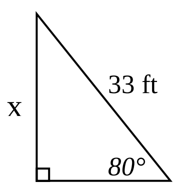


B = 90° - 65° = 25°

$\sin(B) = \sin(25°) = \frac{b}{10}$ => $b = 10\sin(25°) \approx 4.2262$

$\cos(B) = \cos(25°) = \frac{a}{10}$ => $a = 10\cos(25°) \approx 9.0631$

9.

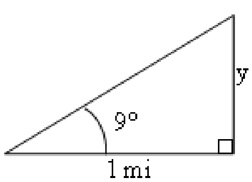


Let x (feet) be the height that the ladder reaches up.

Since $\sin(80°) = \frac{x}{33}$

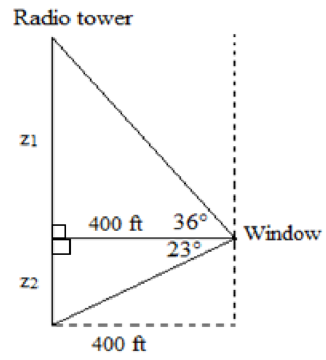
So the ladder reaches up to $x = 33\sin(80°) \approx 32.4987$ ft of the building.

11.



Let y (miles) be the height of the building. Since $\tan(9°) = \frac{y}{1} = y$, the height of the building is y = tan (9°) mi ≈ 836.26984 ft.

13.



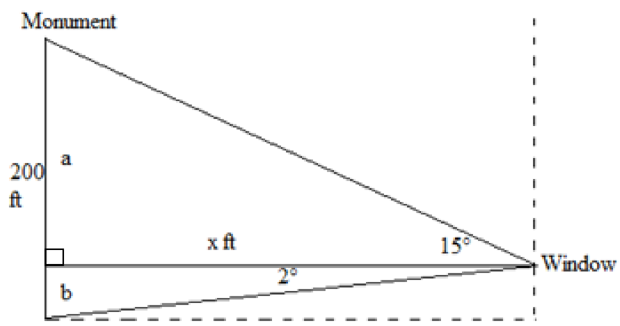
Let z_1 (feet) and z_2 (feet) be the heights of the upper and lower parts of the radio tower. We have

$$\tan(36°) = \frac{z_1}{400} \Rightarrow z_1 = 400\tan(36°) \text{ ft}$$

$$\tan(23°) = \frac{z_2}{400} \Rightarrow z_2 = 400\tan(23°) \text{ ft}$$

So the height of the tower is $z_1 + z_2 = 400\tan(36°) + 400\tan(23°) \approx 460.4069$ ft.

15.



Let x (feet) be the distance from the person to the monument, *a* (feet) and *b* (feet) be the heights of the upper and lower parts of the building. We have

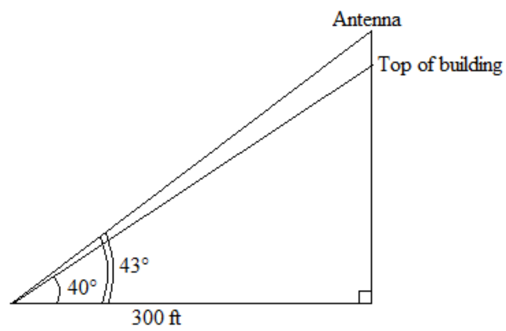
$$\tan(15°) = \frac{a}{x} \Rightarrow a = x\tan(15°)$$

and $$\tan(2°) = \frac{b}{x} \Rightarrow b = x\tan(2°)$$

Since $200 = a + b = x\tan(15°) + x\tan(2°) = x[\tan(15°) + \tan(2°)]$

Thus the distance from the person to the monument is $x = \frac{200}{\tan(15°) + \tan(2°)} \approx 660.3494$ ft.

17.

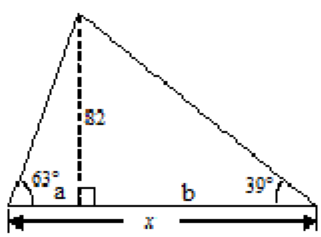


Since $\tan(40°) = \frac{\text{height from the base to the top of the building}}{300}$, the height from the base to the top of the building is 300 tan (40°) ft.

Since $\tan(43°) = \frac{\text{height from the base to the top of the antenna}}{300}$, the height from the base to the top of the antenna is 300 tan (43°) ft.

Therefore, the height of the antenna = 300 tan (43°) - 300 tan (40°) ≈ 28.0246 ft.

19.

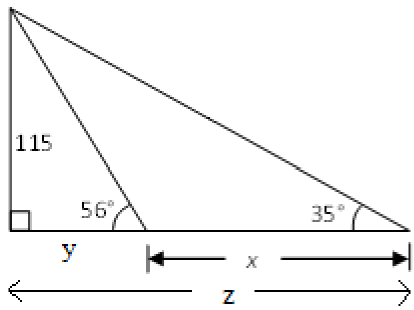


We have $\tan(63°) = \frac{82}{a} \Rightarrow a = \frac{82}{\tan(63°)}$

$\tan(39°) = \frac{82}{b} \Rightarrow b = \frac{82}{\tan(39°)}$

Therefore $x = a + b = \frac{82}{\tan(63°)} + \frac{82}{\tan(39°)} \approx 143.04265$.

21.

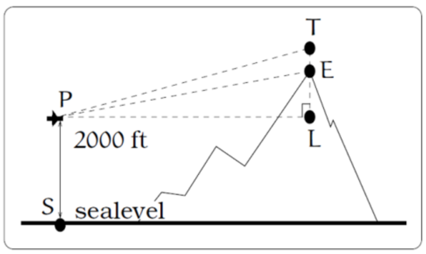


We have $\tan(35°) = \frac{115}{z} \Rightarrow z = \frac{115}{\tan(35°)}$

$\tan(56°) = \frac{115}{y} \Rightarrow y = \frac{115}{\tan(56°)}$

Therefore $x = z - y = \frac{115}{\tan(35°)} - \frac{115}{\tan(56°)} \approx 86.6685$.

23.

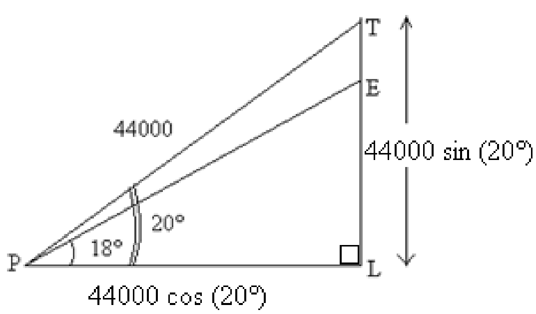


The length of the path that the plane flies from P to T is

$$PT = \left(\frac{100\ mi}{1\ h}\right)\left(\frac{1\ h}{60\ min}\right)(5\ min) = \frac{25}{3}\ mi = 44000\ ft$$

In ΔPTL, $\sin(20°) = \frac{TL}{PT} \Rightarrow TL = PT \sin(20°) = 44000 \sin(20°)$ ft

$\cos(20°) = \frac{PL}{PT} \Rightarrow PL = PT\cos(20°) = 44000\cos(20°)$ ft



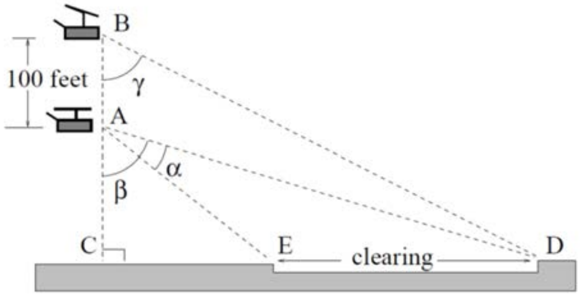
In ΔPEL, $\tan(18°) = \frac{EL}{PL} \Rightarrow EL = PL \tan(18°) = 44000\cos(20°)\tan(18°) \approx 13434.2842$ ft

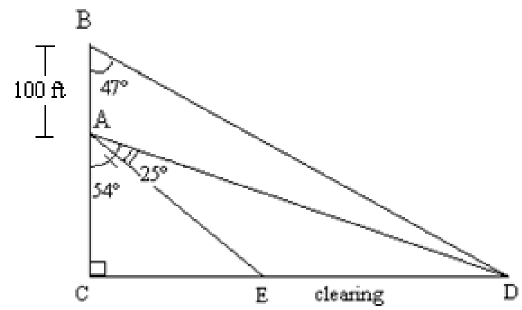
Therefore $TE = TL - EL = 44000\sin(20°) - 44000\cos(20°)\tan(18°)$

$= 44000[\sin(20°) - \cos(20°)\tan(18°)] \approx 1614.6021$ ft

So the plane is about 1614.6021 ft above the mountain's top when it passes over. The height of the mountain is the length of *EL*, about 13434.2842 ft, plus the distance from sea level to point *L*, 2000 ft (the original height of the plane), so the height is about 15434.2842 ft.

25.





We have: $\tan(47°) = \frac{CD}{BC} = \frac{CD}{AC+100} \Rightarrow AC + 100 = \frac{CD}{\tan(47°)} \Rightarrow AC = \frac{CD}{\tan(47°)} - 100$

$\tan(54°) = \frac{CD}{AC} \Rightarrow AC = \frac{CD}{\tan(54°)}$

Therefore, $\frac{CD}{\tan(47°)} - 100 = \frac{CD}{\tan(54°)}$

$$\frac{CD}{\tan(47°)} - \frac{CD}{\tan(54°)} = 100$$

$$CD\left(\frac{1}{\tan(47°)} - \frac{1}{\tan(54°)}\right) = 100 \quad \text{or} \quad CD\left(\frac{\tan(54°)-\tan(47°)}{\tan(47°)\tan(54°)}\right) = 100$$

So $CD = \frac{100\tan(54°)\tan(47°)}{\tan(54°) - \tan(47°)}$

Moreover, $\tan(54° - 25°) = \tan(29°) = \frac{CE}{AC} = \frac{CE}{\frac{CD}{\tan(54°)}} = \frac{CE\tan(54°)}{CD}$

$$= \frac{CE\tan(54°)}{\frac{100\tan(54°)\tan(47°)}{\tan(54°)-\tan(47°)}} = \frac{CE\,[\tan(54°) - \tan(47°)]}{100\tan(47°)}$$

⇨ $CE = \frac{100\tan(47°)\tan(29°)}{\tan(54°)-\tan(47°)}$

Thus the width of the clearing should be $ED = CD - CE = \frac{100\tan(54°)\tan(47°)}{\tan(54°)-\tan(47°)} - \frac{100\tan(47°)\tan(29°)}{\tan(54°)-\tan(47°)}$

$$= \frac{100\tan(47°)}{\tan(54°)-\tan(47°)}[\tan(54°) - \tan(29°)] \approx 290 \text{ ft.}$$